

## SOME REMARKS ON THE KANT-JÄSCHE LOGIC DIAGRAMS

### Algumas observações sobre os diagramas lógicos de Kant-Jäsche

J.-Martín Castro-Manzano

Universidad Popular Autónoma del Estado de Puebla  
josemartin.castro@upaep.mx

Paniel Reyes-Cárdenas

Universidad Popular Autónoma del Estado de Puebla  
panielosberto.reyes@upaep.mx

**Abstract:** In this contribution we pursue a simple goal: to bring more attention on the representative attributes of the Kant-Jäsche diagrams in order to assess their logical properties. After reviewing and reconstructing the diagrams that seem to be available in the *Jäsche Logik*, we provide an assessment of their logical qualities by comparing them with *VENN*.

**Keywords:** Knowledge representation; diagrammatic reasoning; Venn diagrams.

**Resumo:** Nesta contribuição, buscamos um objetivo simples: chamar mais atenção para os atributos representativos dos diagramas de Kant-Jäsche, a fim de avaliar suas propriedades lógicas. Depois de revisar e reconstruir os diagramas que parecem estar disponíveis no *Jäsche Logik*, fornecemos uma avaliação de suas qualidades lógicas comparando-os com *VENN*.

**Palavras-chave:** Representação de conhecimento; raciocínio diagramático; diagramas de Venn.

### 1. Introduction

Although the representative virtues of diagrams have been widely recognized, there is a bias against diagram-based inference (cf. Lagrange, Boissonnade, and Vagliente, 1997, p.vi; Leibniz, Remnant, and Bennett, 1996, p.309; Dieudonné, 1960, p.v; Tennant, 1986, p.303ff; Hammack, 2013, p.20ff). This bias is based upon the assumption that diagrams are naturally prone to fallacies, mistakes, and are not susceptible of generalization. Nevertheless, specially since the work of Shin (1994), and Allwein and Barwise (1996), these objections have been handled with care and the relation between logic and diagrams has been recast with more caution and detail.

Consequently, today we have a research program about diagrammatic inference that promotes different studies and model theoretic schemes that help us represent and better understand diagrams in logical terms, thus allowing logicians to look back into the history of logic for instances of diagrammatic inference (cf. Gardner, 1958; Swoyer, 1991; Shin, 1994; Glasgow, Narayanan and Chandrasekaran, 1995; Stenning and Oberlander, 1995; Allwein and Barwise, 1996; Nakatsu, 2010; Moktefi and Shin, 2013).

This process has proven to be very successful and has produced a revised history of logic diagrams that includes, for example, explanatory diagrams for the square of opposition (Londey and Johanson, 1987, p.109), syllogistic (Hamilton, Mansel, and

Veitch, 1865, p.420), the *pons asinorum* (Hamblin, 1976), and the medieval *summulae logicales* (Murner, 1509). But more importantly, this history also includes inferential diagrams such as Leibniz (linear and regional) diagrams (Leibniz and Couturat, 1961), Lambert diagrams (Lambert, 1764), Euler diagrams (Euler, 1768), Hamilton diagrams (Hamilton *et al*, 1865), Venn diagrams (Venn, 1881), Carroll trilateral diagrams (Carroll, 1887), Peirce's existential graphs (Peirce, 1906), Karnaugh maps (Karnaugh, 1954), Englebretsen diagrams (Englebretsen, 1992), Savio diagrams (Savio, 1998), and Pagnan diagrams (Pagnan, 2012).

Having said all this, it has come to our attention that despite there are logic diagrams within the *Jäsche Logik*, they are not taken into account in this history. Hence, given this situation, in this contribution we review the logic diagrams available in the *Jäsche Logik* (JL, AA 9:1-150) and we provide a brief assessment of their qualities in logical terms, something that, to the best of our knowledge, is yet to be accomplished (cf. Tiles, 2006; Moktefi and Shin, 2012),<sup>1</sup> and could provide some reasons as to why they are not taken into account in this history.

In order to reach this goal we start with a short review of a relation between logic and diagrams, so that we describe the intuitive notion of diagrammatic inference (§2). With that background in mind, we review the diagrams of the *Jäsche Logik* (§3), and finally, we assess their representative and logical attributes by making a comparison with Venn diagrams (§4).

## 2. Logic and diagrams

To briefly describe a relevant relation between logic and diagrams let us consider the nature of logic and diagrams. So, on the one hand, logic is the study of inference, and inference is a relation that produces, so to speak, information given previous data by following certain norms that allow us to describe inference as the unit of measurement of reasoning: inference may be more or less (in)correct depending on the compliance or violation of such norms. On the other hand, in order to represent information we use internal or external representations, and external representations can

---

<sup>1</sup> *Critique of Pure Reason* A716/B744 devotes a good deal of reflection on the epistemological consequences of the use of diagrams in Geometry, but this discussion is a separate matter that has little to do with the use of diagrams as inferential tools. For more on this topic see (Shabel, 2003) and (Friedman, 2012).

be further divided into two kinds: sentential and diagrammatic (Larkin and Simon, 1987).

Diagrammatic representations are sets of diagrams that contain information stored at one particular *locus*, including information about relations with the adjacent *loci*; and diagrams are information graphics that index information by location on a plane (Larkin and Simon, 1987). In particular, it can be said that logic diagrams are two-dimensional geometric figures with spatial relations that are isomorphic with the structure of logical statements (Gardner, 1958, p.28).

Hence, to wrap all this up, if inference is a process that produces information given previous data and information can be represented diagrammatically, it is not uncomfortable to suggest that diagrammatic inference is the unit of measurement of diagrammatic reasoning: diagrammatic inference would be (in)correct depending on the compliance or violation of certain norms (cf. Castro-Manzano, 2017). This relation would define our intuitions about the informal notion of diagrammatic inference and would follow, *ex hypothesi*, classical structural norms, for instance, via Shimojima's notion of free ride (Shimojima, 1996).

To explain all of this let us briefly describe a system that finely captures the notion of free ride: Venn diagrams (or *VENN* for short). *VENN* has a well-defined vocabulary, syntax, and semantics (Shin, 1994, p.48). Its vocabulary is defined by the closed curve, the rectangle, the shading, the X, and the line (Figure 1a); its semantics depends on a homomorphism with sets that helps define a diagram as any finite combination of diagrammatic elements where a region is any enclosed area in a diagram; a basic region is a region enclosed by a rectangle or a closed curve; a minimal region is a region within which no other region is enclosed; an X-sequence is a diagram of alternating X's and lines with an X in each extremal position. Regions represent sets and the rectangle represents the domain. A shaded region represents an empty region and a region with an X represents a non-empty region. With these definitions, a syntax for categorical propositions (*vide* Appendix A) can be elaborated (Fig. 1b).

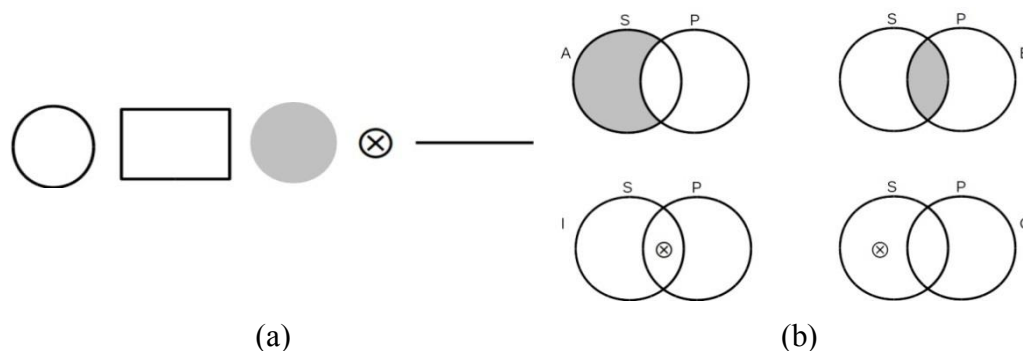


Figure 1. Basic elements of *VENN* (adapted from (Castro-Manzano, 2017))

What is more interesting is that *VENN* provides an essential feature of a well defined diagrammatic logic: a set of rules of introduction, unification, and erasure of diagrammatic objects that helps us determine the correction of an inference (Shin, 1994, p.81; Nakatsu, 2010, p.133). These rules allow us to draw down the diagrams for the premises so that we can check (by mere observation) whether it is possible to “read off” the conclusion: in case we can, we say the inference is correct and there is a free ride; otherwise it is incorrect.

For example, consider a *Darii* syllogism (a summary of syllogistic is shown in Appendix A) (Fig. 2). According to *VENN*, we begin with an introduction of areas (step 1) and then a unification is applied (step 2). After that, we apply an erasure of an X-sequence (step 3) and then a spreading of an X-sequence (step 4). Finally, by the erasure of a closed curve rule, we obtain a final diagram (step 5). Since the conclusion is obtained by drawing down the premises, the diagrammatic inference is valid: this is a fair example of a free ride that we will use later.

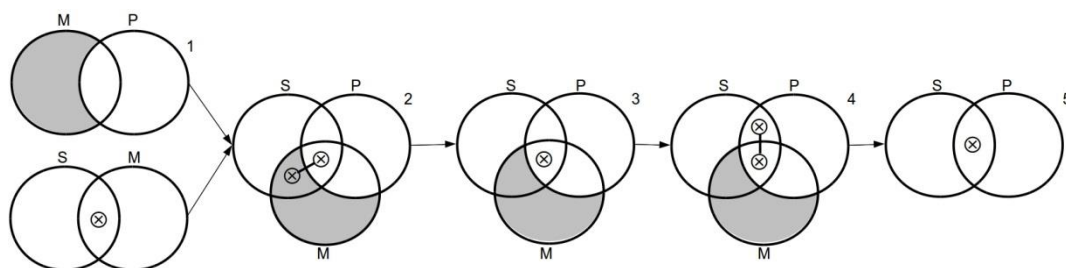


Figure 2. An example of a free ride in *VENN* (adapted from (Castro-Manzano, 2017))

### 3. The Kant-Jäsche diagrams

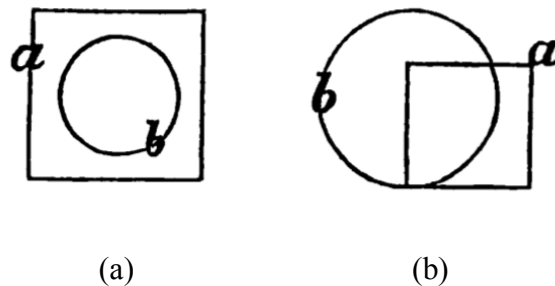
With the previous background in mind, we now proceed to review the diagrams

available in the *Jäsche Logik*, a manual for teaching logic published by Gottlob Benjamin Jäsche under Kant's request.<sup>2</sup> For this purpose we use the diagrams that appear in the *Logik, I. Allgemeine Elementarlehre*, §21-29 (cf. Kant and Young, 1992). And since these diagrams are used to explain the nature of judgments (*Urteile*)<sup>3</sup> with respect to quantity, quality, and relation, we start this section with a description of these aspects.

### 3.1 Judgments with respect to quantity

According to the explanations given in the *Logik*, judgments are divided into three kinds with respect to quantity: universal, particular, and singular. In universal judgments, one concept is wholly enclosed within another concept; in particular judgments, a part of one concept is enclosed under another concept; and singular judgments behave as universal judgments.<sup>4</sup>

The previous descriptions are quite interesting because they implicitly define some sort of diagrams in so far as they suggest a topological clause in terms of the relation of "enclosure." However, the first explicit diagrams we are able to find in the *Jäsche Logik* appear in §20, Note 4 (Fig. 3): in there we can observe a diagrammatic rendition of particular judgments with respect to quantity.



**Figure 3.** Particular judgments with respect to quantity (Kant and Young, 1992, p.599-600)

So, in judgments of this sort the subject-term of the proposition is a broader

<sup>2</sup> This manual cannot be confidently regarded as a definitive version of Kant's views on logic, even though Kant requested its preparation and published it under his own name: for sake of accuracy we call these diagrams *Kant-Jäsche diagrams* as opposed to just *Kant diagrams* or just *Jäsche diagrams*.

<sup>3</sup> We focus on the sentential interpretation of "judgment" as a medium of subject-predicate propositions.

<sup>4</sup> *Logik, I. Allgemeine Elementarlehre*, §20, Note 1 indicates that, with respect to the logical form, singular judgments are to be assessed as universal judgments. This is consistent with the tradition that treats universal and singular quantification as some sort of wild quantity (Englebretsen, 1996).

concept than the predicate-term. Hence, using a square,  $\square$ , to represent the subject-term, and a circle,  $\circ$ , to stand for the predicate-term, Figure 3a represents a particular judgment in so far as a particular judgment states that for some of what belongs under  $a$  is  $b$  but some  $a$  is not  $b$ .<sup>5</sup> This diagrammatic interpretation of particular judgments sounds intuitively right, for we usually say that a particular judgment entails the rejection of the predicate for some other subject terms. For example, when we assert propositions such as “Some philosophers are logicians” it seems we are implying that “Some (philosophers) are not (logicians).” We will return to this issue later.

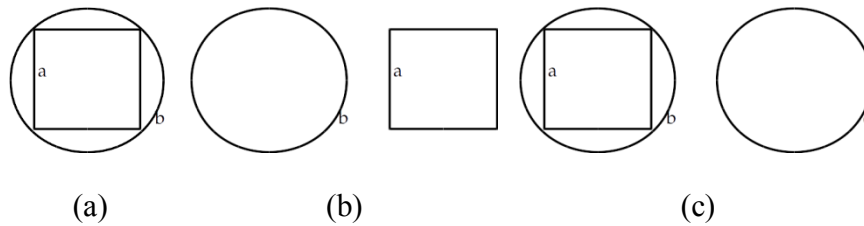
### 3.2 Judgments with respect to quality

With respect to quality, judgments can either be affirmative, negative, or infinite. Unfortunately, the explicit diagrams for these judgments are not to be found in the *Jäsche Logik*; however, we can reconstruct them after the specifications given in §22 of the *Logik*, and from the diagrams Kant sketched in his copy of Meier’s *Excerpts from the Doctrine of Reason* (cf. Tiles, 2006).<sup>6</sup>

According to such specifications, in the affirmative judgments the subject is included in the concept of the predicate; in the negative, the concept of the subject is posited outside the concept of the predicate; and in the infinite, one concept is posited within a concept that lies outside another concept. Diagrammatically, we would say that in an affirmative judgment the subject-term diagram has to be drawn inside the predicate-term diagram (Fig. 4a); in the negative, the subject-term diagram has to be posited outside the predicate-term diagram (Fig. 4b); and in the infinite, the subject-term has to be drawn inside the diagram of a term that lies outside the diagram of another term (Fig. 4c).

<sup>5</sup> A particular judgment by accident (“if at least all  $a$  can be contained in  $b$ , if it is smaller, but not if it is greater”) is represented by the diagram shown in Figure 3b.

<sup>6</sup> Consider the diagram for the *Barbara* syllogism that shows the inclusion of concepts (Kant, 1924, p. 715):  $\gamma? \pi\text{--}\rho? (\kappa\text{--}\lambda?) \eta??$  L 101'. Zu L §. 363:

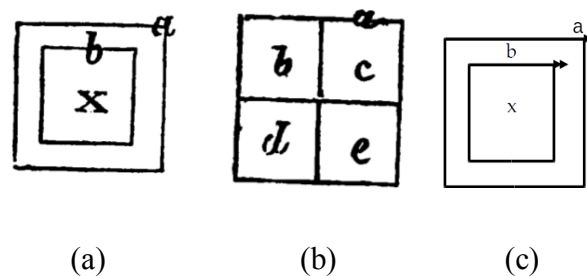


**Figure 4.** Judgments with respect to quality

### 3.3 Judgments with respect to relation

With respect to relation, judgments may either be categorical, hypothetical, or disjunctive (§23). In categorical judgments the terms are subordinated one to another as predicates to subjects; in hypothetical, as consequences to grounds; and in disjunctive propositions, as members of a division to a divided concept.

According to §24, in categorical judgments, if some  $x$  is contained in  $b$ , and if  $b$  is contained in  $a$ , then  $x$  is also contained in  $a$  (Fig. 5a); in disjunctive judgments, if  $x$  is contained in  $a$ ,  $x$  is contained either under  $b$  or  $c$  or  $d$  or  $e$ , provided that all that is contained in a term-diagram is also contained in one of the parts of such term-diagram (§27-29) (Fig. 3b). Unfortunately, there are no explicit diagrams for hypothetical syllogisms, but we can reconstruct them by attending to the specifications given in §25, Note 1: according to this specification, what the *copula* is for categorical judgments, the *consequentia* (which we denote by “ $\gg$ ”) is for hypotheticals (Fig. 3c). The interpretation of *consequentia* we favored here is the traditional one of logical consequence, and it has the strength of a demonstration.<sup>7</sup>



**Figure 5.** Judgments with respect to relation (Kant and Young, 1992, p. 604)

### 3.4 Remarks on the Kant-Jäsche logic diagrams

<sup>7</sup> This is shown by the interchangeable use of the terms “*consequentia*” and the expression “Q.E.D” (*quod erat demonstratum*) in the *Allgemeine Elementarlehre*.

At face value, the previous diagrams may seem to be of logical interest, but there is no free lunch: if we are to logically assess these diagrams, we have to evaluate their representative and logical attributes. In order to review the representative attributes of the Kant-Jäsche diagrams we consider a framework after (Nakatsu, 2010, p.305). According to this framework, the representative qualities of a system of diagrams can be assessed by observing the following attributes: comprehension (whether diagrams promote system understanding), clarity (whether diagrams are unambiguous), parsimony (whether diagrams are explanatory), relevance (whether diagrams support a rational agent in some kind of problem-solving task), and separability (whether diagrams enable multi-layered descriptions). Hence, the question about the representative attributes of the Kant-Jäsche diagrams can be reduced to the question of whether the Kant-Jäsche diagrams follow these requirements. Let us entertain this question.

*Comprehension. Do Kant-Jäsche diagrams promote system understanding?* It seems fair to say that the Kant-Jäsche diagrams do promote the understanding of the internal structure of judgments, for they are advanced as two-dimensional geometric figures with spatial relations that pretend to be isomorphic with the structure of judgments; however, judgments are just a fragment of the system of logic and logic has to do with inference as a whole. Thus, ultimately, we should consider whether Kant-Jäsche diagrams promote inference understanding. In other words, we should ask: do Kant-Jäsche diagrams produce free rides? We will return to this issue later.

*Clarity. Are Kant-Jäsche diagrams unambiguous?* Kant-Jäsche diagrams have a well-defined vocabulary (say, Vocabulary= $\{\square, \circ\}$ ) and some basic semantics defined after the usual relations of inclusion or exclusion of concepts. This seems to imply that the Kant-Jäsche diagrams are not overloaded in the sense that every construct on a diagram means only one thing and one thing only. However, as we will see later, when it comes to inference as a whole, the overall clarity of these diagrams is not necessarily preserved.

*Parsimony. Are Kant-Jäsche diagrams explanatory?* The Kant-Jäsche diagrams seem to be just partially parsimonious because, although they explain the internal structure of judgments, it is not clear they can be used to explain inference writ large, as we will see below. They certainly provide a level of abstraction good enough to explain



that, for judgments, quantity, quality, and relation, are topological relations; but they do not do the same for inference as a whole. This means that the Kant-Jäsche diagrams are successful to show some information, but not all the information that is needed to put together a diagrammatic logical system determined by topological relations.

*Relevance. Do Kant-Jäsche diagrams support a rational agent in some kind of problem-solving task?* If we consider that explaining the internal structure of judgments is a task to be solved, then we could say, with certainty, that these diagrams succeed at a didactic level. However, in a larger picture, the particular problem-solving task for logic diagrams should be at an inferential level.

*Separability. Do Kant-Jäsche diagrams enable multi-layered descriptions?* Due to the notions of exclusion and inclusion of concepts, the Kant-Jäsche diagrams may be joined or separated as to avoid representational issues, that is to say, the relations of inclusion and exclusion allow layered constructions and descriptions of diagrams. This feature would be exemplified below.

To sum this up, we can conclude that, from the standpoint of representation, the Kant-Jäsche diagrams are fairly good diagrams because, with respect to the internal structure of judgments, they do promote comprehension, are unambiguous, parsimonious, relevant, and enable multi-layered descriptions. However, as we stated above, it is not clear that they promote inference understanding as a whole. In order to show this, here we follow the proposal advanced by Larkin and Simon (1987). Thus, we ask ourselves whether the Kant-Jäsche diagrams are informationally equivalent to *VENN* diagrams with respect to a well behaved sentential logical system, namely, syllogistic, provided two diagrammatic systems are *informationally equivalent* if all the information in one is also inferable from the other, and vice versa (Larkin and Simon, 1987).<sup>8</sup> with the aid of a counter-example, here we claim that the Kant-Jäsche diagrams are not informationally equivalent to *VENN* diagrams.

So, suppose they are. Then all free rides in *VENN* (i.e. all valid inferences given a syllogistic base) can be developed within the Kant-Jäsche diagrams and vice versa; but there is a valid inference that is a free ride in *VENN* but is not a free ride in the

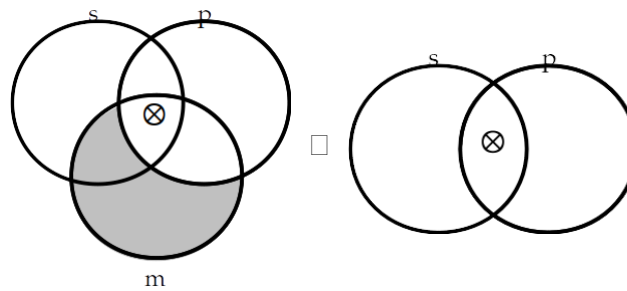
---

<sup>8</sup> Thus, for example, it can be shown that Venn diagrams, Carroll trilateral diagrams, Leibniz diagrams, Peirce's existential graphs, or Pagnan's diagrams are informationally equivalent with respect to a syllogistic base.

Kant-Jäsche diagrams.<sup>9</sup> Consider an instance of a *Darii* syllogism, that is to say, something of the form:

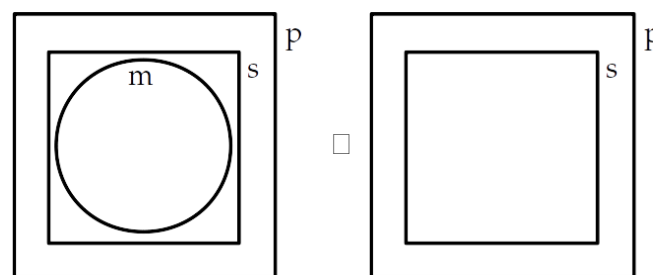
$$\{\text{All } m \text{ is } p, \text{ Some } s \text{ is } m\} \sqsupset \text{Some } s \text{ is } p.$$

A proper diagrammatic rendition of the previous inference in *VENN* would look as follows (Fig. 6):<sup>10</sup>



**Figure 6.** A *Darii* syllogism with *VENN*

Clearly, the conclusion is a free ride: just by drawing down the premises we obtain a proper diagram of the conclusion. However, the representation of the same syllogism using the Kant-Jäsche diagrams produces the following configuration (Fig. 7):

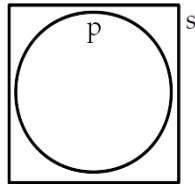


**Figure 7.** A *Darii* syllogism with Kant-Jäsche diagrams

<sup>9</sup> One could ask whether it is fair to compare the Kant-Jäsche diagrams with the good old Venn diagrams. In response to this we could argue that the latter have become such a benchmark for diagrammatic reasoning in general that *VENN* can be regarded as the classical logic of the diagrammatic logical systems (in fact, *VENN* is equivalent to classic monadic first order logic (Hammer, 1995)). So, we do not discard the possibility of comparing the Kant-Jäsche diagrams with alternative diagrammatic systems, but for inferential purposes, a classical diagrammatic system will do.

<sup>10</sup> The full diagrammatic proof of this inference was developed in the previous section.

And as we can see, regardless of the geometry of the figures, the available conclusion (All  $s$  is  $p$ ) is nowhere near the proper conclusion (Some  $s$  is  $p$ ), which should look more or less like this (Fig. 8):



**Figure 8.** “Some  $s$  is  $p$ ” with Kant-Jäsche diagrams

This problem emerges from the diagrammatic interpretation of particular judgments because even if it sounds intuitively right that a particular judgment entails the rejection of the predicate for some other subject terms, this should not be taken as a *bona fide* rule. Certainly, when we assert propositions such as “Some philosophers are logicians” it seems we are implying that “Some (philosophers) are not (logicians),” but nothing assures such an inference. For instance, we could also say “Some cats are animals,” which is true, but that does not imply that “Some (cats) are not (animals)” is also true.

#### 4. Conclusions

In this short contribution we have briefly reviewed the diagrams available in the *Jäsche Logik* in order to evaluate their representative and logical attributes by comparing them with *VENN*. This should be of interest because it might help explain a couple of issues. On the one hand, the fact that the Kant-Jäsche diagrams are not informationally equivalent to *VENN* diagrams may provide reasons to explain why they are not taken into account in the canonical diagrammatic reasoning literature, namely, in the current history of logic diagrams (cf. Gardner, 1958; Moktefi and Shin, 2012), even though they have some interesting representative attributes. On the other hand, the fact that they have some interesting representative properties but lack inferential powers may help us understand why the Kant-Jäsche diagrams remain a didactic tool rather than a logical system, which is fair enough, for the purpose of the *Jäsche Logik* was not to develop a diagrammatic system in and of itself, but to offer comprehensive lectures

on a normative version of general logic.

## References

ALLWEIN, G. and BARWISE, J. (1996). *Logical Reasoning with Diagrams*. New York: Oxford University Press.

CARROLL, L. (1887). *The Game of Logic*. London: Macmillan and Co.

CASTRO-MANZANO, J.M. (2017). Remarks on the Idea of Non-monotonic (Diagrammatic) Inference. *Revista de filosofia Open Insight*, v. 8, n. 14, pp. 249-251.

DESCARTES, R., MILLER, V. R., and MILLER, R. P. (1984). *Principles of Philosophy*. Dordrecht: Reidel.

DIEUDONNÉ, J. (1960). *Foundations of Modern Analysis*. New York: Academic Press.

ENGLEBRETSSEN, G. (1992). Linear Diagrams for Syllogisms (with Relationals). *Notre Dame J. Formal Logic*, v. 33, n. 1, pp. 37-69.

\_\_\_\_\_. (1996). *Something to Reckon with: The Logic of Terms*. University of Ottawa Press.

EULER, L. (1768). *Lettres à une Princesse d'Allemagne*, St. Petersburg; l'Academie Imperiale des Sciences.

FRIEDMAN, M. (2012). Kant on geometry and spatial intuition. *Synthese*, v. 186, n. 1, pp. 231-255.

GARDNER, M. (1958). *Logic Machines and Diagrams*. New York: McGraw-Hill.

GLASGOW, J., NARAYANAN, N. H. y CHANDRASEKARAN, B. (eds.) (1995). *Diagrammatic Reasoning: Cognitive and Computational Perspectives*. Cambridge: AAAI Press/The MIT Press.

HAMBLIN, C.L (1976). An Improved *Pons Asinorum*? *Journal of the History of Philosophy*, v. 14, n. 2, pp. 131-136.

HAMMER, E. (1995). Diagrammatic Logic, in D. M. Gabbay and F. Guenther (eds.), *Handbook of philosophical logic*, v. 4. Amsterdam: Elsevier North Holland.

HAMILTON, W., MANSEL, H. L., and VEITCH, J. (1865). *Lectures on Metaphysics and Logic*. Boston: Gould and Lincoln.

HAMMACK, R. H. (2013). *Book of Proof*. Richmond: Virginia Commonwealth University.

KANT, I., and YOUNG, J. M. (1992). *Lectures on Logic*. Cambridge: Cambridge University Press.

KANT, I. (1924). *Kant's handschriftlicher Nachlas: Logik, Kants gesammelte Schriften, Band XVI, KSniglich Preuflischen Akademie der Wissenschaften*. Berlin: Walter de Gruyter Co.

KARNAUGH, M. (1954). Map method for synthesis of logic circuits, *Electrical Engineering*, v. 73, n. 2, pp. 136-136.

LAGRANGE, J. L., BOISSONNADE, A. C., and VAGLIENTE, V. N. (1997). *Analytical Mechanics*. Dordrecht: Kluwer Academic.

LAMBERT, J. H. (1764). *Neues Organon oder Gedanken Uber die Erforschung und Bezeichnung des Wahren und dessen Unterscheidung vom Irrthum und Schein*. Leipzig: Bey Johann Wendler.

LARKIN, J. H., and SIMON, H. A. (1987). Why a Diagram is (Sometimes) Worth Ten Thousand Words, *Cognitive Science*, v. 11, n. 1, pp. 65-100.

LEIBNIZ, G. W., and COUTURAT, L. (1961). *Opuscules - et fragments inedits de Leibniz, extraits des manuscrits de la Bibliothèqu royale de Hanovre*. Hildesheim, G. Olm.

LEIBNIZ, G. W., REMNANT, P., and BENNETT, J. (1996). *New essays on human understanding*. Cambridge: Cambridge University Press.

LONDEY, D., and JOHANSON, C. (1987). *The logic of Apuleius: including a complete Latin text and English translation of the Peri hermeneias of Apuleius of Madaura*. Leiden: E.J. Brill.

MOKTEFI, A., and SHIN, S. (2012). A History of Logic Diagrams, in D. M. Gabbay and J. Woods (eds.), *Handbook of the history of logic*, v. 11. Amsterdam: Elsevier North Holland.

MOKTEFI, A., and SHIN, S. (2013). *Visual Reasoning with Diagrams*. Heidelberg: Birkhauser.

MURNER, T. (1509). *Logica memorativa*. Argentoratum.

NAKATSU, R. (2010). *Diagrammatic Reasoning in AI*. Hoboken, Wiley.

PAGNAN, R. (2012). A Diagrammatic Calculus of Syllogisms, *Journal of Logic, Language and Information*, v. 21, n. 3, pp. 347-364.

PEIRCE, C. S. (1906). Prolegomena to an Apology for Pragmaticism, *Monist*, v. 16, n. 4, pp. 492-546.

SAVIO, M. (1998). AE (Aristotle-Euler) Diagrams: An Alternative Complete Method for the Categorical Syllogism. *Notre Dame J. Formal Logic*, v. 39, no. 4, pp. 581-599.

SHABEL, L. (2003). *Mathematics in Kant's Critical Philosophy: Reflections on Mathematical Practice*. New York: Routledge.

SHIMOJIMA, A. (1996). Operational constraints in diagrammatic reasoning, in G. Allwein and J. Barwise (eds.), *Logical Reasoning with Diagrams*. New York: Oxford University Press.

SHIN, S. (1994). *The Logical Status of Diagrams*. Cambridge: Cambridge University Press.

STENNING, K. and OBERLANDER, J. (1995). A Cognitive Theory of Graphical and Linguistic Reasoning: Logic and Implementation, *Cognitive Science*, v. 19, pp. 97-140.

SWOYER, C. (1991). Structural Representation and Surrogate Reasoning, *Synthese*, v. 87, pp. 449-508.

TENNANT, N. (1986). The Withering Away of Formal Semantics?, *Mind & Language*, v. 1, n. 4, pp. 302-318.

TILES, M. (2006). Kant: from General to Transcendental Logic, in D. M. Gabbay and J. Woods (eds.), *Handbook of the History of Logic*, v. 3. Amsterdam: Elsevier North Holland.

VENN, J. (1880). On the Diagrammatic and Mechanical Representation of Propositions and Reasonings, *Philosophical Magazine Series 5*, v. 10, n. 59, pp. 1-18.

## Appendix A. Syllogistic<sup>11</sup>

Syllogistic is a term logic that deals with inference between categorical propositions. A categorical proposition is a proposition composed by two terms, a quantity, and a quality. The subject and the predicate of a proposition are called terms: the term-schema S denotes the subject term of the proposition and the term-schema P denotes the predicate. The quantity may be either universal (*All*) or particular (*Some*) and the quality may be either affirmative (*is*) or negative (*is not*). These categorical propositions have a type denoted by a label (either *a* (for the universal affirmative, *SaP*), *e* (for the universal negative, *SeP*), *i* (for the particular affirmative, *SiP*), or *o* (for the particular negative, *SoP*)) that allows us to determine a mood, that is, a sequence of three categorical propositions ordered in such a way that two propositions are premises

<sup>11</sup> This summary is adapted from (Castro-Manzano, 2017).

and the last one is a conclusion. A categorical syllogism, then, is a mood with three terms one of which appears in both premises but not in the conclusion. This particular term, usually denoted with the term-schema M, works as a link between the remaining terms and is known as the middle term. According to the position of this middle term, four arrays or figures can be set up in order to encode the valid syllogistic moods (Table 1).

First figure	Second figure	Third figure	Fourth figure
<i>Barbara</i> MaPSaM□SaP	<i>Cesare</i> PeMSaM□SeP	<i>Disamis</i> MiPMaS□SiP	<i>Calemes</i> PaMMeS□SeP
<i>Celarent</i> MePSaM□SeP	<i>Camestres</i> PaMSeM□SeP	<i>Datisi</i> MaPMiS□SiP	<i>Dimaris</i> PiMMaS□SiP
<i>Darii</i> MaPSiM□SiP	<i>Festino</i> PeMSiM□SoP	<i>Bocardo</i> MoPMaS□SoP	<i>Fresison</i> PeMMiS□SoP
<i>Ferio</i> MePSiM□SoP	<i>Baroco</i> PaMSoM□SoP	<i>Ferison</i> MePMiS□SoP	-

**Table 1.** Valid syllogistic moods

Artigo recebido em: 18.07.2019

Artigo aprovado em: 10.12.2019