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DIALECTICAL RHETORIC: RESPONSE TO ABEL LASSALLE CASANAVE

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Abstract: Abel Lasalle Casanave's comments on the rhetorical and dialectical aspects of the discussion of proof in chapters 19 to 21. I center most of my response on two issues raised in his concluding questions; the first being the hermeneutic aspect of the development of mathematics and of mathematical knowledge, and the second "the symbolic conception of mathematics" and the "algebraic mode of thought".

Keywords: Dialectic. Rhetoric. Hermeneutics. Algebraic mode of thought.

RETÓRICA DIALÉCTICA: RÉPLICA À ABEL LASALLE CASANAVE

Resumo: As considerações de Abel Lasalle Casanave dizem respeito aos aspectos retóricos e dialéticos da discussão da noção de prova nos capítulos 19 à 21, e minha réplica está centrada em duas questões levantadas em suas observações finais. A primeira refere-se ao aspecto hermenêutico do desenvolvimento da matemática e do conhecimento matemático; e a segunda à "concepção simbólica da matemática" e ao "modo algébrico de pensar".

Palavras chave: Dialética. Retórica. Hermenêutica. Modo algébrico de pensar.

Abel's remarks about the rhetorical and dialectical aspects of my discussions in chapters 19-21 are very much to the point, and he is quite right in his surmise that I never considered Aristotle's *Rhetoric* in developing my ideas about proof and deduction. His discussion enriches my own, and his final questioning remarks raise two issues on which I will center most of my response. The first issue is the hermeneutic aspect of the development of mathematics and of mathematical knowledge. The second issue concerns "the symbolic conception of mathematics" and the "algebraic mode of thought", which is also developed at some length in his earlier essay "La Concepción de Demostración de Oswaldo Chateaubriand".

1. PROOF AND REPRESENTATION

As a preliminary caveat, however, I would like to emphasize that a substantial part of my motivation in chapters 19-21 was to distinguish the actual phenomenon of proving from its logical and mathematical representation in a theory of proofs. Many of my observations seem to me to be quite obvious from this perspective—for instance, the claim that in proving things to an audience a mathematician will take into account the nature of the audience. Naturally, this will include the background knowledge that can be expected from the audience, level of experience, maturity, etc. It is one reason I said that the usual logical representation in terms of simple algorithmic rules is not an analysis of the phenomenon of proving. The insistence of many logicians and philosophers that a proof is a finite sequence of transformations over syntactic strings of symbols by means of effectively verifiable rules has considerably muddled the issue, both in relation to the actual phenomenon of proof and in relation to its mathematical representation. In fact, my view is that the usual syntactic analyses are not analyses of *proof*, but

syntactic analyses of the notion of logical consequence for specific logical systems, such as propositional logic, first-order logic, etc.

When I distinguished four main aspects—structural, psychological, social, and ontological—in proof and justification, I was not attempting to formulate a mathematical representation, but, rather, a pragmatic analysis of the phenomenon of proving. Abel’s analysis in terms of rhetorical and dialectical features complements my pragmatic analysis in an insightful way.¹

2. HERMENEUTIC CONSIDERATIONS

From a hermeneutic point of view the example of the Axiom of Choice is very interesting, because it raised issues of interpretation concerning many different aspects of mathematics. In particular, it led to the recognition of *different notions of set*; namely, the notion of set as defined by a rule vs. the notion of set as an arbitrary collection of things. The initial questioning of the axiom derived from the first conception, and constructivists continue questioning it along the same lines. Zermelo’s conception, on the other hand, and Hadamard’s defense of it in the exchange with Borel, Baire, and Lebesgue, was based on the second conception.² I briefly mention this in note 7 (pp. 317-318) and refer to Gödel’s remark in “Russell’s Mathematical Logic” (p. 151) that the axiom of choice is *analytic* for the extensional notion of set. So, in effect, there was a hermeneutic

¹ A suggestion with which I do not agree, however, is that my attempt to combine a realistic approach to mathematics with a non-aprioristic epistemology turns the notion of necessary truth somewhat superfluous. For me the necessary character of logical and mathematical propositions derives from their ontological interpretation, not from our justification of them. Of course, some proofs establish necessary connections whereas others do not.

² See note IV in Borel (1914). The five letters are translated in Appendix I of Moore (1982).

split about the meaning of the notion of set. But, of course, there were other issues as well, such as the seemingly paradoxical results obtained by Tarski and Banach. The sorting out of these issues was not merely a question of obtaining conviction in the truth of the axiom as it was used in mathematics, as I suggest in the passage quoted by Abel, but also—and perhaps more importantly—of getting a better grasp on various different notions of set.

Abel also mentions that “hermeneutic considerations may be necessary to decide whether the *same* assertion is proved by means of diagrams and by means of sentential proofs,” and this connects with his remarks about the symbolic conception of mathematics and the algebraic mode of thought.

Let us consider the symbolic manipulation characteristic of the algebraic mode of thought. Many proofs by induction in arithmetic actually have this character. One does not try to understand what is being proved, but performs operations that lead to the desired result. This is so for my simple example of the sum of the first n positive integers. Once one knows the result of performing certain algebraic operations—that $(a+b)^2 = a^2 + 2ab + b^2$, for instance—one can manipulate the symbols to carry out the inductive proof. I agree this is important, and it is more “efficient” than trying to understand what the result means in every case, but it does substitute computation for understanding. In this sense there is a similarity with the syntactic conception of proof I criticize.

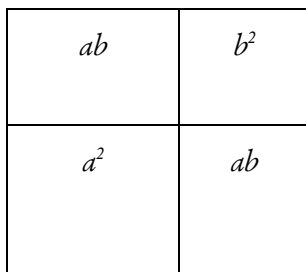
The equation above can itself be used as an illustration. A student who knows how to multiply across sums, can prove it as follows:³

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2.$$

³ The calculation also involves the comutativity of multiplication and the associativity of addition.

In this calculation the equation is treated as uninterpreted, and if one were to ask what it *means*, the student would probably just give a description in terms of the operations performed.

If one interprets the equation geometrically, however, one can prove it very nicely by means of a diagram. Take a segment of length $a+b$, and construct the square with side $a+b$. It is then quite obvious that this square can be divided into a square with side a , a square with side b , and two rectangles with sides a and b —giving, therefore, $(a+b)^2 = a^2 + 2ab + b^2$.



Thus, I think Abel is right that one needs hermeneutic considerations to decide they are the same assertion. Or are they?

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