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THE TRUTHS OF LOGIC AND LOGICAL TRUTH

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Abstract: A principal aim of Chateaubriand's *Logical Forms II: Logic, Language, and Knowledge* is to clarify and defend what Chateaubriand describes as the ontological conception of logic against the standard model-theoretic or "linguistic" view. Both sides to the debate accept that if logic is a science then there must be logically necessary facts that this science discovers, Chateaubriand arguing that because logic is a science, there must be logically necessary facts, and his opponent that because there are no logically necessary facts, logic cannot be a science. I argue that we can go between the horns of this dilemma by showing that, although logic is a science, it does not follow, as Chateaubriand assumes, that there are logically necessary facts. There are truths of (the science of) logic; there are no "logical truths".

Keywords: Chateaubriand. Frege. Inference license. Logical truth. Mathematical intuition. Peirce.

AS VERDADES DA LÓGICA E VERDADE LÓGICA

Resumo: Um dos objetivos principais de *Logical Forms II: Logic, Language and Knowledge* de Chateaubriand é clarificar e defender o que ele descreve como a concepção ontológica da lógica, contra a visão predominante, modelo-teórica ou "lingüística". Os dois lados do debate aceitam que, se a lógica é uma ciência, então deve haver fatos logicamente necessários que esta ciência descobre; Chateaubriand argumenta que, porque a lógica é ciência, deve haver fatos necessários que ela descobre, enquanto seus oponentes argumentam que, porque não há fatos logicamente necessários, a lógica não pode ser uma ciência. Eu argumento que podemos tomar uma via intermediária entre estes dois lados do dilema mostrando que, ainda que a lógica seja uma ciência, não se segue, como Chateaubriand assume, que existem fatos logicamente necessários. Existem verdades da (ciência da) lógica; não existem "verdades lógicas".

Palavras chave: Chateaubriand. Frege. Permissão para inferência. Verdade lógica. Intuição matemática. Peirce.

A principal aim of Chateaubriand's *Logical Form II: Logic, Language, and Knowledge*[1] is to clarify and defend what Chateaubriand describes as the ontological conception of logic against the standard model-theoretic or "linguistic" view. This debate has a very familiar shape. The defender of the ontological view and the defender of the linguistic view are agreed that if there are truths of logic, if, that is, logic is a science answering to something objective, then there are logical truths, that is, logically necessary facts that it is the aim of the science of logic to discover.[2] The defender of the linguistic view of logic argues modus tollens: there are no logically necessary facts, no logical truths, and hence logic itself is not a science. There are, then, no truths of logic; logic concerns the logical *form* of sentences as contrasted with their (non-logical) content and truth. Chateaubriand, in defense of the ontological view, argues modus ponens: because logic is a science (that is, there are truths of logic), there must be logical truths,

[1] Campinas: UNICAMP, Centro de Lógica, Epistemologia e História da Ciência, 2005. References to this work, as well as to the first volume, *Logical Form I: Truth and Descriptions* (2001), with be given parenthetically, by volume and page number, for example, thus: (I.26).

[2] By "logical truth" I mean a truth that is not merely proper to the science of logic (not merely a truth of logic) but somehow "ontologically logical"; a logical truth is a truth about a logical feature of reality. Such a truth would be logically necessary, its content that of a logically necessary fact.

logically necessary facts. The task of logic on Chateaubriand's view is to discover the logically necessary features of reality. I aim to show that both are half right, that there are truths of logic, as Chateaubriand argues, but no logical truths (in our stipulated sense). It is the conditional, agreed on by both, that is the source of the difficulty.

According to the model-theoretic or linguistic view, the concern of logic is logical form as it contrasts with (non-logical) content, and language itself is to be understood in terms of a fundamental dichotomy of form, given by a syntax or grammar together with the logical constants, on the one hand, and non-logical, semantic content, on the other. This form is perspicuously displayed in the standard notation of quantificational logic, in signs such as these: 'Fa', 'Rab', '$(\forall x)(Fx \supset Gx)$', '$(\exists x)Fx$'. Such signs *exhibit* (logical) form. Their (non-logical) content, insofar as they have any, is given by a model or interpretation that assigns a semantic value to the non-logical constants—objects to singular terms and sets of objects, or of ordered n-tuples of objects, to n-ary predicates—and fixes a domain of quantification. Logically valid sentences are those that are true on any interpretation. For example, the sentence '$Fa \vee \sim Fa$' is logically valid, true on every interpretation, and because it is, we can (for the purposes of logic) forget about the interpretation. The sentence is true in virtue of its form. And essentially the same can be said of a logically valid argument. A logically valid argument is one that preserves truth on any interpretation; no matter what the model, if the premises are true then the conclusion is as well. So, again, we can forget about the interpretation (for the purposes of logic). What logic concerns is not truth (about reality, which is a function of content and so a semantic notion) but valid forms, whether of sentences or of strings of sentences conceived as arguments, premises to conclusions. Its aim is to discover (meta-level) laws governing such forms.

As Chateaubriand notes (I.14-15), this conception of logic and language has complex roots in developments of mathematics over the course of the nineteenth century and in mathematical logic in the first half of the twentieth century. But as Chateaubriand also sees, the motivation for the twentieth century model-theoretic view proceeds primarily by way of the rejection of the ontological view, that is, by way of the rejection of the notion of logical truth. The reason is primarily epistemological. Because experience can teach us only what is, not what must be, logic, which (by hypothesis) concerns necessary features of reality, cannot be an empirical science. It must be a priori. But, Quine argues, there cannot be any a priori knowledge.[3] If the judgments of logic are, as for instance the positivist thinks, a priori because analytic, founded on meanings, hence incorrigible and unrevisable, then they are not and cannot be *true*. Alternatively, if they are true (or false), then they are not a priori, founded on meaning alone, because in that case they can be revised as needed. If it really is impossible to get it wrong (save by merely making a mistake in one's formal reasoning, in one's manipulation of signs according to rules), then there is no objective or truth-evaluable content to a claim at all. If Quine is right, the only truth is empirical truth, the only science empirical science.

Quine argues that there is no sense to be made of the idea that we have a priori access to logically necessary features of reality, and concludes on that basis that logic cannot be a science properly speaking. Its concern is grammatical form. But, as Chateaubriand argues, this cannot be right. On the linguistic conception, logic, like mathematics generally, "is the study of abstract mathematical structures" (II.116). It is essentially no different from, say, abstract algebra. In both cases, one begins with some definitions, say, of a group, or of some truth-functional connectives, and then one proves

[3] See "Two Dogmas of Empiricism".

theorems on the basis of those definitions. As a mathematical investigation, this study would seem to be unproblematic. The problems arise, as Chateaubriand sees, when the *philosopher mis*takes this *mathematical* study as a study that reveals the essence of logic and language. The problem is that "from the point of view of formal languages [of the sort that mathematicians study] there is no reason to single out the usual logical notions as *the* logical notions, nor to interpret them in terms of a prior conception of what logic is about" (II.117). If, on the other hand, this mathematical investigation does reveal the essence of logic and language then, as Chateaubriand argues, it actually presupposes a much richer, properly philosophical conception, one that does focus on truth. In sum, the linguistic conception "is an attempt to have the benefits of something like attributes and propositions, dubious as they may be, without acknowledging them in the ontology" (II.127); it is "an attempt to have the benefits of talk of truth and reality, while claiming that talk of reality is basically empty talk, and that talk of truth is ultimately eliminable in terms of grammatical talk, at least as far as logic is concerned" (II.128).

The concern of logic is not merely any (consistent) system but instead systems that address distinctively logical notions, and these notions are, as Chateaubriand argues, inextricably tied to the notion of truth. The notion of a predicate, for example, is not merely a grammatical notion but is properly semantic and constitutively related to the notion of truth (see II.122-127). The notion of a proof similarly is not merely syntactic or formal, as the model theorist claims, but is instead "an epistemological notion essentially connected to the quest for knowledge, justification and truth" (I.19). Logic must then be conceived as a science, as an inquiry into the (objective) truth concerning matters in its proper domain. When the mathematical logician or model theorist abstracts from content in the course of an investigation into the forms of reasoning, that abstractive activity does not merely yield grammatical forms; it yields *concepts* of grammatical form. According to Chateaubriand, it is these concepts that provide the subject matter of the science of logic: "the logical forms are just the logical Forms, i.e., the logical properties" (II.132).

Chateaubriand argues that logic is a science, that it has a subject matter, and investigates the objective truth regarding its subject matter. It is not and cannot be merely formal in the way the model theorist supposes because insofar as logic concerns valid reasoning it cannot abstract from the notion of truth. Either one is concerned with mere forms in which case one is not studying patterns of valid reasoning, or one is studying such patterns, in which case one's investigations do answer to something, namely, the truth regarding those patterns. But granting that logic is a science, does it follow that it investigates the most general features of reality, the logically necessary features? And if so, how does the investigation proceed given that, as Quine argues, there can be no *truth* on the basis of meanings alone? What is the nature of our cognitive and epistemic access to these logical truths?

One common answer to such a question, at least in mathematics, is that of Gödel and Hardy, both of whom are important influences for Chateaubriand. According to them, we, or at least mathematicians, have a quasi-perceptual capacity to discern mathematical truth with the "mind's eye". Hardy, for instance, writes in "Mathematical Proof":

> I have myself always thought of a mathematician as in the first instance an *observer*, a man who gazes at a distant range of mountains and notes down his observations ... There are some peaks he can distinguish easily, while others are less clear. He sees A sharply, while of B he can obtain only transitory glimpses. At last he makes out a ridge which leads from A, and following it to its end he discovers that it culminates in B. B is now fixed in his vision, and from this point he can proceed to further discoveries ... If he wishes

> someone else to see it, *he points to it*, either directly or through the chain of summits which led him to recognize it himself. When his pupil also sees it, the research, the argument, the *proof*, is finished.[4]

As a phenomenological description of the experience of mathematicians this may be correct. Nevertheless, such a phenomenon does not belong to the *science* of mathematics (nor, for that matter, to the science of logic). Mathematicians may be quite good at "seeing" connections among the concepts under investigation; nevertheless proof is needed if one is to be said to *know* that which one has seen with one's mind's eye. If one does not have a proof then what one has is a conjecture, not a theorem.

The practice of calculating provides a useful analogy. Before the Arabic numeration system, with its algorithms for all the basic arithmetical operations, was developed, there were "natural calculators", people who just could solve arithmetical problems, without having any way of showing others how it was done or why the result was correct. And of course there are still such people today. With the development of the Arabic numeration system, such results came, for the first time, to be testable, and so to constitute knowledge properly speaking. Just the same is true in modern mathematics. Although mathematical intuition can often provide a useful guide to what it is worth trying to prove, and even ideas about how to go about formulating a proof, it is the proof that is the *sine qua non* of mathematical knowledge. There is no *knowledge* by means of the mathematician's quasi-perceptual intuition of truths.

Chateaubriand assumes that if logic is a science then it is a science of logically necessary features of reality. But we have no adequate account of our epistemic access to such features. Does it follow that logic is not a science? That also does not seem to be

[4] Hardy (1929), p. 18. See also Gödel (1983), pp. 483-484.

right. Neither the linguistic view that denies that logic is a science, nor the ontological view that posits some kind of a priori or quasi-perceptual access to fundamental and necessary features of reality, is satisfactory. What is needed is a conception of logic as a science, but one that stops short of the idea that it is a science of logically necessary features of reality. Peirce and Frege together help us to see how this might go.

Over the course of the nineteenth century, developments in mathematical practice seemed decisively to show that Kant was wrong to think that mathematics constitutively involves constructions in pure intuition. According to Peirce, what this shows is not that constructions are not needed in mathematics (as the positivist argues) but instead that even logic, even reasoning from concepts alone, involves constructions. He explains in 'The Logic of Mathematics in Relation to Education' (1898):

> Kant is entirely right in saying that, in drawing those consequences, the mathematician uses what, in geometry, is called a 'construction', or in general a diagram, or visual array of characters or lines. Such a construction is formed according to a precept furnished by the hypothesis. Being formed, the construction is submitted to the scrutiny of observation, and new relations are discovered among its parts, not stated in the precept by which it was formed, and are found, by a little mental experimentation, to be such that they will always be present in such a construction. Thus the necessary reasoning of mathematics is performed by means of observation and experiment, and its necessary character is due simply to the circumstance that the subject of this observation and experiment is a diagram of our own creation, the condition of whose being we know all about.
>
> But Kant ... fell into error in supposing that mathematical and philosophical necessary reasoning are distinguished by the circumstance that the former uses constructions. This is not true. All necessary reasoning whatsoever proceeds by constructions; and the difference between mathematical and philosophical necessary

> deductions is that the latter are so excessively simple that the construction attracts no attention and is overlooked.[5]

On Peirce's view, the lesson of nineteenth century developments in mathematical practice is not that mathematics, which can involve reasoning from concepts alone, is for that reason analytic, rather than synthetic as Kant thought, nor even that it is ampliative despite being analytic as the positivist argues, but instead that reasoning from concepts alone is, like the rest of mathematical practice, synthetic, that is, ampliative, albeit necessary, because even reasoning from concepts involves constructions.

On Peirce's pragmatic conception of logic, logic is an experimental science involving constructions. One experiments with a (drawn) construction in order to discover "new relations ... among its parts". As such logic is, like any science, inherently fallible, a self-correcting enterprise that is properly described as a science because it is self-correcting. Frege takes an essentially similar view.[6] According to him, we do not know the basic truths of logic indubitably or a priori but only by following out their consequences. As he puts the point in the Introduction to *Grundgesetze*, the test of his "logical convictions" as made explicit in the basic laws of his system lies not in their apparent obviousness to us but instead in their consequences, that is, in the theorems that may be derived from them according to the rules Frege has laid out; those logical convictions can be refuted only by "someone's actually

[5] Peirce (1931), p. 350.

[6] As, in a way, does Quine. Nevertheless, as already indicated (and will be further clarified below), there are fundamental differences between Quine's pragmatism and that of Frege and Peirce. For Quine, logic, insofar as it is a science at all, is a part of the web of belief that as a whole answers to the empirical world. As already noted, for him all truth is empirical truth. For Peirce and Frege, logic is a science with its own subject matter; the experiments it involves are mental experiments, not empirical ones.

demonstrating either that a better, more durable edifice can be erected upon other fundamental convictions, or else that my principles lead to manifestly false conclusions".[7] The test of the truth of one's axioms lies not in whether they seem on the face of it to be true but in their consequences. It is for just this reason that belief in mathematics and logic is inherently provisional: "it not only corrects its conclusions, it even corrects its premises".[8]

As I want now very briefly to indicate, the essential difference between this pragmatist conception of logic and Chateaubriand's ontological conception lies in their understanding of logical generality. For Chateaubriand, logical generality is conceived directly in terms of truth, in particular necessary truth. On Frege's view (as I argue in *Frege's Logic*), logical generality is to be

[7] Frege (1964), p. 25. As van Heijenoort puts it, on Frege's view of logic "the only question of completeness [and, we can add, consistency] that arises is, to use an expression of Herbrand's, an *experimental* question. As many theorems as possible are derived in the system. Can we exhaust the intuitive modes of reasoning actually used in science? ... The two volumes of *Grundgesetze der Arithmetik* ... can be regarded as a step in an ever renewed attempt at establishing completeness [and consistency] experimentally." (Van Heijenoort 1967, p. 327.)

[8] Peirce (1992), p. 165. Russell's derivation of a contradiction from Frege's basic laws in *Grundgesetze* is an obvious example of the point. We assume that a logically adequate concept invariably determines an extension, or as Frege would put it, a course of values, but as Russell's paradox shows, it turns out that we were wrong: that assumption leads to a contradiction. For further discussion of the point see Chapter Five of my *Frege's Logic* (2005). See also my "Pragmatism and Objective Truth" (2007), for further discussion of similarities in the views of Frege and Peirce regarding the science of logic.

understood first and foremost in terms of the idea of an inference license, and only derivatively in terms of truth.[9]

The distinction between a rule of inference and a premise is that between something according to which one reasons and something from which one reasons, and as Lewis Carroll argues, any inference must involve both, both premises from which to infer and also a rule according to which one reasons.[10] An inference is in this regard like a journey: as any journey has a starting point, an ending point, and the passage from the one to the other, so an inference has a starting point, the premise or premises, an ending point, the conclusion, and the passage or inference from the one to the other that is governed by the principle or rule according to which one reasons. As Carroll shows, one can make the principle according to which one reasons explicit in one's premises. But if one does, then another principle is required to ground the passage to the conclusion from the newly augmented premises. It is logically impossible to make all one's principles of inference explicit in one's premises. As Ryle has put the point, "conclusions are drawn from premises in accordance with principles, not from premises that embody those principles."[11]

Now, unlike a necessary truth, which can be singular or general, atomic or compound, a principle of reasoning is at once inherently conditional (obviously so given that it governs the passage from a premise to a conclusion) *and* constitutively general. As Peirce explains, "*whenever we draw a conclusion*, we have an idea, more or less definite, that the inference we are drawing is only an example of a whole class of possible inferences, in each of which

[9] Peirce, it should be noted, did not quite win through to this insight about logical generality, despite its centrality to the overall pragmatist project he was pursuing.

[10] Carroll (1895), pp. 278-280.

[11] Ryle (1950), p. 328.

from a premise more or less similar to the actual premise there would be a sound inference of a conclusion analogous to the actual conclusion".[12] Ryle puts the same point in terms of the idea of a statement specification. A statement of an inference warrant, he suggests, is something roughly to the effect that if you have a statement of sort P then you can infer another statement of the sort C. To state a rule of inference is to give, in a kind of dummy inference, the motions one would go through had one the appropriate premises, that is, statements meeting certain specifications.

Suppose, for example, that I infer from the fact that Felix is a cat that Felix is a mammal. The rule or inference license in this case is not merely the particular truth-functional conditional that if Felix is a cat then Felix is a mammal, that is, the truth-function that either Felix is not a cat or Felix is a mammal. Nor can the rule take the form of a quantified generality, that everything in the domain of quantification is a mammal if a cat. Although such a quantified generality could serve as a premise from which to reason, it does not express a principle according to which to reason. It has the form of a fact, not that of a rule or principle of reasoning. Nor finally, and for the same reason, can our rule be expressed as a Fregean generality using the concavity, which, as Chateaubriand notes, functions as a second-level concept. What is needed is something with the form of a *rule*. Following Ryle's suggestion about statement specifications, it is natural to combine a sign for the conditional (an arrow, say) and some kind of dummy sign (*not* a variable) lending the requisite generality, '*', say, thus: '* is a cat → * is a mammal'.[13] This generalized conditional does not state something that is the case; in particular, it does not state that all or any cats are mammals or

[12] Peirce (1992), p. 131.

[13] In Frege's notation one would use his conditional stroke and Latin italic letters to express this judgment. See *Frege's Logic*, Chapter One, for discussion of Frege's use of Latin italic letters.

ascribe any second-level concept to any first-level concepts. It expresses an inference license, a principle according to which to reason, by providing (through the use of the dummy sign '*') what Ryle thinks of as statement specifications. Such a conditional is essentially general; it does not have the form of a fact, even a general fact, about objects.

We have seen that any inference is governed by a leading principle or inference license. But much as there could be no instruction in (say) cookery prior to and independent of the practice of cooking, so, as Ryle argues, there can be no explicitly formulated rules of inference prior to and independent of the practice of inferring.[14] At the most basic level, then, are contingent judgments concerning ordinary objects, and the a posteriori conclusions that can be drawn (according to rules implicit in the language) from such judgments: Felix is a cat; therefore, Felix is a mammal. The next step is to make those hitherto implicit inference warrants explicit in judgments, judgments that are, we have seen, essentially general and inherently conditional in form. (At first, such licenses will be read off the language as it is actually used. Later, in the course of scientific theorizing, laws of this form can be postulated in theories aimed at explaining what is observed. The theorems of an adequate theory of this sort, one that has been successfully axiomatized, are synthetic a priori in Frege's *Grundlagen* sense. They are the laws of a special

[14] "The activities of asserting and following both hypothetical statements [i.e., inference warrants] and explanations are more sophisticated than the activities of wielding and following arguments. A person must learn to use arguments before he can learn to use hypothetical statements. In arguing (and following arguments) a person is operating with a technique or method, i.e., he is exercising a skill; but in making or considering hypothetical statements and explanations he is, for example, giving or taking instruction in that technique or operation, Roughly and provisionally, he is ... not practicing an art but teaching it or receiving tuition in it." (Ryle 1950, p. 333)

science governing inferences about the particular objects there are in the world.)

Once we have such rules in the form of judgments, another form of inference becomes possible, namely, inferences that involve these rules themselves as premises and conclusions, for example, this: * is a cat → * is a mammal, and * is a mammal → * is warm-blooded; therefore, * is a cat → * is warm-blooded. This inference too has its leading principle, and Frege shows us how to express this principle as a judgment.[15] Such a judgment contains no object names and no first-level concept words; it does not express a fact about particular objects, or even a law of a special science. It expresses a law of logic. The laws of logic are in this sense maximally general; they involve no reference either to any objects or to any first-level concepts.

In Frege's logic, any generalized conditional expressed using his Latin italic letters, that is, any rule of inference expressed in a judgment, can serve as a premise for an inference to the corresponding generalized conditional expressed using the concavity notation.[16] That is, for every inference rule there can be inferred a corresponding fact, an ascription of some higher-level concept to lower-level concepts. It is these higher-level concepts that Chateaubriand focuses on, but *without recognizing* that our epistemic access to them is possible only on the basis of a prior grasp of rules of inference that in turn essentially depends, as Ryle argues, on actual practices of inferring. Chateaubriand thinks that he can focus directly on logical

[15] See §3.1 of my *Frege's Logic* for details.

[16] If it is a good rule of inference to infer (say) from a number's being a prime greater than two to the conclusion that that number is not divisible by two without remainder, then it follows that no primes greater than two are divisible without remainder by two, that is, that the concepts *prime number greater than two* and *divisible without remainder by two* are mutually exclusive, that they are related by the second-level relation of exclusion.

truth, but then, as Quine shows, we can have no understanding of our epistemic access to such truth. Truth, as both Peirce and Frege see, can be understood only by way of the striving for truth, and in particular, our inferential practices.

On the pragmatist view, logic is a science. But it is one that is essentially late, possible at all only in light of our developed capacity to reason on the basis of concepts alone, governed by (strictly logical) principles of reasoning that are at first only implicit in our practice. The task of the science of logic is to make those principles explicit in an axiomatization that enables one to test the adequacy of one's formulations of those principles. The science of logic does not require, then, that we have some sort of a priori access to logically necessary truths. What we have is experimental access (through our work on constructions) to the rules governing the inferences we actually make in the course of reasoning. It is in just this way that we steer a course between the linguistic view and Chateaubriand's ontological view. We admit truths of logic without being committed thereby to logical truths.[17]

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[17] This essay has benefited from very useful comments by Norma Goethe.

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