

INFINITESIMAL METHOD AND JUDGMENT OF ORIGIN

Método infinitesimal e o juízo da origem

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Abstract: The goal of this paper is to investigate the relation between Cohen’s approach to differential calculus and his doctrine of pure thinking. We claim that Cohen’s logic of origin is firmly based on his interpretation of infinitesimal analysis. More precisely, the transcendental method, when applied to differential calculus, reveals the productive capacity of thinking expressed by the judgment of origin.

Keywords: Infinitesimal Method; Cohen; Judgement of origin.

Resumo: o objetivo deste artigo é investigar a relação entre as considerações de Cohen ao cálculo diferencial e sua doutrina do pensar puro. Defendemos que a lógica da origem coheniana é firmemente baseada na sua interpretação da análise infinitesimal. Mais precisamente, o método transcendental, quando aplicado ao cálculo diferencial, revela a capacidade produtiva do pensar expressada pelo juízo da origem.

Palavras-chave: Método infinitesimal; Cohen; Juízo da origem.

Introduction

The goal of this paper is to investigate the relation between Cohen’s approach to differential calculus and his doctrine of pure thinking. We claim that Cohen’s logic of origin is firmly based on his interpretation of infinitesimal analysis. More precisely, the transcendental method, when applied to differential calculus, reveals the productive capacity of thinking expressed by the judgment of origin.

For our purpose, we will begin by briefly considering Cohen’s early interpretation of differential calculus, contained in *The Principle of the infinitesimal method and its history* and *Kant’s theory of experience*. The analysis of these works will provide us with the presuppositions necessary to understand Cohen’s logic of origin (§1). Later, we will analyze Cohen’s mature doctrine of pure thinking in order to show how this doctrine is built upon Cohen’s transcendental approach to infinitesimal analysis (§2).

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§1 Reality and the infinitesimal

According to the philosophical method adopted by Cohen, philosophy should take a certain fact as a point of departure for investigation and proceed by searching for the conditions of possibility of that fact. This is the so-called *transcendental method*. In the case of theoretical philosophy, the fact to be investigated is experience¹.

Experience is understood not merely as objective cognition, but as *scientific knowledge*. Cohen identifies experience with mathematical natural science, which thus turns out to be the departure point for the transcendental method. Cohen claims that experience is given in the mathematical science of nature, as it has been established by Newton². Accordingly, the task of transcendental philosophy will be to determine the conditions of possibility of Newtonian science³.

The key instrument of the mathematical science of nature is differential calculus, since natural laws are expressed by differential equations. Therefore, in Cohen's terms, the foundation of the concept of the infinitesimal becomes a task for the critique of knowledge.

Cohen's transcendental approach to differential calculus can be traced in his discussion of the Kantian principle of the anticipations of perception, where Cohen investigates the relations between the concepts of infinitesimal, extensive magnitude, intensive magnitude, and reality (Realität). We will now turn to this issue.

Kant distinguishes two different kinds of combination (*conjunctio*). These are: a) composition (*compositio*) and b) connection (*nexus*). Composition is the synthesis of a manifold of what *does not necessarily belong to each other*. This is the synthesis of the *homogeneous* in everything that can be considered mathematically. Composition can in

¹ "Die Erfahrung ist gegeben: es sind die Bedingungen zu entdecken, auf denen ihre Möglichkeit beruht. Sind die Bedingungen gefunden, welche die gegebene Erfahrung *ermöglichen*, in der Art ermöglichen, dass dieselbe als a priori gültig angesprochen, dass strenge Notwendigkeit und unbeschränkte Allgemeinheit ihr zuerkannt werden kann, dann sind diese Bedingungen als die konstituierenden Merkmale des Begriffs der Erfahrung zu bezeichnen, und aus diesem Begriffe ist sodann zu deduzieren, was immer der Erkenntniswert objektiver Realität beansprucht. Das allieu ist das ganze Geschäft der Transzendental-Philosophie" (Cohen, 1877, p. 24). Not only will theoretical philosophy be carried out according to the transcendental method, but practical philosophy as well.

² "Die Erfahrung ist also in Newtons Wissenschaft, in der mathematischen Naturwissenschaft gegeben; nicht etwa in Dingen der Erfahrung" (Cohen, 1877, p. 24).

³ Cohen summarizes his interpretation of Kantian philosophy in the following way: "Kants Aufgabe ist also zunächst die Prüfung und Kennzeichnung des Erkenntniswerthes und des Gewissheitgrundes der Newton'schen Naturwissenschaft, welche er bei dem Drohwort der Erfahrung fasste" (Cohen, 1885, p. 66). To this Newtonian science belong not only mathematical physics but also other sciences, such as biology, which Cohen characterizes as descriptive sciences of nature.

turn be further divided into the synthesis of *aggregation*, directed to *extensive* magnitudes, and that of *coalition*, directed to *intensive* magnitudes (A162/B201 footnote). The concept of a magnitude (*Quanti*), Kant explains, is the consciousness of the homogeneous manifold in intuition in general, insofar as through it the representation of an object first becomes possible (A162/B203). Extensive magnitude is the magnitude in which the representation of the part precedes the representation of the whole and makes it possible. The apprehension of extensive magnitudes is always successive, i.e. it takes place according to a successive synthesis of a manifold of antecedently given parts. To the contrary, intensive magnitude is that which can only be apprehended as a unity⁴. In this case, the apprehension is not successive but instantaneous. There is no manifold of antecedently given parts, but just a multiplicity that can only be represented through approximation to negation “= 0” (A168/B210). Every reality in the appearance has intensive magnitude, i.e. a degree.

Extensive magnitudes correspond to the form of intuition, i.e., the pure space-time manifold of appearances, while intensive magnitudes correspond to the matter of intuition, i.e., sensation. The spatiotemporal pure form of appearances is successively apprehended, while their empirical matter is instantaneously apprehended. This difference regarding the apprehension of the form and the matter of appearances has deep consequences for the Cohenian interpretation of extensive and intensive magnitudes.

Extensive magnitudes are comparative magnitudes (*Vergleichungsgrößen*), because their quantity is obtained by comparison with a certain measurement unit. Since this unit is arbitrary, conventional and depends on the peculiarity of our senses⁵, Cohen claims that the object of the extensive magnitude, as a mere result of a synthesis of aggregation, lacks a proper foundation⁶. Thus, Cohen argues that the comparison of an object with the unit of measurement always presupposes something present in itself and by itself, which constitutes the required foundation and which does not depend on the measurement unit adopted. The ground of the extensive magnitude is not the relative and

⁴ On this definition, Cohen claims: “Fehlerhaft bleibt die unabgelöste Beziehung auf die subjectiven Mittel des Bewusstseins, hier also auf die Apprehension” (Cohen, 1885, p. 427).

⁵ “[Die] Vergleichungsgrösse der Extension [scheint] in ihrer willkürlichen Einheit wie in ihrem Koordinaten-Ursprung auf Convention und sinnlicher Relativität gegründet” (Cohen, 1883, p. 70).

⁶ “Der Gegenstand der extensiven Grösse ist doch nur ein Vergleichungsgebild ohne Fundament” (Cohen, 1885, p. 428).

arbitrary unit of comparison, but an absolute unity, which will not be found in pure intuition, but in the *real* in space and time.

If a totality is successively apprehended in order to be represented as an extensive magnitude, then there must be something real to be grasped at every moment. This instantaneous unity is the unity of intensive magnitudes, which is therefore an absolute unity, “for as contained in one moment no representation can ever be anything other than an absolute unity (A99)”⁷.

Cohen maintains that this intensive, absolute, instantaneous unity of the real in space and time is the *infinitesimal*⁸. Differential calculus is the method by means of which intensive magnitudes ground extensive ones⁹. For example, the trajectory of a body $x(t)$ moving with a certain instantaneous velocity $v(t)$ will be obtained by integrating the velocity $v(t)$ from the initial $t = t_0$ to the final $t = t_f$ ¹⁰. Thus, velocity, as intensive magnitude, grounds the trajectory, as extensive magnitude¹¹.

⁷ “Denn wenn die Einheit einer Vielheit gedacht werden soll, so muss vor Allem die Einheit selbst gedacht sein. Das ist die Einheit, die nur als Einheit “apprehendirt” wird” (Cohen, 1885, p. 428). In this connection, Böhme claims: “Wir glauben nach dieser Analyse das Recht zu der Behauptung zu haben, daß das Grundmaß als Größe intensiv erfahren wird [...]. Die extensive Größenschätzung setzt die intensive als Beurteilung des Grundmaßes voraus” (Böhme, 1974, p. 248).

⁸ “Das ist die Einheit der Infinitesimalzahl, die als der Ursprung betrachtet wird, von welchem alle extensive Grösse anhebt, in welchem sie ihr “Fundament” hat” (Cohen, 1885, p. 427).

⁹ In this sense, Kant states: “Qualitative Einheit ist wie der Grund des Ganzen, quantitative wie ein Theil des Ganzen zu betrachten” (*Refl*, 18: 322). Quoted in (Cohen, 1885, p. 431). Cohen argues: “Denn alles Rechnen mit extensiven Zahlgrößen ist und bleibt ein Vergleichen mit dem relativen und willkürlichen Massstabe der *Einheit*, welche selbst freilich in der Einheit des Bewusstseins ihre Gewähr hat, darum aber auch in anderen Arten dieser Einheit des Bewusstseins fordert und findet. Gäbe es nur die gemeinen Zahlen, so gäbe es nur Relativitäten; Dinge wären nirgends begründet” (Cohen, 1883, p. 133).

¹⁰ In this regard, Kant claims: “Man muß daher das Moment der Geschwindigkeit nicht schon selbst als Geschwindigkeit betrachten, sondern bloß als das Bestreben, einem Körper eine gewisse Geschwindigkeit mitzuthemen; nicht als extensive, sondern als intensive Größe, die aber den Grund der extensiven Größe enthält.” (*Refl*, 14: 496). “Erklärt man aber eine doppelte Geschwindigkeit dadurch, daß man sagt, sie sei eine Bewegung, dadurch in derselben Zeit ein doppelt so großer Raum zurückgelegt wird, so wird hier etwas angenommen, was sich nicht von selbst versteht, nämlich: daß sich zwei gleiche Geschwindigkeiten eben so verbinden lassen, als zwei gleiche Räume, und es ist nicht für sich klar, daß eine gegebene Geschwindigkeit aus kleinern und eine Schnelligkeit aus Langsamkeiten eben so bestehe, wie ein Raum aus kleineren; denn die Theile der Geschwindigkeit sind nicht außerhalb einander, wie die Theile des Raumes, und wenn jene als Größe betrachtet werden soll, so muß der Begriff ihrer Größe, da sie intensiv ist, auf andere Art construirt werden, als der der extensiven Größe des Raumes” (*MAN*, 4: 493-494). Quoted in (Cohen, 1883, p. 110) Velocity, as intensive magnitude, grounds the trajectory, as the extension of motion.

¹¹ In the same sense, Maimon states: “Die extensive Größe ist gleichsam das Schema der intensiven Größe, indem diese und ihre Verhältnisse, nicht an sich unmittelbar, sondern bloß vermittelt jener wahrgenommen werden kann, wie z.B. die verschiedene Grade der Wärme und Kälte, durch das Steigen und Fallen des Thermometers, u. dergl.: sie wird als eine Einheit gegeben und durchs Vergleichen als Vielheit gedacht. Die intensive Größe ist bei Quanta das Differential der extensiven, und diese wiederum das Integral von jener” (GW 2: 121 – 122). For a discussion of the relationship between Maimon and Cohen, see: (Bergman,

Cohen maintains the identity between infinitesimal and intensive magnitudes and claims that this identity was universally accepted in Kant's times¹²: the critical doctrine does not bring any new thesis in this respect. For Cohen, the Kantian novel viewpoint is rather the identification of the infinitesimal with the real [*das Reale*] in space and time. However, Cohen claims that reality is not grounded in sensation, as Kant maintains, but in thinking only. More precisely, the principle of the infinitesimal method is a principle of thinking, which provides the pure spatiotemporal extensive magnitude of appearances with real intensive magnitude by generating the former from the latter, just as the trajectory of a body is generated from its instantaneous velocity.

This generation produces space and time as *quanta continua*, i.e. as magnitudes no part of which is the smallest or simple. Cohen indicates that Kant himself claims this generation of the extensive magnitude from the intensive one using Newtonian terms in the following passage¹³:

Space and time are quanta continua. [...] Magnitudes of this sort can also be called flowing, since the synthesis (of the productive imagination) in their generation is a progress in time, the continuity of which is customarily designated by the expression “flowing” (“elapsing”). (A170/B211-212)¹⁴

According to Newton's terminology, the fluent is the extensive magnitude, generated by the fluxion as an intensive magnitude. Thus, space and time are produced as quanta continua by a condition of thinking: reality as an intensive magnitude.

1939) and (Bergman, 1967, 256 ff.). Schulthess underlines that also Lotze and Trendelenburg claim that extensive magnitudes are generated from intensive ones (Schulthess, 1984, pp. 24-25).

¹² See (Cohen, 1883, p. 14) and (Cohen, 1885, p. 427)

¹³ In Newton's terms, the fluent is the extensive magnitude and the fluxion is the intensive one.

¹⁴ “Der Satz der Continuität will nur sagen: Alle diversa sind remota, d.i. sie sind nicht anders in Verknüpfung als per intermedia, wozwischen der Unterschied noch kleiner ist. D.i. kein Unterschied ist der kleinste, weil kein Übergang elementar ist und der kleinste ist, also immer eine Größe hat. Es gehört zum Übergang eine Zeit, mithin eine Annäherung zu einem neuen Zustande. Der kleinste Unterschied würde ein Differentiale heißen; weil aber kein kleinster ist, so heißt er Fluxion” (*Refl*, 18: 167). In the same sense: “Nun stellen wir uns eine Linie als durch Fluxion, mithin in der Zeit erzeugt vor, in der wir nichts Einfaches vorstellen, und können 1/10, 1/100 etc. etc. von der gegebenen Einheit denken” (*Refl*, 14: 53). On Kant's reception of Leibnizian differentials and Newtonian fluxions, Büchel claims: “Mit dem Newtonschen Konzept der Fluxion als Geschwindigkeit des Fallens oder Steigens ist wegen der Geschwindigkeitsvorstellung eine Zeitvorstellung nahegelegt, wie es für das Erzeugen einer Linie durch Fluxion dargelegt worden ist. Dadurch ist das Newtonsche Konzept der Grundlegung der Differentialrechnung für die mathematische Beschreibung “fließender” Bewegungsabläufe in der Zeit geeigneter als Leibniz' Konzept der “unvergleichbar kleinen Größe”, der Differentialgröße, die von Leibniz selber schon in die Nähe der unendlich kleinen Größen gerückt wurde” (Büchel, 1987, p. 242).

But Cohen criticizes Kant for maintaining an incorrect relation between reality as intensive magnitude and sensation. In order to make his point, Cohen discusses the two versions of the principle of the anticipations of perception that Kant puts forward in the different editions of the *Critique of pure reason*. In the first edition, Kant claims: “In all appearances the sensation, and the *real* which corresponds to it in the object (realitas phaenomenon), has an *intensive magnitude*, i.e., a degree” (A166).

Against this formulation, which assigns intensive magnitude both to sensation and to the corresponding real in the appearance, Cohen argues that intensive magnitude is an objective determination, which cannot be applied to sensation, because the latter is just a subjective way in which consciousness relates to its content. For this reason, Cohen claims, *only* the real which corresponds to sensation has intensive magnitude. Besides, this version of the principle does not prevent the uncritical illusion of conceiving of the object independently of the real, as if the object were (or could be) present [*vorhanden*] without the real. But, Cohen maintains, it is rather the real which determines the object in the first place (Cohen, 1885, pp. 433-434).

Both difficulties are avoided by a new formulation of the principle, that appears in the second edition of the *Critique*. Now the principle reads: “*In all appearances the real, which is an object of the sensation, has intensive magnitude*, i.e., a degree” (B207).

However, a third and deeper error is still contained in this version. The new formulation arouses the false conception that the real has intensive magnitude *only because* and *insofar as* it corresponds to sensation¹⁵. If this were the case, the real would be grounded in sensation (Cohen, 1885, p. 434). But, Cohen claims, reality is not grounded *in* sensation, but *for* sensation, in the principle of the infinitesimal method (Cohen, 1883, p. 106). The infinitesimal method provides sensation with an object, according to the schema of reality¹⁶, by means of a continuous and uniform generation of reality in time (Cohen, 1885, p. 425).

Differential calculus grounds the generation of intensive magnitudes and thereby the generation of extensive ones, as it may be seen by considering the example of the free fall. In this case, velocity (as intensive magnitude) increases from zero to a certain value in accordance with the law of free fall, which is Newton’s second law specified in the

¹⁵ See also *Refl* 6338 (18: 661).

¹⁶ *KrV*, A143/B183.

situation where the only acting force is gravity. This law states that the acceleration (a) is equal to a constant (g). The continuous and uniform generation of velocity is the integration of its differential ($dv = g dt$) from zero to a value V at time T . The so generated intensive magnitude will be $V = gT^{17}$, i.e., time T will be filled up with a reality of grade V . But, moreover, the generation of the extensive magnitude of the appearance will take place according to the same method. The extension of the free fall, i.e., the falling distance $S = gT^2/2$, will be obtained from the velocity by integration, just as the velocity was obtained from the acceleration¹⁸, The extensive magnitude will be obtained from the intensive one¹⁹.

From this viewpoint, Cohen rectifies the Kantian claim that all appearances whatsoever are continuous magnitudes, either in their intuition as extensive magnitudes, or in their perception as intensive ones (A170/B212). Rather, Cohen affirms, appearances are continuous as intensive magnitudes and *only therefore* they are continuous as extensive ones (Cohen, 1885, pp. 436-437).

Whereas the finite, extensive, spatiotemporal magnitudes may still pertain to a domain of *fictions*, of merely subjective formations, the infinitesimal as intensive magnitude grounds their objective character by generating them according to the rules of differential calculus. The intensive magnitude of reality, as infinitesimal, is the unity of generation of the finite (Cohen, 1883, p. 146). This infinitesimal unity is grounded only in thinking, not in intuition or sensation. Space and time are the forms in which the real is ordered, but this real is not passively received but spontaneously produced by thinking.

The transcendental method applied to differential calculus exposes the productive character of the intensive magnitude and, more generally, the productive character of thinking. For Cohen, this entails that the critique of knowledge should turn into a logic of pure knowledge. With this transformation of transcendental idealism, Cohen will follow the lines established by Kant only to go beyond the point reached by the latter.

§2 From the critique of knowledge to the logic of pure knowledge

In his *Logic of pure knowledge* Cohen deepens his criticism of Kantian transcendental philosophy. Now nothing is given to thinking, not even a pure manifold.

¹⁷ $V = \int_0^V dv = \int_0^T g dt = gT$

¹⁸ $S = \int_0^S ds = \int_0^T ds/dt dt = \int_0^T gt dt = gT^2/2$

¹⁹ On Cohen's discussion of free fall, see (Cohen, 1883, p. 49).

The Kantian given is conceived of as a product: a product of thinking. The Kantian distinction between thinking of an object and cognizing it (B146), based on the consideration of the sensible intuition through which the object is given, is finally abandoned. The doctrine of *thinking* is now at the same time the doctrine of *knowledge* (Cohen, 1902, p. 12). In *The Principle of the infinitesimal method*, intuition is still opposed to thinking, the proper task of the critique of knowledge being the determination of the connection between these two different ways in which consciousness relates to its object. This task cannot be fulfilled by logic, because the latter makes abstraction of intuition and considers the laws of thinking only. To the contrary, in the *Logic* Cohen claims the self-sufficiency of thinking in knowledge, eliminating any contribution of an alleged receptivity. The critique of knowledge then turns into a logic of pure thinking²⁰.

Despite the difference between these two moments of Cohen's philosophical development, the interpretation of the infinitesimal remains the key to Cohen's idealism. In his *Logic*, Cohen claims that Kant did not properly understand the meaning of calculus for transcendental philosophy and therefore he was unable to recognize the productive capacity of thinking. For Cohen, if the infinitesimal principle had found in the *Critique* the place it deserved, then the sensibility would not have been put before thinking and pure thinking would not have been undermined in its autonomy (Cohen, 1902, p. 32). Kant did not see the importance of the infinitesimal principle. In contrast, the logic of pure knowledge underlines the crucial logical meaning of the infinitesimal principle and it may even be characterized as the logic of the principle of infinitesimal calculus (Cohen, 1902, p. 31). Thinking generates by itself its own content according to this principle, i.e. by means of infinitesimal calculus thinking produces the content which for Kant could only be given by sensibility.

For Cohen, thinking is pure because no manifold is given to it. While the Kantian synthesis is to be executed upon the spatiotemporal manifold provided by sensibility, pure thinking does not depend on sensible data. Pure thinking does not merely unify a given manifold, but generates both the unity and the corresponding multiplicity. In order to capture this productive capacity of thinking, Cohen makes use of the term *Erzeugen*. Pure thinking is generation. Since generation is the goal of pure thinking and in this activity

²⁰ See (Edel, 1988).

pure thinking does not come out of itself, Cohen identifies the generation with its product: “Die Erzeugung selbst ist das Erzeugniss” (Cohen, 1902, p. 26)²¹.

Even though the Kantian concept of synthesis does not reach the complete self-sufficiency of pure thought, the productive character of thinking, which does not take an intuitive manifold as presupposition, may still be spotted in the Kantian discussion of intensive magnitudes. The real that fills time is represented as a *quantum* by thinking its continuous and uniform generation in time. Kant claims:

the schema of a reality, as the quantity of something insofar as it fills time, is just this continuous and uniform generation of that quantity in time, as one descends in time from the sensation that has a certain degree to its disappearance or gradually ascends from negation to its magnitude. (*KrV*, B183)

We have already seen that Cohen conceives of this intensive magnitude as an infinitesimal magnitude, but criticizes the relation that Kant establishes between sensation and reality. Now two further points are to be underlined. The first one is that thinking produces not just the unity, but also the manifold to be unified. The real is given instantaneously as an absolute unity. As such, it does not contain any manifold. Only when the real is thought of as a product of a continuous and uniform generation in time may a manifold be ascribed to it, namely the manifold of the intermediate states between the complete absence of the real and its presence²². The real as a quantum will be conceived of as the unity of the manifold of grades that it went through in its generation. This gradation is not given, but is rather a product of thinking. The manifold to be unified is therefore generated by spontaneity.

The second point is that the generation of the real, whose multiplicity can only be represented through approximation to negation, is a production *from* a negation. Cohen stresses that this negation is not a *nihil negativum*, but a *nihil privativum*, which is conceived of as a limit. For this reason, the disappearance of sensation does not

²¹ “Durch diese beiden Momente also, dass das Erzeugen ein Erzeugen der Einheit sei, welche Grundbestimmung nicht minder auch für die Mehrheit gelten müsse, und dass das Erzeugen zugleich das Erzeugniss sei, begründet sich die Tendenz, das Denken als Erzeugung zu bestimmen” (Cohen, 1902, p. 26).

²² See (Böhme, 1974, p. 250). Caimi claims that this synthesis takes place instantaneously: “Nicht im Laufe der Zeit (nicht in einer Mehrzahl von unterschiedlichen, nacheinander folgenden Augenblicken), sondern in jedem nennbaren Augenblick, „in einem Punkte und in einem Augenblicke” (*Prol*, 4: 309, Anm.) der Zeit erfolgt die Synthesis der Realität” (Caimi, 2013, p. 98).

correspond to pure intuition, as Kant states (B208), but to the *origin* of sensation. (Cohen, 1885, p. 435). The concept of origin, already present in Cohen's interpretation of Kant, will turn into the central notion of the logic of pure knowledge²³.

The transcendental method applied to differential calculus disclosed the productive character of thinking in the infinitesimal synthesis corresponding to the Kantian categories of quality. Thinking does not merely unify a given manifold, but produces it: thinking is generation [*Erzeugen*]²⁴. Moreover, this generation takes place *ab nihilo*²⁵: thinking is origin [*Ursprung*]. Cohen conceives of these aspects of the Kantian synthesis that grounds the quality of the *object* as moments of the quality of *thinking* itself.

The quality of pure thinking designates the fundamental ways in which thinking proceeds²⁶. The most basic structure of pure thinking is not expressed by categories, but by judgments that correspond to laws of thinking [*Denkgesetze*]. These are identity, contradiction and origin. The law of thinking of origin is the logically first one. It is "das Denkgesetz der Denkgesetze" (Cohen, 1902, p. 100). The judgment of origin expresses that law of thinking by means of which a determination comes into being a valid one for thinking²⁷. The judgment of identity expresses that law of thinking by means of which such determination gets fixed, i.e. it affirms a content A obtained by continuous generation. Finally, the judgment of contradiction prevents that the negation of such content A may also turn into a valid content for thinking²⁸. In the words of Parmenides,

²³ "Es ist das *Problem des Ursprungs*, welches die neue Rechnung aufgerichtet, und welches zugleich das Denken, als Erzeugung, zur Klarheit und Genauigkeit bringt." (Cohen, 1902, p. 32). Cassirer claims: "Cohens *Logik der reinen Erkenntnis* hat den Gedanken des Ursprungs, auf dem sie sich aufbaut, an den Prinzipien der Infinitesimalrechnung entwickelt. Hier ist in der Tat das erste und markanteste Beispiel der allgemeinen Betrachtungsweise gegeben, die vom Größenbegriff zum Funktionsbegriff, von der »Quantität« zur »Qualität« als dem eigentlichen Fundament zurückleitet. Eine erneute Bestätigung gewinnt sodann das logische Prinzip, das hier festgestellt ist, im Fortgang zu den übrigen Problemgebieten der modernen Mathematik. Sie alle, wie verschieden sie ihrem Inhalt nach sein mögen, weisen in ihrem Aufbau auf den Grundbegriff des Ursprungs zurück. Die Forderung, die dieser Begriff stellt, ist überall dort erfüllt, wo die Glieder einer Mannigfaltigkeit aus bestimmten Reihenprinzipien abgeleitet und durch sie erschöpfend dargestellt sind" (ECW 6: 106).

²⁴ "Auch die Mehrheit ist Aufgabe der Erzeugung" (Cohen, 1902, p. 49).

²⁵ (Cohen, 1902, p. 70)

²⁶ "Die Bedeutung des traditionellen Titels der *Qualität* für diese Arten des Urtheils sehen wir somit darin, dass sie weder allein die Beschaffenheit des Körpers oder auch des Gegenstands bezeichnet, noch die des Seienden; sondern die des reinen Denkens. Also auch das reine Denken hat eine Qualität" (Cohen 1902, p. 101).

²⁷ "Nur das aus seinem Ursprung erzeugte Denken ist als Erkenntnis gültig" (Cohen, 1902, p. 99). "Nur das Denken selbst kann erzeugen, was als Sein gelten darf" (Cohen, 1902, p. 67).

²⁸ In this sense, the judgment of contradiction is "ein Urtheil vor dem Urtheil" (Cohen, 1902, p. 88).

the judgments of identity and contradiction state: “Das Seiende ist. Das Nichtseiende ist nicht” (Cohen, 1902, p. 94).

The judgment of origin is for Cohen an infinite judgment. According to their quality, Kant distinguishes infinite judgments from affirmative and negative ones. An affirmative judgment attributes a predicate to a subject, while a negative judgment asserts that a certain property does not correspond to it. On the contrary, in an infinite judgment a negative predicate is affirmed. For example, in the judgment “the soul is immortal” the non-mortality of the soul is predicated positively. The affirmative judgment corresponds to the category of reality, the negative judgment to that of negation and the infinite judgment to that of limitation. Cohen does not follow this systematic arrangement. E.g., Cohen places reality among the judgments of quantity, as the first judgment of mathematics²⁹. Moreover, just as Kant does³⁰, Cohen criticizes the definition of a judgment as the relation between two concepts: the subject and the predicate. For Cohen, the doctrine that subject and predicate are the elements of a judgment is false and lays on two confusions. The first one, the confusion between a judgment and a grammatical proposition and, the second one, that between a judgment and a psychological connection of representations (Cohen, 1902, p. 81). For Cohen, a judgment is rather a logical act of thinking. The most basic operations of thinking are those of separation [*Sonderung*] and unification [*Vereinigung*]. A judgment expresses the correlation of these operations. In the case of the judgment of origin as an infinite judgment, this correlation is paradigmatically exemplified by the infinitesimal method. Cohen indicates that the infinitesimal is not merely inextensive, but intensive. As inextensive, the infinitesimal is the negation of the extensive, while as intensive the infinitesimal has a tendency towards extension (Cohen, 1883, p. 137). As origin of extension, the infinitesimal is the non-extended that generates extension, or, in other words, it is the non-finite that generates the finite. The underlying structure of this relation between the finite and the infinitesimal is that of the infinite judgment. In the act, the law of which the judgment of origin expresses, thinking separates the extensive from the non-extensive and at the same time unifies both by putting the non-extensive as intensive, i.e. as ground of the extensive.

²⁹ See Pringe (2020).

³⁰ See *KrV*, B140 – 141.

The logically first operation of pure thinking is to separate and unify a content and its ground of determination. The content is not yet conceived of as a determinate A, but only as a determinable X. The X is just a determinable something (Cohen, 1902, p. 68). The ground of determination is separated from this X as its negation. This relative negation must be distinguished from the absolute negation contained in the judgment of contradiction. The former is oriented towards the generation of a X and leads to a “Ursprungs-Etwas” (Cohen, 1902, p. 87). This “Etwas” is meant by the noun *nothing* [*Nichts*]. To the contrary, the contradiction is an absolute negation, which prevents the generation of any content whatsoever. It is the mere *no* [*Nicht*] as activity³¹.

The judgment of origin separates and at the same time unifies something [*Ichts*] with its relative nothing [*Nichts*]. This unification is not a mere static correlation between being and not-being, but the continuous generation of the former from the latter. Therefore, the relative nothing is the transition point that thinking passes through in order to generate the determinable (Cohen, 1902, p. 76). The relative nothing provides thinking with a “springboard” in order to carry out the generation³².

The transition over the relative nothing is the way in which thinking shows its productive capacity, insofar as there remains no element merely given to it. This generation is a continuous process. The principle of continuity is a law of thinking that states that there is no isolated element in pure knowledge³³. Every element is a generated one and is to be connected to the rest of them precisely by means of the process of its generation. The unbroken execution of the productive activity of thinking therefore prevents any given or disconnected element in pure knowledge (Cohen, 1902, p. 76). The principle of continuity guides this transition from *nothing* to *something*³⁴.

Conclusions

³¹ “Der Unterschied von Nicht und Nichts ist wichtig und orientierend. Das Nichts hat die Bildung des Substantivs; denn obwohl es ein Unding ist, ist es doch ein Operationsbegriff. Das Nicht dagegen bezieht sich nur auf die Thätigkeit des Urtheils selbst” (Cohen, 1902, p. 88).

³² “Das Sein selbst soll durch das Nichtsein seinen Ursprung empfangen. Das Nichtsein ist nicht etwa ein Correlativbegriff zum Sein; sondern das relative Nichts bezeichnet nur das Schwungbrett, mit dem der Sprung durch die Continuität ausgeführt werden soll” (Cohen, 1902, p. 77).

³³ “Kraft der Continuität werden alle Elemente des Denkens, insofern sie als Elemente der Erkenntnis gelten dürfen, aus dem Ursprung erzeugt” (Cohen, 1902, p. 76).

³⁴ “Nur das aus seinem Ursprung erzeugte Denken ist als Erkenntnis gültig. Die Fahrt wird von einem sicheren Stern geleitet, vom Denkgesetz der Continuität” (Cohen, 1902, p. 99). Cohen adds: “Sie [die Fahrt] ist aber auch durch die Kenntnis der Klippen gesichert. Diese bildet das Denkgesetz des Widerspruchs” (Cohen, 1902, p. 99).

The judgment of origin separates *something* from *nothing* and unifies both by putting the latter as the origin of the former, i.e. by letting a determinable X arise from its relative nothing. This is the most basic activity of pure thinking. As such, it was not merely postulated, but it was discovered by means of the transcendental method. The application of this method to the fact of the mathematical science of nature disclosed the productive capacity of thinking. More precisely, the transcendental investigation on differential calculus as the main instrument of natural science showed that thinking is generation and origin of pure knowledge.

Once this exposition of the most basic acts of pure thinking has been achieved by the transcendental method and those acts have been identified as the judgments of the laws of thinking, Cohen turns to the problem of being. The judgment of origin states that pure thinking must begin by the generation of its own content in order to turn into pure knowledge, i.e. into thinking of *being*. (Cohen, 1902, p. 113). The judgment of origin establishes the logical structure of a progressive determination of being. This determination may be conceived of as a series. However, by means of the judgment of origin, no individual member of this series is posited, but only its general term: “Alle reinen Erkenntnisse müssen Abwandlungen des Principis des Ursprungs sein” (Cohen, 1902, p. 33).

The first step of this progressive process of pure knowledge is taken by the judgments of mathematics, in particular by the judgment of reality, that posits the infinitesimal as the real in cognition³⁵. The transcendental method showed pure thinking as the condition of possibility of differential calculus. The judgments of mathematics, in turn, show how pure thinking generates being by means of infinitesimal analysis. More precisely, the judgments of mathematics posit the content of knowledge (that according to Kant has to be given by sensibility) as a product of pure thinking and opens a determination process that continues with the judgments of mathematical natural science and the judgments of methodology³⁶.

³⁵ In (Krijnen, 2012), Krijnen claims that Cohen does not offer a logic of origin (Ursprung), but a mere logic of beginning (Anfang), i.e. a logic of the beginning of the determination series of knowledge. Yet, by means of the Cohenian judgment of origin no determination series begins. This beginning rather takes place by means of the judgment of *reality*, the first judgment of mathematics. This process of determination is understood by Krijnen as the subjective process of cognizing (Erkennen) and not as the objective process of knowledge (Erkenntnis), as Cohen rather states (Krijnen, 2012, p. 89).

³⁶ For a discussion of this issue see Pringe (2020).

In summary, the transcendental method applied to differential calculus exposes the productive character of pure thinking, which generates being according to the principle of the infinitesimal method. Thus, whereas the transcendental method reveals the ideal conditions of the real, the infinitesimal method describes how the ideal becomes real by means of a pure generation of being.

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