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LOGICAL TRUTH AND LOGICAL STATES OF AFFAIRS: RESPONSE TO DANIELLE MACBETH

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Abstract: Danielle Macbeth disagrees with the view that there are logical truths in an ontological sense, and argues that we have no adequate epistemological account of our access to such features of reality. In my response I recall some main aspects of my ontological and epistemological formulation of logic as a science, and argue that neither Quine's considerations against meaning, nor Benacerraf's considerations against Gödel's realism, show the untenability of an approach to logical truth in terms of logical propositions that denote logical states of affairs.

Keywords: Logical truth. Logical property. Analiticity. Logical state of affairs. Predication.

VERDADES LÓGICAS E ESTADOS DE COISAS LÓGICOS: RÉPLICA À DANIELLE MACBETH

Resumo: Danielle MacBeth discorda da tese que há verdades lógicas em um sentido ontológico e argumenta que não há um tratamento adequado de nosso acesso epistêmico à tais aspectos da realidade. Em minha réplica relembro algumas características principais de minha formulação ontológica e epistemológica da lógica como ciência, e argumento que nem as considerações de Quine contra a noção de significado, nem as considerações de Benacerraf contra o realismo de Gödel, mostram a invalidade de uma concepção de verdade lógica em termos de proposições que denotam estados de coisa lógicos.

Palavras chave: Verdade lógica. Propriedade lógica. Analiticidade. Estado de coisas lógico. Predicação.

Danielle disagrees with the view that there are logical truths in an ontological sense. To my characterization of logical truths as "logical propositions whose parts denote logical properties that combine necessarily into a logical state of affairs" (p. 253), she objects that "we have no adequate account of our epistemic access to such [logically necessary] features [of reality]" (p. 57). She argues that Quine's challenge "there can be no *truth* on the basis of meaning alone" (p. 56) is not met by an "ontological view that posits some kind of a priori or quasi-perceptual access to fundamental and necessary features of reality" (p. 58)—a view she attributes to Gödel and Hardy, both of whom, as she says, are important influences on me.

1. LOGICAL STATES OF AFFAIRS

Although the view Danielle attributes to me about the ontological character of logical truths and logically necessary facts is quite accurate, our disagreement is not merely as to what follows from logic being a science—i.e., about the conditional statement "if logic is a science (that is, there are truths of logic), there must be logical truths, logically necessary facts" (p. 52)—for her questioning reflects a much deeper disagreement as to what it is for logic to be a science. Her "pragmatist conception of logic", which is developed at some length in the second half of her paper, derives from Peirce and from her interpretation of Frege, and maintains that "logical generality is to be understood first and foremost in terms of the idea of an inference license" (p. 61), rather than in ontological terms.

What I propose, on the other hand, is an ontological and epistemological view of logic as a science, with the concept of truth playing a central metaphysical role for both aspects. To this end I first develop an ontological theory of states of affairs and a theory of truth as identification of states of affairs. States of affairs are part of an ontological hierarchy—inspired by ideas of Plato, Frege, Russell, and Gödel—that also includes particulars and properties. My conception of properties is quite broad, comprising relations of all arities (finite and infinite), cumulative properties, multigrade properties, etc. In particular, I distinguish a category of logical properties, which are properties of a very general character that appear throughout the hierarchy of levels—properties such as Subordination, Identity, Universal Quantification, Reflexivity, Transitivity, and so on.

A state of affairs is a combination of a property of a certain type with entities of appropriate types. Thus, the state of affairs that Socrates was a teacher of Plato may be conceived as a combination of the level one binary property *is-a-teacher-of* with the particulars Socrates and Plato, in that order. Similarly, the state of affairs that all dogs are animals may be conceived as a combination of the level two binary property *is-subordinate-to* with the level one unary properties *is-a-dog* and *is-an-animal*, in that order. In this case the property of Subordination is a logical property that combines with two non-logical properties into a (non-logical) state of affairs. But there is no reason that a state of affairs should not involve exclusively logical properties, and these are the logical states of affairs. An example is the level two state of affairs that first-order binary Identity is reflexive, since both level one binary Identity, and level two Reflexivity are logical properties of the appropriate types.

I also characterize propositions ontologically as properties that are complex combinations of other properties (senses). Again, the properties combined may be logical or non-logical, and a logical proposition is one that consists exclusively of logical properties. A true proposition is one that identifies—or is instantiated by—a state of affairs. A logical truth is a logical proposition that identifies a logical state of affairs.

The epistemological component of my views is the account of knowledge, justification, and proof in chapters 19-25. There I suggest some main lines of a non-aprioristic (fallibilistic) epistemology that would be appropriate for my view of logic, mathematics, science, and our ordinary concerns. This is the view of knowledge as truth justified beyond a reasonable doubt. For logic and mathematics, I hold that the notion of proof is the most central epistemological notion, and try to come to grips with it from several different points of view. But I also hold there may be innate components in our apprehension of such logical notions as, for instance, Identity and Difference, Negation, and Instantiation. Although I do not present my epistemological views as systematically as my ontological views, and in many of the epistemological chapters I am "feeling my way" rather than presenting a fully developed view, I do not think they are correctly characterized as aprioristic and as based on a quasi-perceptual access to reality.

In any case, Danielle's main objection to my view of logical truth is that I do not develop a credible account of our knowledge of logical facts. This is related to Benacerraf's objection to Gödel's Platonism, which I discuss at some length in my paper "Platonism in Mathematics"—although Danielle refers to Quine's views and not to Benacerraf's. In what follows I will discuss some of the issues she raises.

2. ANALYTIC TRUTHS

Although I discuss Quine's indeterminacy of translation argument in Chapter 13 (pp. 32-41), I do not examine the arguments in "Two Dogmas". I take the indeterminacy argument to be Quine's main formulation of his attack on notions of meaning, analyticity, synonymy, etc., and I see it as being essentially a response to objections by Carnap, Mates, and others to the arguments in "Two

Dogmas". My conclusion is that the indeterminacy argument, and with it Quine's attack on meaning, is actually rather vacuous, because it neither has a solid empirical basis nor has a well-developed theoretical content.

I did not argue in my book that our epistemic access to logical truth derives from their analyticity, but I am prepared to do so for certain logical (and non-logical) truths. Gödel (1944, p. 151) also accepts a notion of analyticity in virtue of meaning, and claims that, with the exception of the axiom of infinity, the axioms of *Principia Mathematica* are analytic under certain interpretations of the concepts occurring in them. In fact, later in life, Quine himself acknowledged the analyticity of the logical truths of elementary logic.[1]

One of my examples of a logical truth is the reflexivity of identity I mentioned above. In Chapter 18 (p. 251) I describe it as follows:

> It seems reasonable to assume that logical properties either combine or do not combine necessarily. For example, level 2 Reflexivity for level 1 binary relations and level 1 Identity combine into a logical state of affairs. One can think of the formula
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> (8) $\forall x\, x{=}x$,
>
> or, in the notation of Chapter 6,
>
> (9) $[[\forall x Zxx](Z)]([x{=}y](x, y))$,
>
> as determining (or denoting) that state of affairs. And one could say that (8), or (9), is a logical truth in this sense of denoting a logical state of affairs.

The way I see it is that the reflexivity of identity is an analytic truth, in the traditional sense that the concept of the predicate is contained in the concept of the subject. (It is important to realize that the

[1] Bergström and Føllesdal (1994, pp. 71-72).

predicate in (9) denotes a property in the intensional sense, and, therefore, that the quantifier is not treated "extensionally".)

In fact, since I defend the view that all propositions (sentences, statements, etc.), formal or informal, have subject-predicate structure, the traditional notion of analyticity applies quite generally. Thus, just as the proposition asserting the reflexivity of the identity relation is analytic, so is the proposition asserting the one-to-one-ness of the successor relation among natural numbers, as is the proposition asserting the symmetry of the sibling relation among human beings. And I take it that our epistemic access to these analytic truths derives from our apprehension of the notions involved. But it does not follow from this that we have *a priori* knowledge,[2] or that our knowledge claims are infallible, as Danielle suggests. In fact, in chapters 22-25 I argue quite strongly for fallibilism.[3]

3. PROOF

Of course, our epistemic access to logical and mathematical truth is not generally by direct apprehension, but proceeds by means of proof. Everybody agrees to this, including Gödel and Hardy. I think the passage from Hardy (1929) quoted by Danielle (p. 56-57) is misleading in this respect. He is not arguing there for a quasi-perceptual access to mathematical truth, but for the phenomeno-

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[2] Although we may say we have *analytic knowledge*, in that our knowledge derives from our understanding (including definitions) of the concepts involved. In this sense, I always thought it misleading for Kripke (1980, pp. 56-57) to characterize as *a priori* the knowledge of the reference fixer that the stick he used to fix the reference of 'one meter' at time t_0 is one meter long at time t_0. It seems much more appropriate to characterize it as analytic knowledge, based on the introduction of the expression 'one meter' at time t_0.

[3] Otávio Bueno gives an accurate summary of my position in section 4 of Bueno (2008).

logical character of the actual process of proof and of its transmission. The 'seeing' to which Hardy appeals is not a quasi-perceptual seeing but a seeing of the understanding, informed by background knowledge and previous experience. The 'chain of summits' are the steps in a proof, which, as I argue at length in my book, is relative to the knowledge and experience of the audience to whom it is addressed—including the case of a proof (or discovery) addressed to oneself. Thus, a few pages later Hardy remarks (p. 23):

> This is plainly not the whole truth, but there is a good deal in it. The image gives us genuine approximation to the processes of mathematical pedagogy on the one hand and of mathematical discovery on the other ...

And in another passage I quote from Hardy (1940, p. 16) he formulates the conventional (mathematical) view of proof:

> I am not going to get entangled in the analysis of a particularly prickly concept, but I think that there are a few points about proof where nearly all mathematicians are agreed. In the first place, even if we do not understand exactly what a proof is, we can, in ordinary analysis at any rate, recognize a proof when we see one. Secondly, there are two different motives in any presentation of a proof. The first motive is simply to secure conviction. The second is to exhibit the conclusion as the climax of a conventional pattern of propositions, a sequence of propositions whose truth is admitted and which are arranged in accordance with rules. These are the two ideals, and experience shows that, except in the simplest mathematics, we can hardly ever satisfy the first ideal without also satisfying the second. We may be able to recognize directly that 5, or even 17, is prime, but nobody can convince himself that
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> $$2^{127-1}$$
>
> is prime except by studying a proof. No one has had an imagination so vivid and comprehensive as that.

In Gödel's case the situation is more complex, for he does explicitly postulate a notion of mathematical intuition that is a mathematical counterpart of the physical notion of perception. This does not mean, however, that Gödel would not agree with the remarks about proof I just quoted from Hardy. I am quite sure he would, because mathematical intuition for him does not give a direct access to the mathematical and logical aspects of reality. He says (Gödel 1947, p. 484):

> It should be noted that mathematical intuition need not be conceived as a faculty giving an immediate knowledge of the objects concerned. Rather it seems that, as in the case of physical experience, we form our ideas also of those objects on the basis of something else which is immediately given. Only this something else here is not, or not primarily, the sensations. That something besides the sensations actually is immediately given follows (independently of mathematics) from the fact that even our ideas referring to physical objects contain constituents qualitatively different from sensations or mere combination of sensations, e.g., the idea of object itself, whereas, on the other hand, by our thinking we cannot create any qualitatively new elements, but only reproduce and combine those that are given. Evidently the "given" underlying mathematics is closely related to the abstract elements contained in our empirical ideas. It by no means follows, however, that the data of this second kind, because they cannot be associated with actions of certain things upon our sense organs, are something purely subjective, as Kant asserted. Rather they, too, may represent an aspect of objective reality, but, as opposed to the sensations, their presence in us may be due to another kind of relationship between ourselves and reality.

In fact, for Gödel the notion of proof enters into our justification of mathematical and logical principles not only in the standard way described by Hardy, but also in a more indirect way akin to the justification of physical principles. He derives this idea from Russell's justification of the Axiom of Reducibility in *Principia Mathematica*, and argues (Gödel 1947, p. 521):

> ... even disregarding the intrinsic necessity of some new axiom, and even in case it has no intrinsic necessity at all, a probable decision about its truth is possible also in another way, namely, inductively

> by studying its "success". Success here means fruitfulness in consequences, in particular in "verifiable" consequences, i.e., consequences demonstrable without the new axiom, whose proofs with the help of the new axiom, however, are considerably simpler and easier to discover, and make it possible to contract into one proof many different proofs. … There might exist axioms so abundant in their verifiable consequences, shedding so much light upon a whole field, and yielding such powerful methods for solving problems … that no matter whether or not they are intrinsically necessary, they would have to be accepted at least in the same sense as any well-established physical theory.

This is a central feature of Gödel's epistemology for mathematics (and logic), and it shows clearly it is not an aprioristic epistemology —a point emphasized by Bernays (1946) in his review of Gödel's paper.[4]

My view, therefore, is that neither Quine's arguments against meaning, nor Benacerraf's arguments against Gödel's realistic views show the untenability of a realistic approach to logical truth. What I tried to do in my book is to develop a basis for both the ontological and the epistemological aspects of such a realistic approach.

REFERENCES

BENACERRAF, P. "Mathematical Truth". *The Journal of Philosophy*, 70, pp. 661-679, 1973. Reprinted in Benacerraf and Putnam, 1983.

BENACERRAF, P., PUTNAM, H. (eds.). *Philosophy of Mathematics: Selected Readings*. Englewood-Cliffs, NJ: Prentice-Hall, 1964. Second edition, Cambridge: Cambridge University Press, 1983.

[4] This led Bernays to reconsider his arguments in Bernays (1935) concerning the impossibility of a strong form of Platonism as a philosophy of mathematics.

BERNAYS, P. Review of Gödel 1944. *The Journal of Symbolic Logic*, 11, pp. 75-79, 1946.

BERNAYS, P. "Sur le Platonisme dans les Mathématiques". *L'Enseignement Mathématique*, 34, pp. 52–69, 1935. Translated as "On Platonism in Mathematics" in Benacerraf and Putnam.

BERGSTRÖM, L., FØLLESDAL, D. "Interview with Willard Van Orman Quine in November 1993". *Theoria*, 60, pp. 65-78, 1994.

BUENO, O. "Truth and Proof". *Manuscrito*, 31, pp. 419-439, 2008.

CHATEAUBRIAND, O. "Platonism in Mathematics". *Manuscrito*, 28, pp. 201-230, 2005.

GÖDEL, K. "Russell's Mathematical Logic". In P. A. Schilpp (ed.). *The Philosophy of Bertrand Russell*, pp. 126-153. Evanston and Chicago: Northwestern University Press, 1944. Reprinted in Benacerraf and Putnam.

GÖDEL, K. "What is Cantor's Continuum Problem". *American Mathematical Monthly*, 54, pp. 515-525, 1947. Revised and expanded in Benacerraf and Putnam.

HARDY, G. A. "Mathematical Proof". *Mind*, 38, pp. 1-25, 1929.

HARDY, G. A. *Ramanujan*. Cambridge: Cambridge University Press, 1940.

KRIPKE, S. *Naming and Necessity.* Cambridge, Mass.: Harvard University Press, 1980.

QUINE, W. V. "Two Dogmas of Empiricism". *The Philosophical Review*, 60, pp. 20-43, 1951.