

CDD: 149.9

QUANTIFICATION, SENTENCES, AND TRUTH-VALUES*

THOMAS RICKETTS

*Department of Philosophy
University of Pittsburgh
Pittsburgh, PA 15260
U.S.A
ricketts@pitt.edu*

Abstract: The paper maintains (1) that Frege's quantification of sentence positions motivates his identification of sentences as proper names of truth-values; (2) that this identification is fully compatible with the Context Principle; (3) that the relation of a thought to its truth-value is the primary case of the relation of sense to meaning. The paper offers a reconstruction of Frege's defense of (1) in pp. 33-5 of "On Sense and Meaning"

Key-words: Sense-meaning distinction. Truth-values. Context principle. Logical segmentation. Truth. The True. Generality.

QUANTIFICAÇÃO, SENTENÇAS E VALORES DE VERDADE

Resumo: O artigo sustenta (1) que a quantificação Fregeana sobre posições de sentença motiva sua identificação de sentenças como nomes próprios de valores de verdade; (2) que esta identificação é inteiramente compatível com o Princípio do Contexto; (3) que a relação de um pensamento com o seu valor de verdade é o caso primário da relação do sentido para com o significado. O artigo oferece uma reconstrução da defesa de Frege de (1) nas pp. 33-5 de "Sobre o Sentido e o Significado".

Palavras-chave: Distinção sentido-significado. Valores de verdade. Princípio do Contexto. Segmentação lógica. Verdade. O Verdadeiro. Generalidade.

No feature of Frege's philosophy meets with more incredulity from students than his conception of sentences as proper names of truth-values. This conception has no intuitive resonance. Frege himself feared that it would be dismissed as an artificial contrivance. History has confirmed Frege's fears. Frege's extensionalist approach to logic is

* Originally published in *Manuscrito*, v. 26, n. 2, p. 389-424, 2003.

widespread. On this approach, the truth-value of a sentence is preserved under substitution both of co-referring singular terms and of component sentences by sentences of like truth-value. In this way, the relation of a sentence to its truth-value is analogous to that between a singular term and the thing it designates. Nevertheless, very few extensionalist logicians take sentences to be designations of truth-values, and go on, like Frege, both to permit quantification of sentential positions within compound sentences and to use sentences as the terms of equations to express the identity of their truth-values. There is, I think, a tendency among contemporary readers of Frege to take his assimilation of sentences into the logical classification of proper names and the accompanying posit truth-values as the objects designated by sentences to be an inessential idiosyncrasy that arises from Frege's function-argument analysis of logical structure combined with his penchant for notational economy.

Frege's own view of the matter is different. The conception of sentences as proper names of truth-values first appears along with the sense-meaning distinction in Frege's 1891 lecture "Function and Concept". To all appearances, he did not quickly or easily embrace it. However, once he arrives at it, he adheres to it for the rest of his career, and ranks it among his enduring contributions to logic.¹

I shall argue that Frege's conception of sentences as proper names of truth-values represents a well motivated, deeply coherent development of his understanding of quantificational generality within his universalist

¹ In "Was kann ich als Ergebnis meiner Arbeit ansehen?", *NS* p. 200 (184), Frege presents "concepts conceived as functions" as the first of his enduring results. See also the discussion of truth-values as the objects meant by sentences in "Einleitung in die Logik", *NS* p. 211 (194) and in "Aufzeichnungen für Ludwig Darmstaedter", *NS* p. 276 (255). (Parenthetical references following page references to Frege's *Nachgelassene Schriften* are to the English translation, *Posthumous Writings*. Page references for Frege's papers are to their original publication, marginally indicated in many German and English editions. The translations of passages from Frege's writings are my own, made consulting the common English editions of Frege's writings.)

conception of logic. On this conception of logic, Frege finds himself quantifying sentential positions in order to formulate, as we would put it, truth-functional logic. The conception of sentences as proper names of truth-values is how Frege, in the context of his overarching views of logical segmentation, makes sense of this quantification.

The account I offer of sentences as proper names of truth-values will also address a central issue in Frege interpretation. In *The Foundations of Arithmetic*, Frege sets forth his famed Context Principle: only in the context of a sentence does a word have meaning.² This principle, whatever its exact import, assigns some kind of priority to sentences vis-à-vis their subsentential components. The later assimilation of sentences to proper names, a category whose paradigm representatives are subsentential expressions, looks to be at odds with the Context Principle, and perhaps to represent its rejection.³

I will argue that there is no tension or shift here in Frege's philosophy. As I understand Frege, it is our implicit grasp of the inference from generalization to instance in conjunction with Leibniz's law that enables us to recognize in sentences, and the thoughts they express, a segmentation into complete and incomplete parts. Frege's Context Principle encapsulates this view, a view that persists in Frege's philosophy through the adoption of the sense-meaning distinction until the end of his life. The first section of the paper summarizes this interpretation of Frege's view of logical segmentation. The second section of the paper explores the problems that quantification of sentential positions pose for

² This approximates Frege's formulation in *GLA* §60, p. 71. Other formulations of the Context Principle are found in *GLA* p. x and §106, p. 116.

³ Michael Dummett early on argued that Frege's assimilation of sentences to proper names gives up the priority of sentences vis-à-vis their component names set forth in Frege's Context Principle, and so represents a major shift in Frege's thought. See Dummett, 1981, pp. 182-4 and 643-5. However, Dummett's views on the Context Principle and its place in the development of Frege's thought have changed over the years. For Dummett's more recent views on the topic, see Dummett, 1995.

Frege, and the resolution that the identification of sentences as proper names of truth-values promises. On Frege's approach to logic, this identification cannot be stipulated, for the problems it addresses are not merely notational. In the final section of the paper, I consider Frege's defense of his controversial thesis in his celebrated essay "On Sense and Meaning". This paper introduces the sense-meaning distinction with respect to subsentential proper names like "the Morning Star" and the "the Evening Star", and in doing so appears to take the relation of designation between proper name and thing for granted. The order of Frege's exposition here can be misleading. I contend that Frege's defense of his thesis in the middle pages of the paper (pp. 32-5) invokes the Context Principle at a crucial point. When we appreciate the force of this invocation, and the way that it draws on the conception of judgment and truth that frames Frege's philosophy of logic, we will see that the distinction between the thought a sentence expresses and its truth-value is the fundamental application of the sense-meaning distinction that guides the extension of the distinction to subsentential expressions.

I

Frege takes as given our capacity for objective knowledge, our capacity to recognize cognition-independent truths. This capacity for knowledge includes a capacity for logical inference whose exercise enables us to recognize one truth on the basis of others. Frege aims to codify principles for logical inference in such a way that their application in any stated proof will force the explicit statement of any premise on which any conclusion or subconclusion depends. In order to be applicable in proofs across the sciences, these principles of inference must abstract from the content that distinguishes the various sciences. Frege conceives this abstraction substantively. Logical laws are maximally general truths – unrestricted generalizations whose statement requires only that topic-universal vocabulary required to express the results of any science, e.g., an expression for negation. The relation of logic to other sciences is then that

of a more abstract, less detailed science to a more detailed one. Logical laws are applied in the sciences when we infer from the law to an instance of the law containing designations of the objects and concepts under consideration. Of course, this inference from general to specific cannot itself be captured by a generalization. Frege captures it by his Substitution rule. Apart from a rule like Modus Ponens for inferring simpler truths from compound truths⁴ and certain bookkeeping rules for moving among notationally alternative expressions of truths,⁵ other inference-modes can be captured by logical laws inferable by Substitution and Modus Ponens from self-evident maximally general axioms. The capacity for inference thus is the capacity to recognize one truth on the basis of others in accordance with logical laws, as Frege puts it.⁶

On Frege's approach to logic, the inference from generalization to instance is fundamental. He says, "The person who knows how this inference goes has also grasped what generality is (in the sense of the word here intended)";⁷ and he remarks in several places that it is generality that compels the analysis of thoughts and sentences expressing thoughts into parts none of which is a thought or thought-expressing

⁴ Indeed, Frege folds Substitution and Modus Ponens into a single inference rule in his 1879 system in *BEG* §6, p. 8, subsequently referring to it in "Booles rechnende Logik" as the inference rule (*NS*, p. 44 (39)). It should be noted that Frege introduces his inference rules in *BEG* by schematic examples and does not provide syntactic descriptions of the permitted manipulations as in *GGA*.

⁵ These 'bookkeeping' rules include Universal Generalization and Relettering that move among different begriffsschrift expressions of the same thought, and cannot be formulated by single generalizations. In *GGA*, Frege does introduce further inference rules to cut back on tedious repetition of deductive routines in formal derivations. See *GGA*, p. vi.

⁶ For Frege's characterizations of inference, see "On the Foundations of Geometry" (1906), pt. ii, p. 387 and Frege to Dingler, 31.1.17, *WB*, p. 30. Compare Frege's talk of inference modes ("*Schlussweisen*") in *GLA* §90.

⁷ "Logical Generality", *NS*, p. 278 (258).

sentence.⁸ The simplest example of such analysis is the division of a sentence like

Socrates is mortal,

into a proper name, “Socrates”, and the part that remains when this proper name is removed,

ξ is mortal.

Thus analyzed, our sentence says (expresses the thought) that a particular individual, Socrates, is mortal. By replacing “Socrates” with other proper names, we obtain sentences that say that various other things are mortal. To analyze these sentences in this way is to grasp the contents they express as instances of the corresponding generalization expressed by “Everything is mortal”.

These simple examples of the logical segmentation induced by generality give Frege the pattern for his *begriffsschrift*. Colloquial language is variously ambiguous, irregular, redundant, and limited in its expression of topic-universal notions, above all in its expression of generality. The segmentation of sentences into proper names and leftover expressions that analyzes them as expressions of instances of generalizations enables Frege to extend colloquial mathematical usage by using letters in the positions of proper names in sentences, as Frege puts it, to confer generality of content on those sentences.⁹ So, “ x is mortal” replaces “Everything is mortal”. Here we have a uniform way of

⁸ “Aufzeichnungen für Ludwig Darmstaedter”, *NS*, p. 274 (254). See also “Einleitung in die Logik”, *NS*, p. 203f. (187); “Kurze Übersicht meiner logischen Lehren”, *NS*, p. 217 (201).

⁹ Frege uses this rhetoric in “Einleitung in die Logik”, *NS*, p. 204 (188) and 206f (190); “Kurze Übersicht meiner logischen Lehren”, *NS*, p. 215 (199); and “Foundations of Geometry” (1906), p. 307. See also “Begründung meiner strengeren Grundsätze des Definierens”, *NS*, p. 166f (154); *GGÄ* §17, p. 31f; “What is a Function?” p. 659f.

expressing generalizations that makes their instances notationally recognizable.

The use of letters in proper name positions to express generality is Frege's first decisive step toward constructing a begriffsschrift to serve as a medium for the expression of gap-free proofs.¹⁰ In this enterprise, he does not take the notion of an object and of a name's signifying an object to be an independently available basis for introducing the use of letters as variables to express generalizations. Frege takes the inference from generalization to instance itself to be basic. Frege begins with thoughts expressed by sentences, and it is generality – the inference from generalization to instance – that prompts the recognition of parts of sentences that are not themselves sentences. In speaking of proper names as designations of objects and of sentences containing the names, in contrast to the corresponding generalizations, being about the named objects, Frege seeks to awaken an explicit awareness of this distinctive inference-mode.

Quantificational generality is generality over a multiplicity of discrete, determinate items, determinate quanta so to speak. Discreteness is thus built into Frege's conception of an object: no object without identity. In his post-1891 elucidations of identity, Frege says that the identity of objects *a* and *b* is the complete coincidence (*zusammenfallen*) of *a* and *b*.¹¹ If *a* and *b* are one and the same, then there is no difference between them so that whatever holds of *a* also holds of *b*, and vice versa. A grasp of the inference from generalization to instance in connection with proper names thus includes a grasp of objects as discrete, and so a grasp of identity and

¹⁰ His 1879 monograph *BEG* is subtitled “A formula language (*Formelsprache*) for pure thought modeled on the formula language of arithmetic”. On p. iv of the foreword, Frege identifies the use of letters as variables as most direct way in which his notation is modeled on arithmetical notation; the first section of *BEG* presents the distinction between names and variables as a fundamental feature (*Grundgedanke*) of his approach.

¹¹ See *SB*, p. 26, fn. 1, “On Concept and Object”, p. 194, fn. 2, and especially, “Husserl Review”, p. 320.

with it the recognition of the inference-mode captured by Leibniz's law. These two inference-modes come together. It is not the inference from generalization to instance alone, but the interlock of this inference and the Leibniz inference that isolates proper names in sentences.

Frege's second decisive step toward his *begriffsschrift* is his treatment of the expression leftover from the removal of occurrences of proper names from a sentence as itself a name, and doing so regardless of whether the proper name is the grammatical subject of the original sentence and even if that sentence is a compound sentence.¹² For Frege, to treat these leftovers, his predicates, as names is to recognize the sentences in which they occur as instances of generalizations over what predicates signify. Predicates contain blanks, empty positions from which proper names were removed. As a result, variables in proper name positions may not be replaced by predicates to obtain expressions of instances of the generalization: as nothing would fill the blank in the predicate, the replacement would not yield an instance of the original generalization. Similarly, proper names do not fit into predicate positions. Proper names and predicates thus signify differently: proper names mean objects; predicates mean concepts. Frege consequently recognizes two types, two levels of generality – the generality expressed by variables in proper name positions over what proper names mean (objects), and the generality expressed by variables in predicate positions over what predicates mean (concepts).

Frege's broad notion of a predicate and the treatment of these predicates as names enables him to arrive at an adequate notation for the expression of multiple generality over objects. Frege recognizes the expressive limits of the use of indicating letters to express generality: the

¹² As predicates are names that occur in larger sentences, predicates may not be formed by the removal of a proper name from a free variable expression of a generalization. While these function, when prefixed by the judgment stroke, as expressions of thoughts, they, in contrast to their universal closures (to use our terminology), are not names. See *GGA* §17, p. 32f.

notation does not permit the delimitation of the scope of generality within sentences. He accordingly introduces a second-level incomplete expression to provide an alternative expression for generality over objects: “ $(\forall a)\varphi(a)$ ”. This expression, with its empty blank for a predicate, designates that second-level concept under which first-level concepts – the ones designated by predicates – fall, if every object falls under them.¹³ However, for ease and perspicuity in the formulation of inference rules, Frege retains free variable generalizations in his *begriffsschrift*, taking a free variable generalization prefixed by the judgment stroke to express the same thought as its universal closure.¹⁴

Quantificational generality over concepts presupposes that concepts, like objects, comprise a multiplicity of determinate, discrete items. In the case of objects, the identity-predicate gives expression to this determinate discreteness. As a first-level predicate, the identity-predicate cannot perform this service as regards concepts. For concepts are what predicates mean; and predicates, on account of their incompleteness, cannot be the terms of equations. Frege observes that any true sentence analyzable as the completion of a predicate by a proper name will remain

¹³ Similarly, Frege introduces a third-level expression, a designation of a third-level concept that serves as a universal quantifier over first-level concepts. Statement of general laws for this quantifier requires free variable generalizations over second-level concepts. We are at the beginning of a potentially infinite hierarchy, but the generalizations Frege envisions in connection with his foundations for arithmetic do not force ascent to higher levels. See *FB*, p. 31 and *GGA* §25, p. 42. Following Frege’s frequent expository practice, I ignore his assimilation of concepts to functions except where immediately germane to the point under consideration.

¹⁴ See *GGA* §17, p. 31, and *BEG* §11, p. 21. Frege does not explicitly assert in *GGA* that free variable generalizations prefixed by the judgment stroke express the same thoughts as their universal closures, although his rhetoric both in §17 and in §32 suggests it. In later writings, Frege does speak of free variable generalizations as expressing the same thoughts as their colloquial counterparts. See “Einleitung in die Logik”, *NS*, p. 206 (189f); “Kurze Übersicht meiner logischen Lehren”, *NS*, p. 217 (201); and “Logische Allgemeinheit”, *NS*, p. 280 (260).

true under any replacement of the predicate by a predicate designating a concept coextensive with the one designated by the original predicate. On this basis, he takes coextensiveness to be the surrogate for identity for concepts.¹⁵

We can now appreciate the point encapsulated in Frege's Context Principle: namely, a word or phrase functions as a name by virtue of occurring in a sentence that expresses an instance of a corresponding generalization. The inference from generalization to instance, together with the Leibniz inference, isolates proper names in sentences, segmenting sentences into a proper name and the leftover part, the predicate with its blank. These predicates also are names that may recur with the same significance in other sentences and may be replaced by variables to express corresponding generalizations. Thus, the recognition of proper names and generality over objects brings with it the recognition of predicates and generality over concepts. In this way, the view of names and quantificational generality encapsulated in the Context Principle leads to the recognition of levels of generality, leads to the distinction between objects and concepts. Frege adheres to the Context Principle throughout his career. He repeats its basic point in summarizing his life's work in 1919:

What is distinctive about my conception of logic comes out first in that I give top priority to the content of the word "true" and then that I immediately introduce thoughts as that concerning which the question of truth arises. I therefore do not begin with concepts that I put together into thoughts or judgments. Rather, I obtain thought-components [*Gedankenteile*] by analyzing [*Zerfällung*] thoughts.¹⁶

¹⁵ Frege argues along these lines in "Ausführungen über Sinn und Bedeutung", *NS*, p. 128 (118). See also p. 131 (120) and 132 (121); see also "Husserl Review", p. 320.

¹⁶ "Aufzeichnungen für Ludwig Darmstaedter", *NS*, p. 273 (253). Compare "Booles rechnende Logik", *NS*, p. 17 (16), where Frege says, "In opposition to Boole, I begin with judgments and their contents instead of concepts. ... For me the formation of concepts arises only from judgments". See also "On the Aim of the *Begriffsschrift*", p. 5.

II

I noted in §I how Frege says that generality compels the recognition in logic of parts of sentences that are not sentences. There are also a number of elementary inference-modes that require the recognition of sentences as logically relevant parts of compound sentences. Contraposition and hypothetical syllogism are examples here.¹⁷ From the beginning, Frege assimilates this logical segmentation of sentences within sentences to the segmentation of subsentential names within sentences, viewing both through the lens of his quantificational conception of generality. So, among Frege's logical axioms in the 1879 *Begriffsschrift* we find

If a then (if b then a)

and

If (if b then a) then (if not a then not b).¹⁸

The letters that occur in these formulas are genuine variables, not the schematic letters familiar from modern presentations of truth-functional logic.¹⁹ Sentences may be substituted for these variables to form instances of these generalizations. In quantifying sentential positions in compound sentences, Frege takes the component sentences themselves to be compound names. Only by such quantification can Frege frame general

¹⁷ Frege presents a number of these patterns in "Compound Thoughts".

¹⁸ The first law is proposition 1, *BEG*, §14, p. 26; the second is proposition 28, §17, p. 43. Hypothetical syllogism is expressed by proposition 28, a theorem, in §15, p. 32.

¹⁹ The schematic letters "P" and "Q" in a formula like "If P then (if Q then P)" are placeholders that mark the positions of the component sentences of a truth-functionally compound sentence. The formula with these placeholders is not a sentence – it does not say anything; it is not true or false. Instead, it represents a form of truth-functionally compound sentence, and thus gives us a convenient way to specify an infinite class of such sentences, the sentences that result from uniformly replacing "P" and "Q" in our formula by sentences. For a comparison of Frege's conception of logic with the contemporary one that uses schematic letters, see Goldfarb, 2001.

laws in his begriffsschrift that capture such inference modes as contraposition.²⁰

Given that sentences are compound names, what motivation does Frege have for taking them to be proper names like “The teacher of Plato” or “ $3 \times (4 + 1)$ ”? On Frege’s quantificational understanding of generality, no entity without identity or surrogate for identity. Hence, to quantify sentential positions requires that either the identity-predicate, or some surrogate be available to voice claims of identity and difference over what sentences mean. On the view of logical segmentation presented in §I, the only grounds for refusing to count a name as a proper name is its incompleteness, incompleteness that bars it from occupying the argument positions in the identity-predicate. But sentences are complete expressions: together with subsentential proper names, they are the basis for the identification of varieties of incomplete expressions. So, if sentences are to be counted as names, they should be acknowledged as proper names. Of course, this recognition brings with it the requirement to make sense of sentence-termed equations, a linguistic form not found in colloquial language.

This abstract motivation for assimilating sentences to proper names is reinforced by two further considerations. The first arises from Frege’s view of definitions as equations and generalized equations.²¹ To introduce a new, simple proper name as a definitional abbreviation for a compound proper name, definiens and definiendum are joined in an

²⁰ As mentioned in note 4, in *GGA* Frege replaces a number of *BEG* axioms with inference-rules, among them an inference-rule for contraposition, in order to cut back on tedious deductive routines in derivations. I don’t think Frege attaches any significance to this maneuver. Indeed, he voices a preference for the formulation of logic in *BEG* that minimizes inference rules in favor of axioms. See *GGA* foreword, p. vi.

²¹ Here I am indebted to Joan Weiner, who emphasizes the importance of Frege’s use of sentences as terms of equations in Weiner, 1997, especially p. 277.

equation that is put forward as a definition, not an assertion.²² Similarly, to introduce a simple predicate “ $F(\xi)$ ” as an abbreviation for a complex predicate “ $\emptyset(\xi)$ ”, Frege puts the generalized equation

$$F_x = \emptyset_x$$

forward as a definition. Instantiating the variable “ x ” by a proper name yields a sentence-termed equation. To admit sentences as the terms of equations is to make them proper names.

The second consideration is an important insight of Richard Heck. Frege’s construction of number is aided by the introduction of extensions not only of concepts but also of relations. Indeed, for Frege’s envisioned construction of the real numbers, extensions of relations are essential. The introduction of extensions of relations is greatly facilitated, if, as in *Grundgesetze*, the notation for extensions (more generally, value-ranges) is conceived as a second-level incomplete expression, “ $\dot{\epsilon} \emptyset(\epsilon)$ ”, whose empty place may be filled by either “ $1 < \xi$ ” or “ $\dot{\alpha} \Omega(\xi < \alpha)$ ”. Both of these incomplete expressions must then be expressions of the same type, first-level incomplete expressions with a single argument place. If they are of the same type, then their completions must also be of the same type. Hence, sentences and value-range names must be expressions of the same type.²³

²² See *BEG* §23, p. 56. In *GGA* §27, p. 44f, he says, “We introduce by means of a *definition* a new name in that we fix that it has the same sense and meaning as one put together from familiar signs”. See also “Logik in der Mathematik”, *NS*, p. 224f (207f).

²³ See Heck, 1997, pp. 281-5. It may well have been this use of truth-values as objects that prompted Frege to take the audacious, unfamiliar sounding step of taking sentences to be proper names of truth-values. Heck’s point is not, however, the entire story here. It does not explain why Frege, after abandoning efforts to rehabilitate value-ranges in the wake of Russell’s paradox, continues to take sentences to be proper names of truth-values and in 1906 lists the assimilation of concepts to functions as among his chief logical achievements. See the references in note 1.

Although Frege is not explicit on the topic, before 1891 he seems to have taken sentences to designate judgeable contents (*beurteilbarer Inhalt*), which he explains as he later explains thoughts.²⁴ This interpretation fits well with the metalinguistic content Frege gives to his identity-sign in *Begriffsschrift* §8, when we fill the argument-places in the identity-sign by sentences.²⁵ It also fits with the use Frege makes of the content stroke in *Begriffsschrift* to block the substitution for the variables in positions following it of signs for non-judgeable contents.²⁶

²⁴ In particular, a judgeable content is an objective content shareable by several thinkers concerning which the question of truth arises. Judgeable contents are then what are recognized to be true or rejected as false in acts of judging. See “Logik” (before 1891), *NS*, p. 8 (7f).

²⁵ Frege says there that an equation whose terms are signs *A* and *B* means that “...the sign *A* and the sign *B* have the same conceptual content (*begrifflichen Inhalt*) so that *B* can everywhere be placed in the position of *A*”.

²⁶ In *BEG* §2, Frege introduces the content stroke with the stipulation, “What follows the content stroke must have a judgeable content”. See also Frege’s description of the 1879 notation in “Booles rechnende Logik”, *NS*, p. 11, fn. *** (10). On p. 44 (39) of this paper, Frege lists the stipulation among the rules for the 1879 system.

It is unclear whether Frege takes variables following the content stroke in *Begriffsschrift* to generalize only over judgeable contents. In *BEG*, Frege uses small latin letters from the beginning of the alphabet as (free) variables in sentential positions; he uses “*x*”, “*y*”, and “*z*” as variables in proper name positions. This usage is not, however, enshrined in any stipulation. Furthermore, in *Begriffsschrift*, Frege’s Substitution rule is folded together with Modus Ponens into an inference rule that is explained only via examples, and sentences are the only complete expressions in the formalism. Frege’s immediate application of the 1879 system to prove theorems in the theory of sequences does not then require him to be explicit about the relationship between variables in sentential positions and proper name variables. In *GGA*, Frege gives exacting notational specifications of inference rules, including a separate treatment of Substitution. Moreover, Frege introduces nonsentential complete expressions (value-range names) into the formalism. In 1893 then, Frege had to deal explicitly with issues not treated in 1879.

The natural enough assumption that sentences designate judgeable contents clashes with the assimilation of sentences to compound proper names that may then be used as the terms of equations. Let “ $\emptyset(a)$ ” be any compound proper name containing proper name “ a ”; let “ $\emptyset(b)$ ” be the corresponding proper name, with proper name “ b ” replacing “ a ”. The logic of identity that Frege incorporates into his formulation of logic commits him to

$$\text{If } a = b, \text{ then } \emptyset(a) = \emptyset(b).$$

In this way, the designation or meaning of any compound proper name remains unchanged under replacement within it of component proper names by proper names that designate the same thing. In “Function and Concept”,²⁷ Frege observes that the two sentences

The Morning Star is a planet whose orbital period is less than that of the Earth,

and

The Evening Star is a planet whose orbital period is less than that of the Earth,

express different thoughts: a person who does not know that the Morning Star is the same as the Evening Star might, nonetheless, understand both sentences and hold the one true and the other false. However, since “the Morning Star” and “the Evening Star” both designate Venus, these two sentences must designate the same thing.²⁸

²⁷ *FB*, p. 14. See *SB*, p. 32. See also Frege to Russell, 28.12.02, *WB*, p. 235, where Frege explicitly links the point with the use of sentences as terms of equations.

²⁸ Matters are particularly striking when we reflect that equations themselves are compound names. Here we get such instances of Leibniz’s law as

$$\text{If } 2^4 = 4^2, \text{ then } (2^4 = 4^2) = (4^2 = 4^2).$$

In this way, any true equation designates the same as an instance of the principle of identity, “Everything is self-identical” (“ $x = x$ ”).

Frege concludes that what a sentence expresses – that concerning which the question of truth arises, what he after 1891 calls a thought – is not what the sentence designates. He had then combined in his notion of a judgeable content what sentences express and what those sentences designate.²⁹ Now he introduces truth-values to be what sentences designate or mean, and calls the thought a sentence expresses its sense. Sentence-termed equations assert the identity of the truth-values meant by their sentential terms. Furthermore, uncertainties surrounding the use of variables to quantify sentential positions are resolved. Frege now unambiguously uses a single vocabulary of variables to quantify all proper name positions.³⁰

Frege extends the distinction between sense and meaning to names generally. The meaning of a proper name is the object it designates. As for sense, consider how the sentences “ $2^4 = 4^2$ ” and “ $4^2 = 4^2$ ” express different thoughts. Since the sentences differ only in that the one is the completion of the predicate “ $\xi = 4^2$ ” by “ 2^4 ” and the other the completion of this same predicate by “ 4^2 ”, any difference in the thoughts expressed by the two sentences exhibits a difference between these two co-designating proper names. The difference here cannot just be the syntactic difference between the two proper names: definitions yield equations of the form “ $a = b$ ” that are just as trivial as corresponding ones

²⁹ See “On Concept and Object”, p. 198; Frege to Husserl 24.5.91, *WB*, p. 96; and *GGA*, p. x. In these places, Frege describes himself as having combined under the term “judgeable content” what he now separates into thought and truth-value.

³⁰ Frege still uses the horizontal to mark positions for sentences, but he now explains the horizontal as a name of the first-level function that maps the True to itself and everything else to the False. By use of the horizontal, Frege accommodates the use of any proper name, sentential or nonsentential, in any proper name position.

of the form “ $a = a$ ”.³¹ Frege calls this nonsyntactic difference a difference in the senses of the two proper names, and says that the sense of a name is what that name contributes to the thoughts expressed by the sentences in which it occurs.³²

III

In “Function and Concept” and *Grundgesetze*, Frege’s explanation of his conception of sentences as proper names of truth-values is terse and peremptory.³³ He presents a series of equations of the same form, for example

$$(\xi)^2 = 1,$$

noting that some of these equations are true and others false. He then imposes a novel redescription, treating the above incomplete expression as the designation of a function whose values for numbers as arguments are the truth-values, the True and the False. So, just as “ 1^2 ” means 1, so “ $1^2 = 1$ ” means the True and “ $2^2 = 1$ ” means the False. Thus, “ $(2^2 = 4) = (2 > 1)$ ” is a correct equation, just as “ $(2^2 = 4)$ ” is.

Frege fears that this conception of sentences as designations of truth-values will strike his audience as arbitrary and contrived (*willkürlich und künstlich*).³⁴ In both “Function and Concept” and *Grundgesetze*, he refers his readers to “On Sense and Meaning” for a more thorough justification of it. The core explanation of the conception of sentences as proper names of truth-values comes in the pivotal, tangled middle pages

³¹ This paragraph summarizes the argument in the opening paragraph of “On Sense and Meaning”. Even in *BEG*, Frege thinks that after an equation has been used to introduce a name by definition, it becomes an analytic triviality. Compare *BEG* §24, p. 56f and *SB*, pp. 25-6.

³² See *Grundgesetze* §32, p. 51; “Logik in der Mathematik”, *NS*, p. 250 (231); and Frege to Linke 24.8.19, *WB*, p. 156.

³³ See *FB*, pp. 13-4, and *GLA* §2.

³⁴ *FB*, p. 14, fn. 6.

(pp. 32-5) of “On Sense and Meaning”.³⁵ Although Frege goes on to devote the rest of the paper to a defense of this conception against putative counterexamples, it is these middle pages that give content and plausibility to the conception.³⁶ Here is where he answers the charge that his conception of sentences as proper names of truth-values is arbitrary and contrived.

“On Sense and Meaning” opens with an explanation of the sense-meaning distinction in application to subsentential proper names. On p. 32, Frege asks whether the distinction is applicable to sentences. He introduces the notion of the thought expressed by a sentence as “the objective content [of an act of thinking] that is capable of being the common possession of many”.³⁷ Frege assumes that, analogous to the way in which a proper name is a part of sentences, the sense of the proper name is a part of the thoughts expressed by sentences containing the name. He repeats the argument from “Function and Concept” that precludes taking the meaning of a sentence to be the thought it expresses. He then asks whether sentences might generally have sense – that is, express thoughts – while not having a meaning. Frege does not here

³⁵ He repeats the line of thought presented here in several subsequent writings. See “Einleitung in die Logik”, *NS*, pp. 210-1 (194); “Logik in der Mathematik”, *NS*, pp. 250-1. Frege also discusses the point in his correspondence with Russell in his letters dated 20.10.02 (*WB*, pp. 231-2); 28.12.02 (*WB*, pp. 234-5); 21.5.03 (*WB*, p. 240f.); and 13.11.04 (*WB*, p. 245-8).

³⁶ In his subsequent discussions of the sense-meaning distinction in application to sentences in “Mr. Peano’s *Begriffsschrift* and my Own”, the Russell correspondence, “Einleitung in die Logik”, and “Logik in der Mathematik”, Frege recurs to ideas from these pages, and does not mention his defense of the claim that replacement of the component sentences in expressions of compound thoughts by sentences with the same truth-value preserves the truth-value of the entire sentence.

³⁷ *SB*, p. 32 fn. 5. Although Frege only here in “On Sense and Meaning” explicitly introduces his notion of a thought, I take it to be part of the backdrop for Frege’s introduction of the notion of sense in application to subsentential proper names at the beginning of the paper.

mention quantification of sentential positions as a warrant for taking sentences to have a meaning, for it is, in the end, the coherence of such quantification in the context of Frege's view of generality that is at issue.

Frege observes that a sentence's expressing a particular thought is distinct from, independent from, its component names' designating anything.³⁸ Given this independence of thought from meaning, Frege urges that there must be something connected with a sentence other than the thought it expresses for which the meaningfulness of its component names is essential. For if we were interested just in the thought a sentence expressed, there would be no need to bother whether those names have a meaning at all. He suggests that this additional something will clue us into the meaning of sentences.

So far in the paper, Frege has taken the notion of designation or meaning for subsentential proper names for granted. Now he asks after the point of this notion.

Frege's answer invokes the Context Principle: the meaningfulness of the component names in a sentence matters to us, when we inquire after the truth of the thought expressed by those sentences.³⁹ When we inquire after the truth of the thought expressed by a sentence, we

³⁸ Frege is not very precise about this independence. For his purposes, I don't think he has to be. He observes that we take many personal proper names that occur in sentences in fiction not to designate any actual people, while, nevertheless, taking those sentences to express thoughts. He also opines that should we come to believe that such a name – his example is “Odysseus” – did designate a historical personage, we would not thereby take the thoughts expressed by the sentences in which this name occurred to be different. Cf. Frege's presentations of these points in “Einleitung in die Logik”, *NS*, p. 208 and 210 (191 and 194) and in “Logik in der Mathematik”, *NS*, p. 250 (232).

³⁹ Tyler Burge notes that Frege's argument here invokes the Context Principle in “Frege on Truth”, in Burge, 1986, p. 103f. I differ with Burge as regards the content of the Context Principle and the substance of Frege's argument here.

confront a requirement to recognize the thought to be true or to reject it as false.⁴⁰ As we saw in §I, to analyze the sentence

Odysseus was set ashore at Ithaca while fast asleep,

as the completion of the predicate “ξ was set ashore at Ithaca while fast asleep” by the proper name “Odysseus” is to recognize the thought it expresses as the instance of a generalization: someone who recognizes the thought to be true ipso facto affirms that a certain individual – the one designated by “Odysseus” – falls under the concept *set ashore at Ithaca while fast asleep*, and someone who rejects the thought as false thereby denies that the man falls under the concept. So, if there is no such man, if the word “Odysseus” does not designate any individual in our sample sentence, then the thought the sentence expresses can neither be recognized to be true nor rejected as false.

In general, to inquire after the truth of a thought expressed by a sentence is to presuppose that the thought is true or false and so amenable to logical analysis as a generalization or an instance of various generalizations. We can then go on to avail ourselves of instances of logical laws in whose expression the names discovered by logical analysis figure. In doing so, we presuppose that those names are meaningful. Frege begins with truth, and introduces thoughts as the objective contents of thinking for which the question of truth arises. The exercise of the ability for logical inference, for recognizing one truth on the basis of others, leads us to discern in thoughts and the sentences expressing them a segmentation into complete and incomplete parts. To recognize these parts in the course of drawing inferences is to recognize these parts to be

⁴⁰ See “Negation”, p. 143.

meaningful, to designate something.⁴¹ As Frege puts the point in “On Sense and Meaning”, “It is the striving after truth that everywhere drives us forward from sense to meaning”.⁴²

To presuppose in inquiry that a sentence expresses a true or false thought is to presuppose that its component names are meaningful. So, sentences’ being true or false corresponds to their component proper names’ having a meaning. Frege proposes to press this correspondence into an identification:

We are in this way impelled to recognize the *truth-value* of a sentence as its meaning. I understand by the truth-value of a sentence the circumstance that it is true or that it is false. There are not further truth-values. For the sake of brevity, I call the one the True and the other the False. Each indicative sentence, when the meanings of its words matter, is therefore

⁴¹ What about thoughts that are neither true or false, thoughts that, as Frege puts it, belong to fiction (*Dichtung*) not to science? In “Thoughts”, p. 63, Frege speaks of an actor’s pseudo-assertions (*Scheinbehauptung*) of sentences expressing thoughts that neither actor nor audience presuppose to be true or false. I suggest that in such cases there is a pseudo-presupposition, a pretense of presupposing that the sentence is true or false. With this pretense, comes a pretense of exercising our logical ability on such thoughts and so a pretense of presupposing that that the parts that this pretence at inference discerns are meaningful.

⁴² *SB*, p. 33. See also “Ausführungen über Sinn und Bedeutung”, *NS*, p. 133, where Frege says:

They [intensional logicians (*Inhaltslogiker*) as opposed to extensional logicians (*Umfanglogiker*)] do not consider that for logic it does not matter how thoughts proceed from thoughts without regard to truth-value, that the step from thought to truth-value, more generally that the step from sense to meaning, must be taken, that logical laws are first and foremost laws for the realm of meanings and only indirectly [*mittelbar*] concern sense.

For a very early expression of this attitude, see the pre-1884 “Dialog mit Pünjer”, *NS*, p. 67.

to be conceived as a proper name, and its meaning, if one is present, is either the True or the False.⁴³

At this stage of Frege's explanation, the conclusion is proleptic. He has yet to confirm that sentences behave like proper names of truth-values under substitution of their co-designating component proper names, both subsentential and sentential. But even apart from this task, there remains a fundamental difficulty with Frege's elucidation of the meaning of sentences.

I noted in §II that to sustain the recognition of sentences to be proper names, Frege must make sense of sentence-termed equations, a form of words absent from colloquial language. The sense Frege makes of these equations cannot be a matter of stipulation: it must be fixed by the sense of the identity-sign as it occurs in other equations and by the thoughts expressed by its sentential terms. The explanations of "Function and Concept" and *Grundgesetze* do not appear by themselves to convey any such sense. These explanations transparently depend on the use of the predicates "true" and "false". On this point, the explanation we have just examined from "On Sense and Meaning" is no improvement. This feature of Frege's explanation is problematic. For Frege, a sentence, a series of marks or sounds, is true or false only by dint of expressing a thought that is true or false. Application of the predicates "true" and "false" to sentences is then derivative on the primary application of these predicates to the thoughts that sentences express.⁴⁴ The words "true" and "false" are predicates, so truth and falsity present themselves as properties of various thoughts. To use more Fregean terminology, as "true" and "false" are predicates, they must mean concepts under which thoughts

⁴³ *SB*, p. 34. Frege brings the observation that the truth-value of sentences is preserved under substitution of subsentential proper names into his explanation of sentences as proper names of truth-values in "Logik in der Mathematik, *NS*, p. 251 (232f) and in Frege to Russell 28.12.02, *WB*, p. 235. In *SB*, he presents it on the next page as a validation of his conclusion.

⁴⁴ See "Thoughts", p. 60 and "Logik" (1897), *NS*, p. 140 (129).

fall. Truth-values thus appear to be concepts, not objects meant by proper names; the relation of a thought to its truth-value is that of subsumption of an object under a concept, not that of the sense of a name to the name's meaning. Frege's proposal for finding sense in sentence-termed equations is thus threatened. At best, Frege seems to have stipulated for sentence-termed equations a sense that has nothing to do with identity.⁴⁵

Having identified sentences as proper names of truth-values, Frege confronts this objection in the next paragraph "On Sense and Meaning". Alluding to the use of "true" as a predicate, he says:

It is tempting to see the relation of a thought to the True not as that of sense to meaning, but as that of subject to predicate. We can indeed even say, "The thought that 5 is a prime number is true". However, on closer examination, we notice that with this sentence no more is really said than in the simple sentence "5 is a prime number". In both cases the assertion (*Behauptung*) of truth lodges in the form of an assertoric sentence (*Behauptungssatz*). Where this has lost its usual force – for example, in the mouth of an actor in a play – the sentence "the thought that 5 is a prime number is true" only contains a thought and indeed the very same thought as the simple sentence "5 is a prime number". From this it may be gathered that the relationship of a thought to the True is not to be compared at all to that of subject to predicate.⁴⁶

This argument must be placed in the context of Frege's conception of judgment. I said in §I that Frege takes as given our capacity for judgment, our capacity to arrive at objective knowledge. In the preceding paragraph of "On Sense and Meaning", Frege had said that in a judgment a step is taken from "the level of thoughts to the level of meanings (the

⁴⁵ This is in essence the problem Frege raises in *GLA* §56 with the proposal to analyze statements of number by means of their quantificational paraphrases. The problem is that these paraphrases do not respect the sense that statements of numbers, as equations ("The number of $F = n$ "), have.

⁴⁶ *SB*, pp. 34-5. See also "Einleitung in die Logik", *NS*, p. 211 (194). Here it is explicit that Frege, in speaking of the relation of subject to predicate, has in mind subsumption under a concept. Frege also treats this objection in "Logik in der Mathematik", *NS*, p. 251f.

objective)”; and in a footnote to this passage, he presents his standard characterization of judgment, repeated over most of his career: judgment is the recognition of the truth of a thought.⁴⁷ The dictum itself, by its grammar, suggests that being true is a condition that thoughts may or may not satisfy – a concept under which thoughts fall or fail to fall.⁴⁸ The relation of thought to truth-value would then be subsumption of an object under a concept. But then, Frege holds, putting a thought forward as true would have to involve a predication of truth. So, by arguing that the sentences used to make assertions do not predicate truth of thoughts,

⁴⁷ This dictum is also stated in *Grundgesetze* §5, p. 9. It is prominent in “Thoughts”, p. 62, and in the four antecedents to “Thoughts” in *NS*: “Logik” Before, 1891, p. 2 (2); “Logik” (1897), p. 150 (139); “Einleitung in die Logik”, p. 201 (185); “Kurze Übersicht”, p. 213 (197). It also appears in “Meine grundlegenden logischen Einsichten”, p. 271 (251); “Darmstaedter Aufzeichnungen”, p. 273 (253), Frege to Dingler (6.2.17), *WB*, p. 33; Frege to Jourdain (1912?), *WB*, p. 120; Frege to Russell (13.11.04), *WB*, p. 245.

⁴⁸ Frege may have conceived of judging in this way in *BEG*. There he suggests that the combination of judgment stroke-content stroke (“┌”) that prefixes every *Begriffsschrift* assertion is a common predicate for all judgments, and can be read as “is a fact (*Tatsache*)”. See *BEG*, §3, p. 4. It is not clear how seriously Frege intends this suggestion. He makes it in the course of urging that the grammatical subject-predicate distinction is of no logical importance; and in *Begriffsschrift* itself, Frege uses “┌” only to prefix asserted sentences, and so never in components of compound sentences. In any event, Frege does not repeat this characterization of “┌” as a predicate in later pre-1891 writings. For Frege’s other pre-1891 explanations of the judgment stroke, see: “Applications of the *Begriffsschrift*”, p. 33; “On the Aim of the *Begriffsschrift*” p. 5; and “Booles rechnende Logik”, *NS*, p. 11 (11). I believe that Frege gives up thinking of truth as any kind of property well before he comes up with his conception of truth-values as objects. After 1891, Frege explicitly distinguishes the judgment stroke from the predicates (incomplete expressions) of *Begriffsschrift*, thus sharply distinguishing the asserting force it expresses from predication.

Frege seeks to refute the view that the relation of a thought to its truth-value is that of subsumption of an object under a concept.⁴⁹

Frege's argument turns on a particular alleged superfluity of the grammatical predicate "true" exhibited in the pair of sentences:

(A) *The thought that 5 is a prime number is true,*

and

(B) *5 is a prime number.*

Frege takes it to be evident that (A) and (B) say the same thing, express the same thought,⁵⁰ and concludes that the use of (A) to put this thought forward as true does not involve any supplement to the thought expressed by (B). Consistent with this synonymy, might not (A) make explicit a predication of truth implicit in (B)?⁵¹ If so, then presumably

(C) *The thought that the thought that 5 is a prime number is true is itself true,*

would make explicit a predication of truth implicit in (A). We have now embarked on a regress that would preclude the entirely explicit expression of thoughts. Frege's conclusion is that assertions do not involve even implicit predications of truth.⁵² Putting a thought forward as true is a

⁴⁹ I treat this point and the argument in the next three paragraphs more fully in Ricketts, 1996, §II.

⁵⁰ Frege presents the superfluity thesis in connection with the application of the sense-meaning distinction to sentences in *SB*, pp. 34-5, "Einleitung", *NS*, pp. 210-1 (194); "Logik in der Mathematik", *NS*, pp. 251-2 (233). He presents it apart from the sense-meaning distinction in "Logik" (1897), *NS*, p. 140 (129), Frege to Russell 13.11.04, *WB*, p. 245, "Meine grundlegenden logischen Einsichten", *NS*, pp. 271-2 (251-52), and "Thoughts", p. 61 and 63.

⁵¹ I am grateful to Paul Guyer for urging this question on me.

⁵² See also "Thoughts", p. 63. In "Meine grundlegenden logischen Einsichten", *NS*, p. 272 (252), Frege says that the predicate "true" is a failed attempt to make "what corresponds to asserting force" into a part of a thought.

matter of uttering an expression of the thought with asserting force, and the conventional indicator of asserting force is the grammatical form of an indicative sentence.

Use of the grammatical predicate “true” in sentences like (A) is then deceptive.⁵³ It suggests that the relation of a thought to its truth-value is subsumption under a concept. The superfluity of the predicate “true” exhibited in (A) and (B) shows this suggestion to be mistaken. Truth-values are not properties of thoughts, are not Fregean concepts.

Once the distortions engendered by the predicate “true” have been eliminated, Frege can deploy the notion of a thought and its truth-value to explain the sense-meaning distinction with reference to sentences. We saw that the fulcrum of Frege’s explanation is his observation that the meaningfulness of the names in a sentence matters to us, when we inquire after the truth of the thought the sentence expresses so that it is the truth-valuedness of thought-expressing sentences that corresponds to the meaningfulness (meaning something) of their component names. Frege does not then explain or elucidate the relation of thought to truth-value as a special case of the independently graspable relation of designation. Quite the contrary, the relationship of thought to truth-value is the more basic, independently graspable relationship: it is the relationship recognized in acts of judging and linguistically expressed by the grammatical indicators of asserting force.⁵⁴ Frege takes the notion of the truth-value of a thought to be available to his audience by reflection on

⁵³ In “Einleitung in die Logik”, *NS*, p. 211 (194), Frege says: If we say ‘The thought is true,’ we seem to attribute truth as a property to the thought. ... Here, however, language deceives us”.

⁵⁴ In *SB*, p. 35, Frege says that “Judgment can be conceived as a stepping forward from a thought to its truth-value. Of course, this should not be taken for a definition. Judgment is indeed something entirely unique and incomparable”. See also “Ausführungen über Sinn und Bedeutung”, *NS*, p. 133 (122) and Frege to Husserl, 24.5.91, *WB*, p. 32. This characterization of judging cancels the misleading suggestion that truth is a property of thoughts carried by Frege’s dictum that judgment is the recognition of the truth of a thought.

their engagement in the singular activity of judging.⁵⁵ Frege then uses this notion to present thought-expressing sentences as proper names of truth-values, i.e., as capable of occurring as the terms of equations to express the identity of their truth-values. He thus explains what the sense-meaning distinction comes to as regards its fundamental application to expressions of thoughts, from which its application to names generally is, via the Context Principle, derivative.⁵⁶ In this way, Frege seeks to disperse the appearance of artificiality surrounding his conception of sentences as proper names of truth-values.⁵⁷

To take thought-expressing sentences to mean truth-values is to take sentence-termed equations to express the identity of the truth-values of the thoughts expressed by the terms. This construal of sentence-termed equation meshes with the sense-expressed by the identity sign, for Leibniz's law guarantees that the substitution within a sentence of codesignating subsentential proper names preserves the truth-value of the original sentence.⁵⁸ Of course, if sentences are to be proper names of truth-values, Leibniz's law also requires that replacement of a component sentence in a compound sentence by a sentence of like truth-value must preserve the truth-value of the original compound sentence. The clausal structures of colloquial language provide numerous putative counter-

⁵⁵ He thus says in "On Sense and Meaning", p. 34 of the two truth-values: "These two objects are recognized, if only implicitly, by everyone who judges, who holds something true or false, and so even by a skeptic".

⁵⁶ On this point, I agree with Ruffino, 1997, p. 147. See also Gabriel, 2000, p. 27.

⁵⁷ Just here Wittgenstein criticizes Frege at *Tractatus* 4.063 and 4.431. I see Wittgenstein's points here to be part of a critique of Frege's views on quantificational generality and logical segmentation. For some details, see Ricketts, 2002.

⁵⁸ I take Frege to begin with the recognition of the truth of Leibniz's law, and relying on it, to take putative counterexamples to it to reveal an ambiguity between the proper name as used in the original sentence and the proper name as it appears in the equation that formulates the premise for the Leibniz inference.

examples to this attempted use of sentences as designations of truth-values to form instances of Leibniz law. Indeed, nearly half of “On Sense and Meaning” is devoted to this topic. Frege’s general tactic is to provide paraphrases of the offending clausal structures in order to establish that the component sentence does not contribute to the sense of the compound sentence just by expressing a thought, a thought that could be affirmed or denied by use of the component sentence standing alone. In effect, these paraphrases indicate that the grammatical structure of these compound sentences does not track the structure of the thoughts they express.

Frege realizes that some may balk at calling truth-values objects. However, Frege understands objects to be what proper names mean. Once sentences are conceived as playing the logical role of proper names in compound sentences, and sentence-termed equations are understood to say that the truth-values of the thoughts expressed by their terms are the same, the status of truth-values as objects is settled.⁵⁹ Frege’s treatment here of the objecthood of truth-values is then parallel to his treatment of the objecthood of numbers in *Foundations* §§55-61.

How does Frege’s opening discussion of the sense-meaning distinction for subsentential proper names, especially his talk of the sense of a proper name as containing a way of being given the object the name means, fit with the interpretation of the sense-meaning distinction I have developed here?

Frege does not elaborate on this notion of *a way of being given an object*; I believe that he places no explanatory weight on it. His rhetoric points toward the role of proper names in the expression of thoughts, especially the thoughts expressed by equations. Suppose we have recognized as true the thought expressed by a nontrivial equation “ $a = b$ ”. I have already remarked how Frege argues from the difference between this

⁵⁹ On p. 34 of *SB*, Frege says, “What I call an object can be more precisely discussed only in connection with concepts and relations. I will reserve this topic for another essay”. Frege here anticipates “On Concept and Object”.

thought and the thought expressed by " $a = a$ " to the conclusion that names of the same object may make different contributions to the thoughts expressed by the sentences in which they occur. In this argument, Frege begins with a difference in thought, in sentence-sense, and moves to a difference in the sense expressed by subsentential proper names, logically segmented units, within these expressions of thoughts. Moreover, it is the role of " a " and " b " as proper names in these known-to-be-true equations that gives content to the claim that their senses contain different ways of being given the same object.⁶⁰ Finally, on this view of matters, there is no difference in meaning without a difference in sense. Proper names " a " and " b " mean different things only if " $a = b$ " expresses a false thought. But if " a " and " b " express the same sense, then " $a = b$ " expresses the same thought as the trivial truth " $a = a$ ". In these ways, Frege's elucidation of the sense-meaning distinction for subsentential proper names in the opening pages of his famous essay draws on an implicit grasp of the relation between thought and truth-value, of the sense-meaning distinction for sentences.⁶¹

⁶⁰ Besides the opening of *SB*, see "On Mr. Peano's *Begriffsschrift* and My Own" p. 369. See also Frege to Russell 28.12.02, *WB*, p. 234; Frege's discussion of the "Ateb" – "Afla" example in Frege to Jourdain, undated draft from 1914, *WB*, p. 128; Frege to Peano undated probably from 1896, *WB*, p. 196f; and "Einleitung in die Logik", *NS*, p. 209.

⁶¹ I am indebted to Enzo DePellegrin, Warren Goldfarb and Peter Hylton for comments on earlier drafts of this paper. I have also profited from conversations about the sense-meaning distinction with Michael Kremer and from his paper, "Sense and Meaning: the Origins and Development of the Distinction", to appear in *The Cambridge Companion to Frege*, ed. T. Ricketts (Cambridge: Cambridge University Press, to appear). I presented this paper first at a Frege workshop at the University of Leyden and later at the Boston University Colloquium for the Philosophy of Science, and benefited from discussions at both occasions.

REFERENCES

- BURGE, T. "Frege on Truth". In: L. Haaparanta and J. Hintikka (eds.). *Frege Synthesized*. Dordrecht: D. Reidel, pp. 97-154, 1986.
- DUMMETT, M. *Frege: Philosophy of Language*. Cambridge, MA: Harvard University Press, 2nd ed., 1981.
- DUMMETT, M. "The Context Principle: Centre of Frege's Philosophy". In: I. Max and W. Stelzner (eds.). *Logik und Mathematik: Frege-Kolloquium, Jena 1993*. Berlin: De Gruyter, pp. 3-19, 1995.
- GABRIEL, G. "Logik und Metaphysik in Freges Philosophie der Mathematik". In: G. Gabriel and U. Dathe (eds.). *Gottlob Frege: Werke und Wirkung*. Paderborn: Mentis, pp. 25-38, 2000.
- FREGE, G. *Begriffsschrift, eine der Arithmetik nachgebildete Formelsprache des reinen Denkens*. Halle: Verlag von Louis Nebert, 1879. (Cited as *BEG*)
- . "Applications of the Begriffsschrift" (German title: "Anwendungen der Begriffsschrift"). *Jenaische Zeitschrift für Naturwissenschaft*, 13, supplement 2, pp. 29-33, 1879b.
- . "On the Aim of the Begriffsschrift" (German title: "Über den Zweck der Begriffsschrift"). *Jenaische Zeitschrift für Naturwissenschaft*, 16, supplement, pp. 1-10, 1883.
- . "Function and Concept" (German title: "Function und Begriff"). Jena: H. Pohle, 1891. (Cited as *FB*)
- . "On Sense and Meaning" (German title: "Über Sinn und Bedeutung"). *Zeitschrift für Philosophie and Philosophische Kritik*, 100, pp. 25-50, 1892. (Cited as *SB*)

- . “On Concept and Object” (German title: “Über Begriff und Gegenstand”). *Vierteljahrsschrift für wissenschaftliche Philosophie*, 16, pp. 192-205, 1892b.
- . *Grundgesetze der Arithmetik*. Jena: H. Pohle, vol. I, 1893. (Cited as *GGA*)
- FREGE, G. “Review of E. G. Husserl”. *Philosophie der Arithmetik I. Zeitschrift für Philosophie und philosophische Kritik*, 103, pp. 313-32, 1894. (Cited as *Husserl Review*)
- . “On Mr. Peano’s *Begriffsschrift* and My Own”. (German title: “Über die Begriffsschrift des Herrn Peano und meine eigene”. *Berichte über die Verhandlungen der königlich sächsischen Gesellschaft der Wissenschaften zu Leipzig. Mathematisch-Physische Klasse*, 48, pp. 361-78, 1897.
- . “What is a Function?” (German title: “Was ist eine Function?”). In: *Festschrift Ludwig Boltzmann gewidmet zum sechzigsten Geburtstage, 20 Februar 1904*. Leipzig: J.A. Barth, pp. 656-66, 1904.
- . “On the Foundations of Geometry.” (German title: “Über die Grundlagen der Geometrie”). *Jahresbericht der Deutschen Mathematiker-Vereinigung*, 15, pp. 293-309; 377-403; 423-30, 1906.
- . “Thoughts”. (German title: “Der Gedanke”). *Beiträge zur Philosophie des deutschen Idealismus*, 1, pp. 58-77, 1918.
- . “Negation”. (German title: “Die Verneinung”). *Beiträge zur Philosophie des deutschen Idealismus*, 1, pp. 143-57, 1918b.
- . “Compound Thoughts”. (German title: “Gedankengefüge”). *Beiträge zur Philosophie des deutschen Idealismus*, 3, pp. 36-51, 1923.

- . *The Foundations of Arithmetic*. Trans. by J. L. Austin. Evanston: Northwestern University Press, 1953. (Original publication 1884; cited as *GLA*)
- FREGE, G. *Posthumous Writings*. Edited by H. Hermes, F. Kambartel, and F. Kaulbach and translated by P. Long and R. White. Chicago: University of Chicago Press, 1979.
- . *Nachgelassene Schriften*. Edited by H. Hermes, F. Kambartel and F. Kaulbach. 2nd revised edition. Hamburg: Felix Meiner Verlag, 1983. (Cited as *NS*)
- . *Wissenschaftlicher Briefwechsel*. Edited by G. Gabriel *et al.* Hamburg: Felix Meiner Verlag, 1976. (Cited as *WB*)
- GOLDFARB, W. "Frege's Conception of Logic". In: J. Floyd and S. Shieh (eds.). *Future Pasts: Reflections on the History and Nature of Analytic Philosophy*. New York: Oxford University Press, pp. 25-41, 2001.
- HECK JNR, R. "The Julius Caesar Objection". In: R. Heck Jnr (ed.). *Language, Thought, and Logic: Essays in Honour of Michael Dummett*. Oxford: Oxford University Press, pp. 273-308, 1997.
- RICKETTS, T. "Logic and Truth in Frege". *The Aristotelian Society*, supplementary volume 70, pp. 121-40, 1996.
- . "Wittgenstein against Frege and Russell". In: E. Reck (ed.). *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy*. New York: Oxford University Press, pp. 227-51, 2002.
- RUFFINO, M. "Wahrheitswerte als Gegenstände". In: G. Gabriel and W. Kienzler (eds.). *Frege in Jena*. Würzburg: Königshausen and Neumann, pp. 139-48, 1997.

WEINER, J. "Frege and the Linguistic Turn". *Philosophical Topics*, 25, pp. 265-88, 1997.

WITTGENSTEIN, L. *Tractatus Logico-Philosophicus*. London: Routledge and Kegan Paul, 1922.

