PROPOSITIONAL LOGIC:
RESPONSE TO KEN LÓPEZ-ESCOBAR

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Abstract: Ken López-Escobar questions the timeless status of various entities—propositions, numbers, etc.—as well as my characterization of pure propositional logic as an ontological theory. In my response I argue that my characterization of propositional logic does not depend on timeless propositions, or on other abstract truth bearers, but is a characterization in terms of truth relations between any truth bearers. I also discuss his views on numbers as cultural constructs, as well as his use of quantification in propositional logic.

Keywords: Propositional logic. Truth relations. Propositions. Numbers. Quantification.

LÓGICA PROPOSICIONAL:
RÉPLICA À KEN LÓPEZ-ESCOBAR

Resumo: Ken López-Escobar questiona o estatuto atemporal de vários entes—proposições, números, etc.— assim como minha caracterização da lógica proposicional pura como teoria ontológica. Na réplica argumento que minha caracterização não depende de proposições atemporais, ou de outros portadores de verdade abstratos, mas é uma caracterização em termos de relações de verdade entre quaisquer portadores de verdade. Examino também suas considerações sobre números como construtos culturais, assim como seu uso de quantificação na lógica proposicional.

Palavras chave: Lógica proposicional. Relações de verdade. Proposições. Números. Quantificação.

Ken questions the timeless status of various entities—propositions, numbers, etc.—as well as my characterization of pure propositional logic as an ontological theory. At the end of the paper he outlines an interesting and powerful theory based on the Extended Propositional Calculus. I will first discuss his arguments concerning abstract entities, and then comment on his use of quantification in propositional logic.

1. PROPOSITIONAL LOGIC AS AN EXTENSIONAL THEORY

The second passage Ken quotes from my book is misleading in suggesting that whereas ontological logic is a purely abstract theory about timeless entities, a theory of deduction is a human activity about concrete entities.

Although I do take propositions to be timeless abstract entities, which I characterize as senses,[1] my ontological treatment of propositional logic in Chapter 16 does not depend on treating propositional logic as a theory of abstract propositions. On the contrary, it applies to any truth-bearers, whether abstract propositions, thoughts, beliefs, sentences, statements, utterances, or anything else that can properly be said to be true or false. The basic point of my account is that classical propositional logic is a theory of truth, falsity, and truth-relations between truth-bearers. I argue also that the structure of the truth-bearers is totally irrelevant to propositional logic in this ontological sense, and reject the claim by Quine, Mates, and others, that since propositions (statements, thoughts, beliefs, etc.) do not have a clear structure, they should be replaced by sentences as the subject matter of propositional logic.[2]

A logical theory of deduction, on the other hand, does depend on the structure of the entities to which it applies. One way in

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[1] See Ruffino (2008) and my response.

[2] These are the conclusions summarized in Chapter 16 (p. 183).

which we can formulate the difference is that whereas as an ontological theory of truth-relations between truth-bearers, propositional logic is a purely extensional theory, depending exclusively on truth-values, as a theory of deduction it is an intensional theory, with rules of inference formulated in structural terms.

What I mean by "a purely extensional theory" in this context is explained in some detail on pp. 168-182. The basic idea is that what matters to negation, conjunction, disjunction, etc., are not some structural relations between the truth-bearers, but only their truth-values. Consider negation, for instance. We normally assume that a truth-bearer that negates another truth-bearer must include some "part"—such as the word 'not' in a sentence—which "expresses the negation operation". This is the main reason for insisting that sentences are more appropriate than other truth-bearers as the subject matter of propositional logic. For, if we don't have a clear account of the structure of propositions, thoughts, beliefs, etc., then how do we identify the expression of the negation operation? My view is that this objection is entirely beside the point, and that *any* false truth-bearer negates *any* true truth-bearer, and *vice-versa*. What we have in the ontological treatment of classical propositional logic is a *negation relation* operating exclusively on truth-values—and, of course, this extends to all the other propositional "connectives".

2. LAWS AND THEORIES

I certainly agree with Ken that our theories and their laws are revisable in the light of new developments. I also agree with his claim, supported by the quotations from Crossley (1987), that the emergence of number "was a great cultural event". The question, however, is whether the fact that the notion of number is a human

cultural development has any bearing on the status of numbers as entities.

In Chapter 20 (p. 318, note 8) I reject as fallacious the implication that because mathematics is a product of human socio-cultural development, its objects do not have an independent existence. My argument was simply that *all* our theories—be they mathematical, or physical, or biological, or of any other kind—are products of our cultural development, and it does not follow from this that their objects do not have an independent existence. Anthropological questions about the genesis of the notion of number are very interesting, and are still surrounded by a fair amount of mystery and controversy, but I consider it a fundamental error to argue, as Crossley does in the passage quoted by Ken, that "what numbers there are at any point in history depend on the state of knowledge at that time". Substitute other terms in place of 'numbers'—'physical objects', 'planets', 'elementary particles', 'biological species', etc.—and consider whether the resulting statement follows from the fact that the corresponding notions are the result of cultural development.

It is interesting, in this connection, that Ken appeals to Brouwer's views as a main challenge to the status of traditional mathematical and logical laws. Brouwer does indeed question some of these laws, and argues that their postulation was a historical mistake, but if we take Brouwer's idealistic views seriously, there were no numbers before *he* created them—which I do not expect to be very convincing to anthropologists. Brouwer's position is philosophical, and derives from a specific idealistic and mystical conception of reality.

3. EXTENDED PROPOSITIONAL LOGIC

The very interesting theory Ken briefly outlines at the end of his paper has been presented in several of his publications—see, e.g., his (2005)—and it is not my purpose to discuss it here. As I argue in Chapter 16 (pp. 153-156), however, the use of quantification in propositional logic seems to me to raise difficulties of interpretation, and I will recall some of my misgivings.

Since quantifiers normally operate on predicates and not on terms, expressions such as '$\forall pp$' and '$\forall p(p \equiv p)$' do not have a natural interpretation. Thus, unless one reads '$\forall pp$' as involving an implicit predicate 'is true' (or 'is the case'), and reads '$\forall p(p \equiv p)$' as asserting that every proposition is materially equivalent to itself, it is not clear to me what these formulas mean. Ken introduces '0' as an abbreviation for '$\forall pp$' and '1' as an abbreviation for '$\forall p(p \equiv p)$', referring to these as '*the FALSE*' and '*the TRUE*', respectively. He also introduces the notion of *subordination*, expressed by the symbol '$\lhd$', which would naturally be interpreted as a relation—but this does not mesh with its use in a formula such as '$\forall p((1 \lhd p) \equiv 0)$', where '$1 \lhd p$' plays the role of a term in the biconditional.

Although Ken does interpret his symbolism through the English readings he suggests, I find these readings somewhat problematic. Let me illustrate with his first axiom:

(*a*) $((0 \lhd 1) \equiv 1)$,

which is read as

(*b*)   *The FALSE is subordinate to the TRUE.*

It would appear, however, that

(*c*) $0 \lhd 1$

should also be read as (*b*), because '0' and '1' are abbreviations for two propositions, and, according to an earlier formulation we have

(*d*) *The statement that a proposition P is subordinate to a proposition Q is itself a proposition.*

Why, then, do we need to state the first axiom using the biconditional '≡'? Is it because '$\lhd$' is not a relational symbol that (*c*) does not express the relation expressed by (*b*)? But since '≡' is not a relational symbol either, the same could be said of (*a*).

Thus, although I am sure that formally it all makes sense, I have some difficulties giving a precise meaning to these formulas.

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