CDD: 511.3

PROOF AND EXPLICATION: RESPONSE TO JOSÉ SEOANE

OSWALDO CHATEAUBRIAND

Department of Philosophy
Pontifical Catholic University of Rio de Janeiro
Rua Marquês de São Vicente, 225, Gávea
22453-900 RIO DE JANEIRO, RJ
BRAZIL

oswaldo@puc-rio.br

Abstract: José Seoane centers his commentary on my critique of the standard formal analysis of proof as an elucidation of the informal notion of proof, and I basically agree with his considerations throughout the paper. In my response I argue that the notion of formal proof is fundamentally an analysis of the notion of logical consequence, rather than an elucidation of the informal notion of proof.

Keywords: Proof. Explication. Formalism.

PROVA E EXPLICAÇÃO: RÉPLICA À JOSÉ SEOANE

Resumo: O comentário de José Seoane está centrado na crítica da análise formal de prova como elucidação da noção informal de prova, e estou basicamente de acordo com suas considerações. Em minha réplica argumento que a noção de prova formal é fundamentalmente uma análise da noção de conseqüência lógica ao invés de uma elucidação da noção informal de prova.

Palavras chave: Prova. Explicação. Formalismo.

José centers his commentary on Chapter 19, which he considers a philosophical critique of the standard formal analysis of proof as an elucidation of the informal notion of proof, and I basically agree with his considerations throughout the paper. With respect to his final questions and conclusions, I did not intend my arguments in Chapter 19 as an attempt of elucidation of the informal concept, and I quite agree with his final conclusion that the point of my critique was not to propose a new technical concept, but to point to serious difficulties in the traditional analysis as an elucidation of the informal notion of proof.

1. FORMAL PROOF AS AN ANALYSIS OF LOGICAL CONSEQUENCE

I argue in Chapter 20 (p. 315, note 4) that the analysis of formal proof by Frege was not meant as an explication of the informal notion of proof, but as an analysis of logical consequence in terms of a sequence of regimented elementary steps whose validity is recognized as uncontroversial. I think the same may be true of the work of other pioneers of modern logic, and that it was only with the introduction of logic textbooks that people began to interpret formal proofs as an explication[1] of the notion of proof. Lakatos (1976) and others reacted strongly against this interpretation, but by then the practice was so entrenched it could not be dislodged by arguments. It is remarkable, in fact, as I mention

[1] In the sense of Carnap (1950, pp. 3-15), who says (p. 3): "By the procedure of explication we mean the transformation of an inexact, prescientific concept, the explicandum, into a new exact concept, the explicatum. Although the explicandum cannot be given in exact terms, it should be made as clear as possible by informal explanations and examples."

at the beginning of Chapter 19, that until very recently there were so few attempts to discuss and analyze the practice of proving.

My view is that the various formal analyses of proof for first-order logic—axiomatic, natural deduction, sequent calculus, etc.—are in fact rather abstract syntactic analyses of logical consequence for first-order logic. In most systems we have obviously valid axioms and/or rules of inference structured in ways that guarantee the validity of the conclusions reached. We also have abstract analyses of logical consequence in terms of models, and completeness theorems that show their extensional equivalence to the syntactic analyses in terms of axioms and/or rules. But whereas the syntactic analyses have a finitary and effective character, the characterization of the model-theoretic analyses is infinitary. Although both approaches to logical consequence have an intuitive basis, I think neither can claim to be the more accurate analysis of the informal notion of logical consequence.

My example of the proof that it is a logical consequence of the infinitely many premises

(*i*) $\forall x \forall y \forall z\ ((Rxy \wedge Ryz) \rightarrow Rxz)$
(*ii*) $\forall x \neg Rxx$
(*iii*) Ra_1a_2
(*iv*) Ra_2a_3
:
:

that R has an infinite domain, was meant to suggest that just as we can analyze logical consequence in terms of finite sequences, we can also analyze logical consequence in terms of infinite sequences—and it is also quite obvious that in this case the infinite analysis reflects the informal argument more accurately.

2. PROOF AND JUSTIFICATION

To some extent I do try to elucidate the informal notion of proof, partly in Chapter 19, but mostly in the following chapters. My aim in Chapter 19 was to criticize the central assumptions of the usual accounts of formal proof in logic textbooks. I initially based my considerations on the remarks in Enderton (1972) because he gives a succinct formulation of the view that takes finiteness and effectiveness to be essential characteristics of proof. This view is so widespread we find it presupposed not only in textbooks written by mathematicians, but also in those written by philosophers—including even textbooks on the philosophy of logic. An important exception is the philosophical discussion and justification of the formal notion of proof in the Introduction of Church (1956).

My conclusion in Chapter 19 was that we must distinguish our acts of proving—which are not only expressed by finite sequences of statements, but by very short sequences of statements—from a mathematical representation of their logical structure as a (possibly infinite) sequence of inferences establishing logical consequence. This distinction between formal proof structures and actual proofs (provings) is explored in the next two chapters with the aim of obtaining a formulation of the aims and character of actual proofs. This formulation, in terms of structural, psychological, social, and ontological conditions, places the proofs within the broader epistemological context of justifications, and is further explored in Chapter 24.

REFERENCES

CARNAP, R. *Logical Foundations of Probability*. Chicago: University of Chicago Press, 1950.

CHURCH, A. *Introduction to Mathematical Logic*. Princeton: Princeton University Press, 1956.

ENDERTON, H. *A Mathematical Introduction to Logic*. New York: Academic Press, 1972

LAKATOS, I. *Proofs and Refutations*. Cambridge: Cambridge University Press, 1976.