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CHATEAUBRIAND ON PROPOSITIONAL LOGIC

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Abstract: In *Logical Form*s *Par*t *II*, Chateaubriand begins the Chapter on "Propositional Logic" by considering the reading of the 'conditional' by 'implies'; in fact he states that:

> *There is a confusion, as a matter of fact, and it runs deep, but it is a confusion in propositional logic itself, and the mathematician's reading is a rather sensible one.*

After a careful, erudite analysis of various philosophical viewpoints of (two-valued propositional) logic, Chateaubriand comes to the conclusion that:

> *Pure propositional logic, as just characterized, belongs to ontological logic, and it does not include a theory of deduction as a human activity. This is a part of epistemological logic, and is more closely connected to the applications of pure propositional logic.*

An implicit assumption in Chateaubriand's reasoning appears to be that propositions (logic, number, etc.) have a timeless status. I will present arguments for the opposite viewpoint which leads to an analysis of Propositional Logic not covered under Chateaubriand's monograph and perhaps resolves some conflicts therein; much as the conflict between the Intuitionist and Classical Mathematician on whether every function on the Reals is continuous is resolved by the realization that they are talking about different "entities".

Keywords: Propositional logic. Extended propositional logic. Number theory. Set theory.

CHATEAUBRIAND SOBRE A LÓGICA PROPOSICIONAL

Resumo: Em *Logical Forms* II, Chateaubriand inicia o capítulo "Lógica Proposicional" considerando a leitura do 'condicional' como 'implica'. De fato, ele diz o seguinte:

> *Na verdade, existe uma confusão, e ela é profunda, mas é uma confusão na lógica proposicional ela mesma, e a leitura de um matemático é bastante sensível.*

Depois de uma análise cuidadosa e erudita dos vários pontos de vista filosóficos da lógica (proposicional bivalente), Chateaubriand chega à conclusão que:

> *A lógica proposicional pura, tal como aqui caracterizada, pertence à lógica ontológica, e não inclui uma teoria da dedução como atividade humana. Isto é parte da lógica epistemológica, e é mais intimamente conectada às aplicações da lógica proposicional.*

Uma premissa implícita no raciocínio de Chateaubriand parece ser a de que proposições (assim como a lógica, os números, etc.) têm um estatuto atemporal. Eu argumentarei em favor da visão oposta, que leva a uma análise da Lógica Proposicional não abordada no texto de Chateaubriand e que talvez resolva alguns conflitos. Muito do conflito entre Intuicionistas e Matemáticos Clássicos sobre se toda função sobre os números reais é contínua é resolvido pela compreensão de que eles estão falando de "entidades" diferentes.

1. LAWS ARE NOT ETERNAL

In my schooldays (in Shropshire, England) the "Laws" of Newtonian mechanics had the same status as, say, the Ten Commandments. In fact, whenever the word "Law" was attached to a physical equation, we were to understand that the equation was a timeless truth not open for any further discussion. The extraordinary success[1] of Newtonian mechanics on terrestrial and extraterrestrial situations entrenched the timelessness of all such "Laws".

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[1]That is, the agreement of theoretical prediction with experimental evidence.

Towards the end of the nineteenth century observations were made that could not be explained by Newtonian methods; for example, the amount[2] of the precession of the perihelion of the planet Mercury. It took the advent of Einstein's theories of Relativity, based on quite distinct principles, to "explain" the discrepancy and yet maintain all the successes of Newtonian mechanics.

Einstein's theories did not only account for the additional 43 seconds of arc per century, but in addition showed that a "*physica*l *law*" was no longer (necessarily) a timeless, universal truth but rather either (i) an oft recurring observable event or (ii) a theory which could be used to predict or explain (lots of) observable events. Since the word "*Law*" has either Biblical or legal connotations, suggesting that disobedience of them have dire consequences, scientists nowadays often use "*Theory*"[3] instead.

The challenge to "Laws" in Logic and Mathematics took a little longer to arrive. Although he was not the first to challenge them, L. E. J. Brouwer was the first to challenge them and to propose an alternative. In [1] we find:

> *The historical development of the mechanism of mathematical thought is naturally closely connected with the modifications which ... have come about in the prevailing philosophical ideas firstly concerning the origin of mathematical certainty, secondly concerning the delimitation of the object of mathematical science.*
>
> ...
>
> *Exact knowledge of these properties was called mathematics, and was generally pursued in the following way: for some regularities of (outer or inner) experience which, with any attainable degree of approximation,*

[2] The discrepancy is of 43/3600 degrees per century.

[3] This practice fits in with Wolfgang Pauli's comment "*This [theory] isn't right. It is not even wrong*".

> *seemed invariable, complete invariability was postulated. These regularities were called axioms...*
>
> ...
>
> *During the observational period mathematics was considered functionally, if not existentially, dependent on logic, and logic itself was considered autonomous.*

Although Brouwer disregarded Logic in his development of Intuitionistic Mathematics, later philosophers introduced an "Intuitionistic Logic" in which the *Law of the Excluded Middle* had lost its status as a *Law*.

1.1. Laws for Numbers

Perhaps because no lesser authority than Kronecker had decreed:

> *Die ganzen Zahlen hat Gott gemacht, alles andere ist Menschenwerk.*[4]

the Natural numbers maintained their timelessness even though no one knew *what they were*; what was known were *properties* about—or between—them[5].

It is only lately that there have been challenges to the timelessness of numbers; a very readable introduction to those challenges can be found in J. N. Crossley's *The Emergence of Number*, [2]. In the Prologue we find:

[4] "God made the integers, all the rest is men's work." References obtained from [2].

[5] For example that $(2^{67} - 1)$ is composite—i.e. not a prime—.

> *That numbers have a timeless status is a view held both by most mathematicians and the world at large. It is a view that we shall challenge. ... it will also appear in the course of this book that what a number is and what numbers there are at any given point in history also depend on the state of knowledge at that time and on what human beings have done. ... the concepts of number have continually developed from their earliest beginnings.*
>
> *Even the simplest of numbers, the natural numbers 1, 2, 3, ... will be seen to have emerged only slowly into the abstraction we have today. ... Certainly the view that they are innate has been held by distinguished anthropologists since the beginning ... On reflection they appear in a very different light as a phenomenon which has slowly grown and developed as the need has arisen.*
>
> ...
>
> *Already one can see that they are not innate, though they are cultural universals*[6]*, they are present to a greater or lesser extent in all cultures.*
>
> ...
>
> *... We shall see the precise rôle of infinite processes in the description of the real numbers. We shall also see that what appears intuitively obvious actually requires a new axiom and that the connexion between arithmetic and geometry which concerned the ancient Greeks has not yet been completely resolved.*

There is one point on which there is universal agreement; namely that it was a great cultural event[7] when *Hom*o *sapien*s realized that there was an object, namely a number, that could be used to count both '*a pair of doves*' and '*a brace of greyhounds*'.

[6] References omitted.

[7] Of the same, or greater, magnitude than the one that takes place in S. Kulbrick's movie "*2001*".

2. PROPOSITIONS

It would appear sensible that in determining what is *Propositional Logic* or the *Logical theory of propositions*, one should first determine what is a *proposition*. However that has not been the case in most areas; for example, in a course in Number theory the last thing one discusses—if ever—is *what is a number*. In Calculus courses the Real numbers are defined as points on the Real line (and the Real line is the collection of Real numbers); interestingly that much of an "understanding" of the ontology of the Real numbers suffices in order to develop modern Analysis with all its practical techniques such as carbon dating, designing bridges or planning the trajectories for Mars' probes.

Analogously in Propositional logic a *propositio*n is often defined as a declarative statement, something like *Snow is white*. Then it is "obvious" that you can form the conjunction, conditional *etc*. of propositions obtaining further propositions.

Once that much has been accepted about propositions (or declarative sentences) one can proceed as, for example, in the book: *Logic. The Techniques of Formal Reasoning* by D. Kalish and R. Montague, in which it is shown how to construct logically valid arguments[8], and how to, sometimes, show that an argument is not valid.

And just as in the case of numbers, not knowing what exactly we are talking about, has not prevented Propositional logic in being applicable in our daily lives of the XXI*st* Century.

One of the most visible applications of Propositional Logic is the ubiquitous computer and it is rewarding to see that Computers have helped in clearing up some problems that philosophers had been considering long before there were computers. A particularly

[8] Where an argument is non-empty sequence of sentences in which the last sentence is the *conclusio*n and the other ones the *premises*.

simple example is the traditional problem between *use and mention*; a modern student who has doing a little computing programming would never confuse the *variable*—or *identifier*—with its content[9] .

Of course Logicians and Philosophers are not satisfied in knowing that something is useful, they must find out *why* it is so. And this usually involves trying to explain what you are talking about. For example in Russell [3], in the Chapter "*On propositions: what they are and how they mean*", we find:

> A PROPOSITION *may be defined as*: What we believe when we believe truly or falsely. ... *I take it as evident that the truth or falsehood of a belief depends upon a fact to which the belief 'refers'. Therefore it is well to begin our inquiry by examining the nature of facts.*

I do not believe that Russell would have approved of the following consequence of the explication given above, namely that the nature and existence of *propositions* depend on *Homo sapiens* and thus may not be timeless entities.

3. PROPOSITIONS IN HISTORY

Unlike for Numbers, we do not have any Upper Palaeolithic bones to suggest that Man was making use of propositions about 20,000 years ago. On the other hand he must have used some kind of elementary reasoning since he obtained his food by hunting and thus must have had to think along the lines of:

> "If *the wildebeest goes in such a direction*, then *I should do ...* ".

[9] The Computer Scientist sometimes goes to far; I remember reading in a CS textbook that *to exist* is *to be a location in memory*. I suppose similar views were contained in the movie "*Matrix*".

However even such simple[10] conditionals would be different from the ones in *Principia Mathematica* since they would be interpreted as time dependent and would be undefined (or irrelevant) for the Palaeolithic man when the antecedents are not satisfied.

Probably one of the earliest recordings of conditionals must have been in the famous *Code of Laws* of Hammurabi[11] since it contains the punishments for various transgressions; in other words:

"If *you do* ... then *you will be* ..."

Those Babylonian's conditionals have a similarity (or as Wittgenstein might have said: *have a family resemblance*) to the conditionals of Classical Propositional Logic; but are not identical to them.

There is a quotation from Wittgenstein in [2] concerning the concept of *number* which, I believe, could also be applied to the concept of *proposition*:

> *... And we extend our concept of number as in spinning a thread we twist fibre on fibre. And the strength of the thread does not reside in the fact that some fibre runs through its whole length, but in the overlapping of many fibres. But if someone wished to say: "There is something common to all these constructions — namely their disjunction of all the common properties" — I should reply: Now you are only playing with words.*

4. ON SYMBOLIZATION OF PROPOSITIONS

If we consider a proposition to be like a thread of propositional fibres, then it is very unlikely that any one symbolization would capture all aspects of the Calculus of Propositions.

[10] It is doubtful that he would have considered iterated conditionals.
[11] Sumeria, ca. 1700 BCE.

On the other hand there is no doubt that there are extremely successful[12] formalizations of Propositional Calculi. However none of those formalizations can (or are likely to) be the complete formalization to the *thread proposition*.

So what would be a reasonable strategy to enlarge the scope of those successful formal Propositional Calculi? The Einsteinian/ Newtonian theories give us a precedent.

The essential points about the *new* theory are that (i) the *new* theory should be compatible with the *old* one, and (ii) new methods, concepts or operations be introduced.

How is the formalization of the new theory to be obtained? Basically as all formalizations have been obtained in the past; or to quote Brouwer's previous quote:

> ... *for some regularities of (outer or inner) experience which, with any attainable degree of approximation*, seemed invariable, complete invariability was postulated. *These regularities were called* axioms...

4.1. An Example of a New Theory

Let us take the Extended Intuitionistic Propositional Calculus with the Universal Quantifier '$\wedge$' and the biconditional '$\equiv$' as its only primitive terms, as the *old theory*[13].

The fundamental idea of the "new theory"[14] is that

- *a proposition may be* subordinate *to another proposition*.
- *The statement that a proposition P is subordinate to the proposition Q is itself a proposition.*

[12] Both practically and theoretically.

[13] It is well known that it is **not** equivalent to the Extended Intuitionistic with all the traditional connectives.

[14] Formally it isn't new.

Let use the symbol '$\lhd$' to represent *subordination*. Also let '0' be an abbreviation for '$\wedge pp$' and '$\mathbf{1}$' be an abbreviation for '$\wedge p(p \equiv p)$'.

Then as axioms for $\lhd$ take:

$$(0 \lhd \mathbf{1}) \equiv \mathbf{1}$$

$$\wedge p\ (\mathbf{1} \lhd p) \equiv 0$$

$$\wedge p\ (p \lhd 0) \equiv 0$$

The axioms attempt to say that:

- *The FALSE is subordinate to the TRUE.*
- *The TRUE is not subordinate to any proposition.*
- *No proposition is subordinate to the FALSE.*

Then define the CONJUNCTION of p and q by the formula:

$$\wedge r \left[p \equiv [\wedge s\ (p \equiv (s \lhd r)) \equiv \wedge s(q \equiv (s \lhd r))]\right]$$

In the new theory we can prove that CONJUNCTION has the usual properties of the traditional connective. And once we have the connective for conjunction we can then obtain all the remaining traditional connectives.

This new theory is more than a variation on Tarski's fundamental result of 1923 since by the addition of one axiom schema and finitely many axioms[15] we can obtain well known impredicative set theories.

[15] Without any additional primitive terms.

REFERENCES

[1] BROUWER, L. E. J. "Historical background, principles and methods of Intutitionism". *South African Journal of Science*, pp. 139-146, 1952.

[2] CROSSLEY, J. N. *The Emergence of Number*. New Jersey: World Scientific, 1987.

[3] RUSSELL, B. *Logic and Knowledge. Essays 1901-1952*. New York: George Allen & Unwin Ltd, 1956.