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# MATHEMATICS AND LOGIC: RESPONSE TO MARK WILSON

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**Abstract:** Mark Wilson argues that in order to make physical first-order properties suitable for inclusion in the bottom levels of a logical hierarchy of properties, their proper treatment must take into account the methods of applied mathematics. I agree that the methods of applied mathematics are essential for studying physical properties, and in my response focus on the nature of the logical hierarchy and on the requirements of classical logic.

**Keywords:** Logical forms. Applied mathematics. First-order properties.

# MATEMÁTICA E LÓGICA: RÉPLICA À MARK WILSON

**Resumo:** Mark Wilson argumenta que um tratamento adequado para tornar as propriedades físicas de primeira ordem apropriadas para inclusão nos níveis inferiores de uma hierarquia de propriedades lógicas deve levar em consideração os métodos da matemática aplicada. Concordo que os métodos da matemática aplicada são essenciais para estudar as propriedades físicas, e em minha réplica enfoco a natureza da hierarquia lógica e os requisitos da lógica clássica.

**Palavras chave:** Formas lógicas. Matemática aplicada. Propriedades de primeira ordem.

Mark argues that the proper treatment of physical first-order properties must take into account the methods of applied mathematics in order to make these properties suitable for the requirements of classical logic. Although I agree that the methods of applied mathematics are essential for studying physical properties, and the specific examples he discusses are extremely interesting, I do not view the logical hierarchy as being incompatible with his approach.

## 1. THE TWO PICTURES

Mark's descriptions of the logicist's and the applied mathematician's pictures do not seem to me as divergent as he suggests—not, at any rate, as far as my own logicist picture is concerned. In fact, I agree with Mark's criticism (p. 334):

> ... an "applied mathematics" critic will complain that Oswaldo has terminated his story prematurely, for the mild improvement processes he discusses cannot possibly bring Victor's loosely specified traits to the acme of perfection required in the bottom floor of an acceptable logical hierarchy. Orthodox classical logic demands first-order traits with crisp and well-defined extensions and such exactness cannot be reached simply through the humble improving processes that Oswaldo catalogues. To properly satisfy logic's demands, Victor probably needs to go to the university, where he can acquire the requisite improvement tools, including a good deal of contemporary set theory. And so we critics complain to Chateaubriand: "You've stopped detailing the story of epistemological improvement with respect to physical concepts at exactly the point when important mathematical issues become philosophically salient."

I did indeed terminate my story prematurely, insofar as I did not discuss the lower levels of the hierarchy. Aside for the few remarks about Victor's learning the beginnings of language, there is

practically no discussion in my book of the level of objects, processes, events, etc., and very little of the level of their (first-order) properties. My idea when writing *Logical Forms* was that there would be later volumes on ontology and on philosophical aspects of mathematics where some of these questions would be treated in detail. So, in this volume, I concentrated on the upper echelons of the hierarchy, and especially on the logical properties and logical states of affairs.

I do hold that the logicist picture need not satisfy the demands of orthodox classical logic for crisp and well-defined extensions. I think it is obvious that properties at lower levels are quite often not well defined, and do not satisfy the principle of excluded middle. I assumed in Chapter 9 that they do, but this was only because I did not want to deal with the issue at that point. Which does not mean we have to abandon classical logic, because, as I have repeatedly emphasized, what characterizes classical logic for me is a commitment to a realistic metaphysics and to an objective notion of truth, not to sharpness of concepts.

When I place the logical (and mathematical) properties at the higher levels of the hierarchy, essentially beginning at level 2, I am not suggesting the lower levels are (somehow) determined "first", and the logical properties "build" on them. On the contrary, I take the logical properties to be totally independent of what happens at the lower levels, and to be essential for characterizing and understanding the lower levels. Consider, for instance, Frege's second-order relation Equinumericalness, which relates two first-order properties that apply to the same number of things. This is a logical (and mathematical) relation, the nature of which is completely independent of there being any things, or any specific properties of things, but which is necessary for characterizing things and their properties. It is part of those improvement tools to which Mark refers.

I am all in favor of Mark's suggestion that the improvement tools include a good deal of set theory, and I take set theory to be part of the logical properties in the upper levels of the hierarchy. Instead of viewing sets as a kind of logical or mathematical *objects* at the bottom of the hierarchy, I take sets (or extensions) to be abstracted from properties *via* the second-order logical relation Coextensiveness. Just as we can characterize the cardinality of a (first-order) property as a second-order property applying to all properties Equinumerical with it, we can characterize the extension of a (first-order) property as a second-order property applying to all properties Coextensive with it. In other words, I see the process of abstraction as going *up* the hierarchy of properties rather than *down*.

To the extent the logicist project is successful in developing mathematics from the abstract logical properties in the hierarchy, it will provide all the necessary tools needed by the applied mathematician—including Mark's beloved p.d.e.s—to study the phenomena of the physical world. But is it successful?

## 2. THE SUCCESS OF THE LOGICIST PROJECT

Many of those frigates to which Mark refers in note 4 were built from the alleged wreck of the logicist ship. As I argue in Chapters 10 and 23, Quine's indispensability argument derives partly from his nominalistic convictions based on Ockham's razor, and partly from his conviction that the paradoxes put an end to any natural development of the logicist program and of set theory. He accepts sets *a faute de mieux*, because science "needs" mathematics, and mathematics can be built from sets alone—but the fewer the better. Besides, abstract entities are a-causal and are claimed not to fit

into a strictly empiricist (naturalistic) epistemology, adding to the image of doom for the logicist program—as argued by Benacerraf.[1]

I see things differently, and a large part of my motivation in writing *Logical Forms* was to develop an approach to abstract logical properties along lines suggested by the original logicists, and by Gödel's formulation of a realistic philosophy of logic and of mathematics. This involved developing an ontological reformulation of such notions as state of affairs, truth, logical property, logical truth, etc., and an epistemological reformulation of the notions of proof, justification, knowledge, etc.

One of the reasons for leaving the discussion of the bottom part of the hierarchy for (some possible) later volumes, was the complexity of the ontological issues for the non-logical entities, processes, and phenomena which may be placed there, including the various kinds of phenomena Mark discusses in his paper. The only partial exception to this policy was the inclusion of the discussion of language in chapters 13 and 14, which was actually due to the insistence of students, who argued that the treatment of senses and propositions in Chapters 11 and 12 was too abstract, and should be connected to a broader discussion of language. In fact, I viewed (and view) the book as being essentially a philosophical work about logic, and even the various discussions of issues in the philosophy of mathematics were subordinate to that aim.

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[1] See Benacerraf (1973) and my discussion of Platonism in (2005).