Logic, Language, and Knowledge. Essays on Chateaubriand's **Logical Forms**
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LOGICAL TRUTH AND SECOND-ORDER LOGIC: RESPONSE TO GUILLERMO ROSADO-HADDOCK

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Abstract: In my response to Guillermo Rosado-Haddock I discuss the two main issues raised in his paper. The first is that by allowing Henkin's general models as a legitimate model-theoretic interpretation of second-order logic, I undermine my defense of second-order logic against Quine's views concerning the primacy of first-order logic. The second is that my treatment of logical truth and logical properties does not take into account various systems of logic and properties of systems of logic such as the Löwenheim-Skolem property.

Keywords: Logical truth. Logical property. Second-order logic. General models.

VERDADE LÓGICA E LÓGICA DE SEGUNDA ORDEM: RÉPLICA À GUILLERMO ROSADO-HADDOCK

Resumo: Em minha réplica à Guillermo Rosado-Haddock discuto as duas questões centrais levantadas em seu artigo. A primeira é que ao permitir modelos gerais de Henkin como uma interpretação legítima da lógica de segunda ordem, desvirtuo minha defesa da lógica de segunda ordem contra a visão de Quine respeito à primazia da lógica de primeira ordem. A segunda é que meu tratamento da verdade lógica e das propriedades lógicas não leva em consideração diversos sistemas de lógica e propriedades de sistemas de lógica tais como a propriedade de Löwenheim-Skolem.

Palavras chave: Verdade lógica. Propriedade lógica. Lógica de segunda ordem. Modelos gerais.

Guillermo has written extensively about my book, and I am very grateful to him for his detailed comments on many of the issues discussed therein. His present paper is concerned with two main issues; namely, the interpretation of second-order logic, and the interpretation of the notion of logical truth. I begin with some preliminary remarks.

1. PRELIMINARY REMARKS

Some apparent disagreements Guillermo points out are due to differing uses of the terms 'logic' and 'logical property'. Guillermo talks about 'logics' in a general sense (p. 177), and of certain properties of a logic as logical properties. Thus, we can say that first-order logic has the Löwenheim-Skolem property, whereas second-order logic does not. This is an established way of speaking about logical systems, but is not the sense in which I use the term 'logical property'.[1]

I have (or postulate) an absolute ontology consisting of particulars, properties (in the general sense that includes relations), and states of affairs. Among the properties in this hierarchical

[1] In other words, the Löwenheim-Skolem property is a property that logical systems may or may not have, but it is not a logical property in my sense.

In connection with the Löwenheim-Skolem theorem I also agree with Guillermo that just as one can obtain denumerable models by the downward Löwenheim-Skolem theorem, one can obtain models of higher cardinalities by the upward Löwenheim-Skolem theorem. However, I did not maintain that it follows from the downward Löwenheim-Skolem theorem that "the structure of the real world does not differ essentially from that of the natural numbers", as Guillermo says on p. 167; I was a little more cautious, and maintained only (p. 72) that by pushing Skolem's ideas a bit further, we can hold that reality may be like that. Guillermo's point, with which I agree, is that we may hold other views as well.

ontology—which is a cumulative type theory and not a ramified type theory—are the properties (and relations) I characterize as logical properties. These are properties that appear at every level of the hierarchy after the first level in which they appear. With some exceptions, logical properties appear first at the second level, and "repeat" at higher levels. Thus the logical relation Instantiation first appears as a level 2 relation that relates level 1 properties with their instances. Evidently, there are indefinitely many Instantiation relations at every level greater than 1 (relating entities of lower levels) and although they cannot all be "collected together",[2] there are partial "collections" of them. I take logic (in an absolute sense) to be a theory of such logical properties.

Evidently, one can still distinguish various logical systems as theories of specific classes of logical properties. Thus, Aristotelian logic is essentially a theory of the four extensional logical relations Subordination, Exclusion, Partial Subordination, Partial exclusion. Propositional logic is a theory of Truth and Falsity (which are also logical properties on my account) and of infinitely many truth relations between propositions.[3] First-order logic is a theory of infinitely many logical properties, including Instantiation properties, Quantification properties, the Aristotelian relations, etc.

2. SECOND-ORDER LOGIC

I agree with Guillermo that my speculative discussion of the questions concerning general models for second-order logic is not clear enough. In fact, I had some doubts when writing those pages and was aware of the conflicts he points out. I will try to spell out my view (and intuitions) a little more carefully.

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[2] This is the sense in which I say there are no "absolute" logical properties.

[3] See Chapter 16.

I see model-theoretic interpretations of logical systems as a way of conceptualizing the notions of validity, logical consequence, etc., in terms of certain algebraic structures, but I do not see model theory as providing the "true" characterization of these notions. Hence, when it comes to formulating the allowable model-theoretic interpretations for second-order logic—and for the higher levels of the hierarchy—I do not see the "absolute" model-theoretic interpretation as being privileged from a logical point of view. If one interprets the quantifiers extensionally—or objectually, as it is usually said—the *absolute* interpretation for second-order logic is the one in which the individual variables range over *all particulars* (whatever they may be) and the first-order variables range over *all properties of particulars* (whatever they may be), which was essentially Frege's view.

What happens in practice is that model-theoretic interpretations are taken to be the basic semantic and ontological interpretation of logic, with models coming in all sizes and shapes. *Any* set is allowed as the universe of individuals, and in the "absolute" model-theoretic interpretation for second-order logic the first-order variables range over *all* subsets (and relations) of the chosen universe. But from a model-theoretic point of view I do not see any clear justification for saying that this is the "correct" interpretation of second-order logic. For, if one can choose different sets as the domain of the individual variables, why can't one choose different sets as the domain of the predicate variables? Or, in other words, what makes the notion of an n-ary ($n \geq 1$) relational set over the domain of individuals an absolute notion?[4]

[4] Henkin's general models reject this absoluteness while maintaining some conditions that must be satisfied by the domain of the predicate variables. I think this is entirely justified from a purely model-theoretic point of view, and the fact that with this interpretation second-order logic is equivalent to first-order logic seems to me to speak against the model-theoretic interpretation as the semantics and ontology of logic.

Although I do not follow Russell's ideas in the ramified hierarchy, my view is akin to his in the sense that I see the ontological hierarchy as being absolute. For all I know, the level of individuals may well be finite, and I do not see any logical reason for thinking it infinite. Of course, if one were to follow Frege in postulating logical objects (extensions, numbers, etc.), then there may be a logical reason for claiming the level of individuals to be infinite. On the other hand, I hold that the first level of properties is infinite because there are infinitely many logical properties. The basic logical properties at this level are Existence, Non-existence, Identity (binary) and Diversity (binary). But there are also infinitely many Pairwise Diversity relations as well as mixed Identity-Diversity relations of all arities (finite or infinite). In fact, as I see it, every level of the hierarchy is absolutely unlimited (by any cardinality) in the logical properties it contains.

Just as I hold it is not the proper business of logic to decide which individuals there are, I also hold it is not the proper business of logic to decide which non-logical properties appear at the various levels. Nor, as I argue on p. 270, is it a logical matter whether for any plurality of individuals there is a property having that plurality as its extension, even if this were justified as a general metaphysical principle—i.e., one should distinguish the logical characteristics of the ontological hierarchy from its more general metaphysical characteristics.

Although I would agree with Guillermo that there is a lot more to be sorted out about these issues, I do not think my remarks on pp. 270 ff. "neutralize" my position with respect to second and higher-order logics as absolute theories. This would only be the case if I took the purely model-theoretic point of view, according to which the proper interpretation of logic is given by set-theoretic structures.

3. LOGICAL TRUTH

Guillermo and I are basically in agreement on the characterization of logical truth I develop in Chapter 18, and he points out an important connection with Husserl's characterization of analytic laws and analytic necessities. He voices a doubt whether my characterization of logical truth would apply to any "logic", and I certainly agree with this, but I am not trying to characterize logical truth in the extended sense he suggests in the last paragraph of his paper.

As I mentioned earlier, many logical systems we normally study—Aristotelian logic, propositional logic, monadic first-order logic, first-order logic, second-order logic, and so on—may be considered theories of specific classes of logical properties in the absolute hierarchy, and for these we can use the notions of logical truth, logical state of affairs, and logical proposition as I have developed them. But there are logical systems with an altogether different character. An important example, which I discuss in several chapters, is intuitionistic logic, as interpreted within the context of Brouwer's idealistic philosophy.[5]

What characterizes classical logic is a commitment to an objective notion of truth and reality, and the notions of logical property, logical state of affairs, logical truth, etc., I develop are relative to this objective metaphysical perspective. Intuitionistic logic, on the other hand, at least according to Brouwer's philosophical outlook, is committed to a purely subjective interpretation of the fundamental logical notions, where reality is conceived as the mental constructions of a creating subject, and truth resides in the present and past experiences of the creating subject. Evidently, my account of logical truth does not fit in with such a conception.

[5] See the end of Chapter 24, for example.