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CHATEAUBRIAND ON LOGICAL TRUTH AND SECOND-ORDER LOGIC: REFLECTIONS ON SOME ISSUES OF *LOGICAL FORMS II*

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Abstract: In this short paper I am concerned with basically two especially important issues in Oswaldo Chateaubriand's *Logical Forms II*; namely, the dispute between first- and higher-order logic and his conception of logical truth and related notions, like logical property, logical state of affairs and logical falsehood. The first issue was also present in the first volume of the book, but the last is privative of the second volume. The extraordinary significance of both issues for philosophy is emphasized and, though there is a basic agreement with Chateaubriand's views, some critical remarks are interspersed.

Keywords: First-order logic. Second-order logic. Logical truth. Logical falsity. Logical property.

CHATEAUBRIAND SOBRE VERDADE LÓGICA E LÓGICA DE SEGUNDA ORDEM: REFLEXÕES SOBRE ALGUMAS QUESTÕES EM *LOGICAL FORMS II*

Resumo: Neste pequeno artigo considero basicamente duas questões particularmente importantes em *Logical Forms II* de Oswaldo Chateaubriand; a saber, a disputa entre a lógica de primeira e de segunda ordem e sua concepção de verdade lógica e noções relacionadas, como as de propriedade lógica, estado de coisas lógico e falsidade lógica. A primeira questão também estava presente no primeiro volume do livro, mas a última apenas aparece no segundo volume. Enfatizo o significado extraordinário de ambas as questões para a filosofia e, embora haja uma concordância básica com as visões de Chateaubriand, algumas observações críticas são inseridas.

Palavras chave: Lógica de primeira ordem. Lógica de segunda ordem. Verdade lógica. Falsidade lógica. Propriedade lógica.

1. INTRODUCTION

Since I have already dealt at length in this same journal with some issues of Logical Forms I1 and in an extensive critical study of the two volumes of Oswaldo Chateaubriand's outstanding book,² I will presently only discuss more thoroughly a few issues treated in Volume II of that work, which I consider especially significant for rigorous philosophy. I will mostly be concerned with two extremely important issues, namely, (i) the debate between first- and secondorder logic, in which Chateaubriand sides against the received view propounded very forcefully by Skolem and Quine, and (ii) the characterization of logical truth and its instances, an issue that gives rise to comparisons with some views of Husserl, as already occurred with other issues discussed in my commentary to the first volume of Chateaubriand's monumental book. Some reference to other related issues will be unavoidable, especially to Chateaubriand's decisive criticism of Quine and his emancipation from the chains of what since Russell has been assumed as the first commandment of analytic philosophy, namely, Ockham's razor, according to which entities are not to be postulated unnecessarily, a commandment that in the hands of empiricists and nominalists has served an ideological purpose and stymied the development of rigorous philosophy.

The criticism of Quine and, in particular, that of his stance concerning second-order logic is present in Chateaubriand's book from the very beginning, as stressed in my commentary to the first volume. Nonetheless, the culmination both of Chateaubriand's general criticism of Quine and of the latter's very influential view of second-order logic as disguised set theory and, thus, as being mathematics belongs to the second volume. As is well known, besides Ockham's razor, Quine's criticism of analyticity, the

¹ Rosado Haddock, 2004.

² Rosado Haddock, 2007.

predominance of the indispensability argument for (a physicalist) realism in mathematics, and the view that logic is essentially first-order logic, constitute the core of analytic philosophizing in most English speaking circles. Thus, Chateaubriand questions very forcefully two of those commandments.

2. PRELIMINARIES

Before expounding Chateaubriand's stance in the debate on second-order logic, it seems pertinent to mention that he also criticizes another important thesis of Quine's philosophy, though one that has had less general acceptance, namely, the double-headed indeterminacy thesis, which concerns, on the one hand, the indeterminacy of physical theories and, on the other hand, the indeterminacy of language. With respect to the latter thesis, Chateaubriand questions the very soundness of Quine's approach to language. Thus, Chateaubriand argues on the basis of the most recent theories of language and linguistic acquisition that Quine's conception of language, based on behaviourism, is demonstrably false. He appeals³ to the current view originating with Chomsky and his co-workers that there are basic innate components in language learning. Moreover, he stresses4 that in view of such results, Quine's views on language learning can be empirically refuted, and, in particular, that the latter's contention about the basic role presumably played by one-word sentences in the acquisition of language is demonstrably false. With respect to the first thesis, a very decisive passage occurs in an extensive footnote on pp. 101-102. In that footnote Chateaubriand compares the extreme rigour and precision, as well as the clear delimitation of the range of application with which Gödel formulates his epoch-making

³ See, for example, p. 37.

⁴ See p. 38.

incompleteness results, with the lack of rigour and precision of Quine's indeterminacy thesis and the presumption of an almost unlimited range of application, including our whole science. In particular, Chateaubriand stresses that if Gödel had been somewhat less rigorous in his argumentation - for example, if he had just mentioned but not carried out the arithmetization of syntax - he would not have been taken seriously, as precisely occurred with Finsler's imprecise and general argumentation. As Chateaubriand stresses, his criticism of Quine on this last point by no means should be rendered as not acknowledging the essentially theoretical nature of most scientific terms. In fact, it is only because of the decisive influence of official logical empiricism in rigorous philosophizing in the last three quarters of a century that scholars tend to forget or ignore that philosophers before Quine, for example, Duhem,5 Poincaré⁶ and Husserl⁷ were perfectly conscious of the indeterminacy and essentially theoretical nature of physical theories, though they, of course, neither made so much fuss about it nor adopted any sort of relativism On the other hand, Chateaubriand does not try to explain why Quine's double-headed indeterminacy thesis and other equally unfounded Quinean theses have had such a wide acceptance in analytic circles. More precisely, it seems perfectly irrational that though (i) Chomsky and Harris had shown the inadequacy of the behaviouristic approach to language, (ii) behaviourism itself as a psychological theory is inadequate as a theory of perception,8 and (iii) the inadequacy of the more liberal

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⁵ See, for example, his "Quelques réflexions au sujet de la physique expérimentale", 1894. In: R. Ariew and P. Barker (eds.) (1996).

⁶ See, for example, Poincaré (1968), especially Chapters IX and X.

⁷ See Logische Untersuchungen I (1900), Chapters V and XI.

⁸ See on this issue Chateaubriand's observations about abstract elements in perception acknowledged by the Gestalt theorists. As usual,

brand of empiricism that was logical empiricism as a philosophy of science makes at least very implausible the adequacy of Quine's views, the latter's views continue to exert a decisive influence on most current analytic philosophy.

3. CHATEAUBRIAND ON FIRST- VERSUS SECOND-ORDER LOGIC

As already stressed in my commentary to the first volume of Logical Forms, Chateaubriand is a decisive opponent of Quine's restriction of logic to first-order logic. Nonetheless, I will argue that there is an important slip in Chateaubriand's defence of second- and higher-order logic. Chateaubriand forcefully and correctly rejects9 the relativism that Skolem and most propounders of the predominance of first-order logic have extracted, as a philosophical "lesson" from the so-called Skolem's Paradox, Moreover, Chateaubriand even argues¹⁰ that Quine and others came very close to consider that since we are able to adequately handle only firstorder theories - which are the only ones for which the syntax perfectly mirrors the semantics -, hence, theories that, in virtue of the Löwenheim-Skolem Theorem, have a denumerable model similar to the structure of the natural number system, the structure of the real world does not differ essentially from that of the natural numbers. However, Chateaubriand points out 11 that since the conclusion is clearly absurd, we should abandon the syntactic standpoint of logic linked to first-order logic. Although I agree with Chateaubriand with respect to the absurdity of the conclusion, I am

Chateaubriand ignores Husserl, though the Gestalt psychologists were probably influenced on this point by the latter.

⁹ See p. 72.

¹⁰ Ibid.

¹¹ Ibid.

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not sure that it follows from the mere adoption of the syntactic standpoint. Certainly, first-order theories have, in virtue of the Löwenheim-Skolem Theorem, a denumerable model. But in virtue of the Upward Löwenheim-Skolem-Tarski Theorem, they also have models of higher cardinalities, and it is by no means excluded that also models of much greater complexity than the structure of the natural number system. Hence, nothing like Chateaubriand's conclusion follows from the adoption of what I also consider an incorrect syntactic standpoint.

Nonetheless, I agree with Chateaubriand's remark on p. 131 that from a philosophical standpoint it is completely arbitrary to limit logic to first-order logic. I would also add that from a purely logical standpoint it is no less arbitrary to equate logic with firstorder logic. Certainly, first-order logic is semantically complete and, thus, the syntactic clothes fit perfectly the semantic body. However, similarly forceful argumentations could be brought to favour either propositional logic or second-order logic. Propositional logic has a decision procedure, whereas full (n-adic) first-order logic does not and cannot have a decision procedure. Thus, one could argue that it is essential for logic to have a decision procedure and, therefore, logic would either coincide with propositional logic or extend at most up to monadic first-order logic. On the other hand, one could argue that logic needs to be powerful enough in order that our most basic mathematical theories, which are intuitively categorical, remain categorical when formulated in logical clothes. Therefore, on such premises, first-order logic does not deserve the role of being logic, a role that should be assumed by second-order logic.

On p. 133, Chateaubriand makes it clear that for him logic should include higher-order logic, logic with infinitely long formulas and logic with partially ordered formulas. This should by no means be rendered as an exclusion of other sorts of extensions of first-order logic. In fact, as he points out on p. 211, for him the theory of types is the core of logic, not the whole logic. I perfectly agree on this point with Chateaubriand, in case he means by the theory of types 'simple type theory', not 'ramified type theory'. Although there is a passage on p. 162, according to which "...the simple theory of types collapsed all the definability conditions that Russell built into the ramified hierarchy" in a similar way to that in which "...two-valued extensional logic collapsed all the distinctions of sense that Frege built into his account of logic", that could be rendered as a preference for ramified type theory versus simple type theory, I opt for a charitable rendering of Chateaubriand's views and consider that for him simple type theory belongs to the core of logic, without taking any stance on the inclusion of ramified type theory.

On pp. 231-232, Chateaubriand emphasizes the semantic commitment of second- and higher-order logic by stressing that quantification in those logics is not over predicates, but over properties or sets – I would, more generally say: relations. He correctly adds that the rendering of higher-order quantification as quantification over predicates, that is, over syntactic constituents of language, is a result of the influence of the syntactic view.

Although I basically agree with Chateaubriand's general stance with respect to the problem of first- versus second- and higher logic, there is a passage on p. 270 of Chateaubriand's book that certainly tends to neutralize his whole argumentation on behalf of higher-order logic. Thus, Chateaubriand – who conceives logic as intimately related to ontology – surprisingly asserts with respect to the so-called different interpretations of second- (and higher-) order logic that he opts for the so-called general interpretation of Henkin, that is, the interpretation that allows for general or truncated models, on the basis that the full or absolute interpretation involves some "metaphysical principles", which he considers that "go beyond the scope of logic". As is well known, the opponents of second- (and higher-) order logic have argued that second order logic allows for

more than one interpretation, namely, (i) the absolute interpretation, in which all models have cardinalitywise all relations that they could have, (ii) Henkin's general models, in which structures are allowed to count as models though they do not have all the relations they could have, for example, all the domains of relations could have the same cardinality as the domain of objects, and (iii) many sorted models. Since the last two interpretations are essentially the same, I will consider only the first two.

Second-order logic with the absolute interpretation has all the many virtues and some defects usually associated with such a logic, namely, it has a much greater expressive power than first-order logic, enabling it to express mathematical notions and theories not expressible at all in first-order logic, while some statements, like the Induction Principle of arithmetic, are more adequately expressed than in its weaker rival logic, and many intuitively categorical mathematical theories are precisely categorical when expressed in second-order logic, whereas they lack categoricity when expressed in first-order logic. On the other hand, in contrast with its weaker rival, second-order logic is neither semantically complete nor compact, nor possesses the Löwenheim Property. In contrast with the absolute interpretation of second- (and higher-) order logic, the interpretation by means of Henkin's general models is such that it preserves those three valuable features of first-order logic, that is, second-order logic with Henkin's interpretation is semantically complete, compact and has the Löwenheim Property, namely, any of its models has a countable elementary submodel. Of course, under second-order logic with Henkin's interpretation the categoricity of many mathematical theories disappears. Thus, for example, both second-order analysis and second-order set theory with Henkin's interpretation have countable models.

The most serious problem with Henkin's so-called interpretation of second-order logic, however, is that it is not an

interpretation of second-order logic at all - since it does not preserve the essential features of that logic -, but a reduction of second-order logic to first-order logic. It is a consequence of Lindström's most famous characterization theorem, according to which every extension of first-order logic for which the Compactness Theorem and the Löwenheim-Skolem Theorem are valid is equivalent to firstorder logic, that second-order logic with Henkin's interpretation is equivalent to first-order logic and, thus, it is not second-order logic anymore. Hence, if one adopts, as Chateaubriand does, Henkin's general interpretation for second (and higher-) order logic, one would not be moving away and up from first-order logic, but would remain chained at the side of Quine and Skolem to first-order logic. Hence, it is a consequence of Lindström's most famous theorem that, contrary to the received view about higher-order logic, there is only one genuine interpretation for full second- (and higher-) order logic, whereas the other two so-called interpretations collapse second-order logic in first-order logic.

4. CHATEAUBRIAND ON LOGICAL TRUTH AND ITS INSTANCES

One of the most important and interesting issues discussed in any of the two volumes of *Logical Forms* is that of logical truth and its instances. In fact, the central issue of both volumes is the search for logical forms and, as we will see, logical truth is intimately related to logical forms. I have already pointed out in my commentary to *Logical Forms I* that for Chateaubriand, who accepts – as I do – the Frege-Husserl distinction between sense and referent, the referents of statements are states of affairs. Thus, without knowing it, he sides with Husserl against Frege on this important point. States of affairs are going to play a pivotal role in the discussion of logical truth. Indeed, states of affairs play such an important role that on p. 229 he states that whereas propositional

logic is essentially an analysis of truth, predicate logic - I suppose, in the broadest sense that includes all of higher-order logic - is an analysis of the structure of states of affairs. Another notion playing a decisive role in Chateaubriand's analysis of logical truth is the notion of logical property. In fact, on p. 132, Chateaubriand identifies logical forms with logical properties, while emphasizing that one uses different systems of notation to represent them. He immediately adds that he by no means presupposes that there is one system of notation that can represent all logical forms. Those assertions are complemented much later - on p. 213 - by a more thorough analysis. Thus, he stresses that though the notion of a logical property - that is, of a logical form - is of one that is present "throughout the whole ontology", "there cannot be any such properties in an absolute sense". We are, thus, compelled, according to Chateaubriand, to represent our ontology by means of a structuring into types. Chateaubriand observes that one can in any case obtain semi-absolute logical properties by means of the structuring into types, since once a logical property is made available at a certain level n, it will continue to appear at any level m such that m>n. As examples of logical properties, Chateaubriand mentions identity, existence, subordination, unity, plurality, diversity and universality. Before continuing my exposition, I would like to insert some critical remarks. First of all, Chateaubriand's assertion that there are not any absolute logical properties can only be true, on the basis of his conception of a sort of 'cumulative hierarchy of logical properties', if there are no logical properties at the first level, that of propositional logic. On the other hand, Chateaubriand seems to be presupposing in this context that the whole of logic is somehow stratified in a hierarchical order similar to that of type theory. However, that is not true, since the different extensions of first-order logic cannot simply be ordered in a hierarchy. Hence, an argument should be given in order to show

that it is impossible that logical properties get lost in no matter what extension of a weaker logic, and such an argument is absent from *Logical Forms*. Moreover, Chateaubriand would have to give some argument in order to establish that the Löwenheim Property and the Hanf Property are not logical properties at all, since they are clearly not preserved in all extensions of first-order logic.

On p. 251, Chateaubriand fixes the meaning of some extremely important notions, namely; (i) logical states of affairs are combinations only of logical properties; (ii) logical properties are such that they either combine necessarily or do not combine necessarily; and (iii) logical truths are (the propositional content of) statements referring to logical states of affairs. Thus, as Chateaubriand puts it on p. 252, a logical truth is "...a proposition that consists exclusively of logical properties", or, as he also states, ¹² they are logical propositions. On p. 253, there is an important passage that resumes the discussion in such a compact way that it deserves being quoted and not simply paraphrased:

Given that logical properties either combine necessarily or necessarily do not combine, it seems reasonable to say that logical propositions whose parts denote logical properties that combine necessarily into a logical state of affairs, are logically true, and to say that logical propositions whose parts denote logical properties that necessarily do not combine, are logically false.

As pointed out on p. 254, on the basis of Chateaubriand's precise characterization of the notion of logical truth and of other related notions, statements like (i) Frege=Frege, (ii) $(\forall x)$ (Human(x) \rightarrow Human(x)), and (iii) Human(Russell) \vee -Human(Russell) are not logical truths, since they do not refer to logical states of affairs. The presence in such statements of non-logical predicates or individual constants amounts to making them in some sense contingent.

¹² Logical Forms II, p. 252.

Nonetheless, they are clearly instances of logical truths. As Chateaubriand puts it, 13 "...they attribute logical properties to certain nonlogical entities" and, of course, "...[such] logical properties apply universally to any entities of the appropriate types". On p. 255, Chateaubriand correctly characterizes such instances of logical truths as applied logical truths. They apply to those contingent entities, provided those entities exist. As he states it:

Given the universal applicability of the logical property involved, which is a logical truth in each case, the truth of those propositions is equivalent to the existence of the objects and properties to which the logical property is applied.

Hence, the statement ' $(\forall x)(x=x)$ ' is a logical truth and, thus true under any circumstance, since, as Chateaubriand puts it on p. 262, it is true "in virtue of the logical features of the world", whereas the statement 'Frege=Frege' is true in every circumstance in which Frege exists, that is, its truth is dependent on the contingency of Frege's existence, though there is no circumstance in which Frege exists and in which it is not true. As Chateaubriand puts it on the same p. 262, "...given Frege's existence, his self-identity is not only a necessary truth, but a logically necessary truth". Thus, though applied logical truths are dependent on the contingency of the existence of the non-logical entities referred to in them, they are logically necessary truths. An interesting case discussed briefly by Chateaubriand is that of statements containing, besides logical properties, non-logical constants that refer to non-logical entities that would necessarily exist, like the God of the Christian theologians, for example, the statement 'God=God'. On the basis of Chateaubriand's characterization of logical truth, such a statement is not a logical truth, but an applied logical truth, while the entity

¹³ Ibid., p. 254.

referred to by the constant 'God' would necessarily but not logically necessarily exist. Moreover, since the entity referred to by the non-logical constant 'God' would necessarily exist, 'God=God' is neither contingent on God's existence, since God necessarily exists, nor is it a contingent statement, but a necessary one, even a logically necessary one. In contrast to the statement 'Frege=Frege', whose logical necessity is contingent on Frege's existence, the logical necessity of the statement 'God=God' would not be contingent on God's existence, since God would necessarily exist.¹⁴

Chateaubriand applies his extremely valuable discussion of logical truth to explain the notions of logical implication and logical equivalence. Thus, on p. 262, he states:

Logical implication...is a relation that holds between propositions, or sentences, independently of whether they denote or not; and, if they denote, independently of the contingency of the states of affairs that they denote.

With respect to logical equivalence, he stresses¹⁵ that unless the coexistence of the entities referred to by two presumably logically equivalent propositions has a logical character, that is, that they exist exactly under the same circumstances, one should refrain from considering them logically equivalent.

There is no doubt about the extraordinary importance of Chateaubriand's insights into the nature of logical truth and related notions. He has probably gone farther than any of his predecessors on this issue. It should be pointed out, however, that, though

¹⁴ Of course, the preceding discussion is based on the false premise that something, for example, the God of the theologians, necessarily exists. Although, contrary to the views of some atheists, modern science has not proved the inexistence of God, it has shown, by means of a coherent model of the origin of the universe and of life, that God does not necessarily exist.

¹⁵ Ibid., p. 262.

Chateaubriand was certainly not acquainted with Husserl's prior definitions, once more there are similarities with some insights of Husserl, in this case with the latter's definition of the notion of analyticity. Thus, in § 12 of the Third Logical Investigation, 16 Husserl defines the notion of analyticity in terms similar to those of Chateaubriand's definition of logical truth, namely, such that analytic statements are true exclusively in virtue of their logical form. Moreover, Husserl distinguishes between analytic laws (or statements) and their applications, which he calls 'analytic necessities', in a similar way to Chateaubriand's distinction between logical truths and applied logical truths. Since, as already pointed out, statements for Husserl, refer to states of affairs, the relation between Husserl's and Chateaubriand's notions and distinctions is far from superficial. It should be stressed, however, that Chateaubriand's discussion is more thorough than Husserl's treatment. Without much doubt, Chateaubriand's investigations on logical truth are not only a significant contribution to analytic philosophy, but also a contribution to the unavoidable approach between the non-ideological features of the two most important schools in recent philosophy, namely, analytic philosophy and Husserl's phenomenology.

As I pointed out elsewhere,¹⁷ I regard the use of the term 'logical truth' more adequate to delimit the sort of truths with which Chateaubriand and Husserl are concerned than the term 'analytic' used by the latter, and would prefer to reserve the term 'analytic' for a wider notion of non-synthetic truths. I have, however, also expressed

¹⁶ Logische Untersuchungen II, U. III, § 12. See also his recently published Alte und Neue Logik: Vorlesungen 1908/09, Dordrecht 2003.

¹⁷ See Chapter Four of my recently published book *The Young Carnap's Unknown Master*, as well as my also recently published papers "Husserl on Analyticity and Beyond" and "Issues in the Philosophy of Logic: an Unorthodox Approach".

with respect to Chateaubriand's notion of logical truth some misgivings similar to my *caveat* with regard to the preservation of logical properties in, for example, all extensions of first-order logic. It is not easy to define what logic means. In abstract model theory one usually defines the extensions of first-order logic following Lindström. Thus, one could say that propositional logic, first-order logic and all so defined extensions of first-order logic are logics. Nonetheless, there are many other possible candidates for logic, for example, modal logic, Lesniewski's systems and many others. It is certainly impossible to put all those logical systems in a hierarchy, and even more difficult to characterize the notion of logical property and, hence, that of logical truth for any possible logic in such a way as to satisfy the prerequisites for Chateaubriand's definitions. I would be delighted, however, if my misgivings are unfounded and Chateaubriand's goal of characterizing logical truth were achieved.

REFERENCES

- ARIEW, R., BARKER, P. (eds.). Pierre Duhem, Essays in the History and Philosophy of Science. Indianapolis: Hackett, 1996.
- CHATEAUBRIAND, O. Logical Forms. Part I: Truth and Description. Campinas: Unicamp, Centro de Lógica, Epistemologia e História da Ciência, 2001. (Coleção CLE, v. 34)
- ———. Logical Forms. Part II: Logic, Language, and Knowledge. Campinas: Unicamp, Centro de Lógica, Epistemologia e História da Ciência, 2005. (Coleção CLE, v. 42)
- DUHEM, P. 'Quelques réflexions au sujet de la physique expérimentale'. English translation: 'Some Reflections on the Subject of Experimental Physics'. In: R. Ariew and P. Barker (eds.) (1996), pp. 75-111.

HUSSERL, E. Logische Untersuchungen, 1900-1901. Husserliana XVIII. Dordrecht: Reidel, 1975. (v.1) . Logische Untersuchungen, 1900-1901. Husserliana XIX. Dordrecht: Reidel, 1984. (v.2) . Alte und Neue Logik: Vorlesungen 1908/09. Dordrecht: Kluwer, 2003. POINCARÉ, H. La Science et L'Hypothèse. Paris: Flammarion, [1902] 1968. ROSADO HADDOCK, G. E. "Chateaubriand on Logical Forms". Manuscrito, 27 (1), pp. 115-128, 2004. . "Critical Study of Oswaldo Chateaubriand's Logical Forms". Manuscrito, 30(1), pp. 185-218, 2007. . The Young Carnap's Unknown Master. Aldershot: Ashgate, 2008. . "Husserl on Analyticity and Beyond". Husserl Studies, 24, pp. 131-140, 2008. . "Issues in the Philosophy of Logic: an Unorthodox Approach". Principia, 11(1), pp. 25-44, 2007.