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A COHERENCE THEORY OF TRUTH*

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Abstract: In this paper, we provide a new formulation of a coherence theory of truth using the resources of the partial structures approach – in particular the notions of partial structure and quasi-truth. After developing this new formulation, we apply the resulting theory to the philosophy of mathematics, and argue that it can be used to develop a new account of nominalism in mathematics. This application illustrates the strength and usefulness of the proposed formulation of a coherence theory of truth.

Key-words: Coherence theory of truth. Partial structures. Quasi-truth.

UMA TEORIA COERENTISTA DA VERDADE

Resumo: Neste artigo, propomos uma nova formulação de uma teoria coerentista da verdade utilizando os recursos da abordagem de estruturas parciais - em particular as noções de estrutura parcial e quase-verdade. Após desenvolver esta formulação, aplicamos a teoria resultante à filosofia da matemática, e argumentamos que aquela pode ser usada para desenvolver um novo tratamento do nominalismo na matemática. Esta aplicação ilustra a força e utilidade da formulação proposta de uma teoria coerentista da verdade.

Palavras-chave: Teoria coerentista da verdade. Estruturas parciais. Quase-verdade.

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1. INTRODUCTION

Scientific practice has two notable features. First, when investigating a particular domain, scientists usually do not possess complete information about it. Typically, a number of results are known to hold, others are known not to hold, and several issues are simply left open. Second, scientific theories are typically valid only in a limited domain, and usually they can't be extended beyond the latter. This is the case, for example, of Newtonian theory in comparison with the theory of relativity (the former only holds under certain conditions).

These two features highlight what can be called the *informational incompleteness of science*. And given the incompleteness, it is difficult to see how we can talk about the *truth* of the relevant scientific theory. For without complete information, how can we claim that a scientific theory is true? Furthermore, if a theory is valid only in a certain domain, how can it also be true *simpliciter*?

This informational incompleteness motivated the introduction of an alternative conception of truth, which is broader than the classical one, and able to do justice to the lack of complete information in science. This is the theory of pragmatic truth, or quasi-truth (see, e.g., Mikenberg, da Costa and Chuaqui (1986), da Costa and French (1989), (1990) and (1993), and da Costa, Bueno and French (1998)). We shall consider below the formal characterization of quasi-truth. What is important to stress here is the following point. A given scientific theory T might not be true if extended beyond a certain domain, and a number of issues (both conceptual and empirical) might be left open by it. However, T might still be quasi-true. In other words, although T might not be true, it can be quasi-true. And in terms of this notion, the development of science can be understood; that is, we can represent science in terms of the search for quasi-true theories (see da Costa and French (1993) and (2003), and Bueno (1997)).

When first introduced, the notion of quasi-truth was called *pragmatic truth*. One of the *motivations* underlying the introduction of the formalism of quasi-truth was the connection it bore with some views put forward by pragmatist philosophers (such as James and Peirce) about the notion of truth (see Mikenberg, da Costa and Chuaqui (1986), and da Costa and French (2003)). However, it was soon realized that pragmatic truth was only a *particular* interpretation of the formalism. The latter could also be interpreted as an *epistemic possibility of truth*; that is, if a sentence α is pragmatically true, the current information doesn't preclude that α is indeed true (see da Costa, Bueno and French (1998), and Bueno and de Souza (1996)). To make room for the different interpretations, the neutral notion of *quasi-truth* has been introduced.

In this paper, we have two aims. We will first provide a new interpretation of the quasi-truth formalism: it is a "syntactic" reformulation of quasi-truth. Just as pragmatic truth was motivated by the writings of pragmatist philosophers, this syntactic reformulation bears some connections with the work of coherence theorists of truth. Secondly, we will argue for the usefulness of the syntactic notion, elaborating, in terms of it, a new nominalist reconstruction of mathematics. In section 2, we review the formal framework of quasi-truth (the framework is called the *partial structures approach*). The syntactic reformulation is then provided in section 3. Finally, in sections 4-6, an application of the syntactic notion to the philosophy of mathematics is put forward.

2. PRAGMATIC TRUTH AND PARTIAL STRUCTURES

The partial structures approach relies on three main notions: partial relation, partial structure and pragmatic truth (or quasi-truth).[1] One of the main motivations for introducing this proposal comes from the need for

[1] This approach was first presented in Mikenberg, da Costa and Chuaqui (1986) and da Costa (1986). For further developments, see da Costa and French (1989), (1990), (1993), (2003), da Costa, Bueno and French (1998), and Bueno (1997), (1999*a*) and (1999*b*).

supplying a formal framework in which the "openness" and "incompleteness" of information dealt with in scientific practice can be accommodated in a unified way (da Costa and French (1990)). This is accomplished by extending, on the one hand, the usual notion of structure – in order to model the partialness of information we have about a certain domain (introducing then the notion of a *partial* structure) – and on the other hand, by generalizing the Tarskian characterization of the concept of truth for such "partial" contexts (advancing the corresponding concept of *quasi-truth*).

In order to introduce a partial structure, the first step is to formulate an appropriate notion of *partial relation*. When investigating a certain domain of knowledge Δ, we formulate a conceptual framework that helps us in systematizing and organizing the information we obtain about Δ. This domain is tentatively represented by a set D of objects, and is studied by the examination of the relations holding among D's elements. However, given a relation R defined over D, often we do not know whether all the objects of D (or n-tuples thereof) are related by R. This is part and parcel of the "incompleteness" of our information about Δ, and is formally accommodated by the concept of partial relation. More formally, let D be a non-empty set; an n-place *partial relation* R over D is a triple $\langle R_1,R_2,R_3\rangle$, where R_1, R_2, and R_3 are mutually disjoint sets, with $R_1 \cup R_2 \cup R_3 = D^n$, and such that: R_1 is the set of n-tuples that (we know that) belong to R, R_2 is the set of n-tuples that (we know that) do not belong to R, and R_3 is the set of n-tuples for which we do not know whether they belong or not to R. (If R_3 is empty, R is a usual n-place relation which can be identified with R_1.)

But in order to represent the information about the domain under consideration, we need a notion of *structure*. The following characterization is meant to supply a notion that is broad enough to accommodate the partiality usually found in scientific practice. The main work is done, of course, by the partial relations. A *partial structure* S is an ordered pair $\langle D, R_i \rangle_{i \in I}$, where D is a non-empty set, and $(R_i)_{i \in I}$ is a family of partial relations defined over D.[2]

Two of the three basic notions of the partial structures approach are now defined. In order to spell out the last one – quasi-truth – an auxiliary notion is required. The idea is to use, in the characterization of quasi-truth, the resources supplied by Tarski's definition of truth. However, since the latter is only defined for full structures, we have to introduce an intermediary notion of structure to "link" full to partial structures. And this is the first role of those structures that extend a partial structure A into a full, total structure (which are called A-normal structures). Their second role is purely model-theoretic, namely to put forward an interpretation of a given language and, in terms of it, to characterize basic semantic notions. But how are A-normal structures to be defined? Let $A = \langle D, R_i \rangle_{i \in I}$ be a partial structure. We say that the structure $B = \langle D', R'_i \rangle_{i \in I}$ is an *A-normal structure* if (i) $D = D'$, (ii) every constant of the language in question is interpreted by the same object both in A and in B, and (iii) R'_i extends the corresponding relation R_i (in the sense that each R'_i, supposed of arity n, is defined for all n-tuples of elements of D'). Notice that, although each R'_i is *defined* for all n-tuples over D', it holds for some of them (the R'^1_i-component of R'_i), and it doesn't hold for others (the R'^2_i-component).

As a result, given a partial structure A, there are *several* A-normal structures. Suppose that, for a given n-place partial relation R_i, we don't know whether $R_i a_1 \ldots a_n$ holds or not. One way of extending R_i into a full R'_i relation is to look for information to establish that it *does* hold, another way is to look for the contrary information. Both are *prima facie* possible ways of extending the partiality of R_i. But the same indeterminacy may be

[2] Notice that the partiality modeled here is due to the incompleteness of our knowledge about the domain under investigation. Given further information, a partial relation may become total. Therefore, the partiality is not understood as a property of the world, but it is the result of our lack of information about the latter. We are concerned here with an "epistemic", not an "ontological" partiality.

found with other objects of the domain, distinct from $a_1,..., a_n$ (for instance, does $R_ib_1...b_n$ hold?), and with other relations distinct from R_i (for example, is $R_jb_1...b_n$ the case, with $j \neq i$?). In this sense, there are *too many* possible extensions of the partial relations that constitute A. Therefore we need to provide constraints to restrict the acceptable extensions of A.

In order to do that, we need first to formulate a further auxiliary notion (see Mikenberg, da Costa and Chuaqui (1986)). A *pragmatic structure* is a partial structure to which a third component has been added: a set of accepted sentences P, which represents the accepted information about the structure's domain. (Depending on the interpretation of science that is adopted, different kinds of sentences are to be introduced in P: realists will typically include laws and theories, whereas empiricists will tend to add certain laws and observational statements about the domain in question.) A *pragmatic structure* is then a triple $A = \langle D,R_i,P\rangle_{i\in I}$, where D is a non-empty set, $(R_i)_{i\in I}$ is a family of partial relations defined over D, and P is a set of accepted sentences. The idea, as we shall see, is that P introduces constraints on the ways that a partial structure can be extended.

Our problem now is, given a *pragmatic* structure A, what are the necessary and sufficient conditions for the existence of A-normal structures? We can now spell out one of these conditions (see Mikenberg, da Costa and Chuaqui (1986)). Let $A = \langle D,R_i,P\rangle_{i\in I}$ be a pragmatic structure. For each partial relation R_i, we construct a set M_i of atomic sentences and negations of atomic sentences, such that the former correspond to the n-tuples that satisfy R_i, and the latter to those n-tuples which do not satisfy R_i. Let M be $\cup_{i\in I}M_i$. Therefore, a pragmatic structure A admits an A-normal structure if, and only if, the set $M\cup P$ is *consistent*.[3]

We can now formulate the concept of quasi-truth. A sentence α is *quasi-true* in a pragmatic structure $A = \langle D,R_i,P\rangle_{i\in I}$ according to an A-

[3] For further discussion, see Bueno (1997, section 3.1.), and Bueno and de Souza (1996).

normal structure $B = \langle D',R'_i \rangle_{i \in I}$ if α is true in B (in the Tarskian sense). If α is not quasi-true in A according to B, we say that α is *quasi-false* (in A according to B). Moreover, we say that a sentence α is *quasi-true* if there is a pragmatic structure A and a corresponding A-normal structure B such that α is true in B (according to Tarski's account). Otherwise, α is *quasi-false.*

Intuitively speaking, a quasi-true sentence α does not necessarily describe, in complete detail, the whole domain to which it refers, but only an aspect of it – the one modeled by the relevant partial structure A. For there are several different ways in which A can be extended to a full structure, and in some of these extensions α may not be true. Thus, the notion of quasi-truth is strictly weaker than truth: although every true sentence is (trivially) quasi-true, a quasi-true sentence is not necessarily true (since it may be false in certain extensions of A).

In order to clarify the concept of quasi-truth, let us consider three objections that could be raised against it. (1) It may be argued that because quasi-truth has been defined in terms of full structures and the standard notion of truth, there is no gain with its introduction. In our view, there are several reasons why this is *not* the case. Firstly, as we have just seen, despite the use of full structures, quasi-truth is weaker than truth: a sentence which is quasi-true in a particular domain – that is, with respect to a given partial structure A – may not be true if considered in an extended domain. Thus, we have here a sort of "underdetermination" – involving distinct ways of extending the same partial structure – that makes the notion of quasi-truth especially appropriate for the empiricist. Secondly, one of the points of introducing the notion of quasi-truth, as da Costa and French (1989), (1990) and (1993) have argued in detail, is that in terms of this notion, a formal framework can be advanced to accommodate the "openness" and "partialness" typically found in science. Bluntly put, the actual information at our disposal about a certain domain is modeled by a *partial* (but not full) structure A. Full, A-normal structures represent ways of extending the actual information which are possible according to A. In this respect, the use of full structures is a semantic expedient of the framework (in order to provide a definition of quasi-truth), but no epistemic import is assigned to them. Thirdly, full structures can be ultimately dispensed with in the formulation of quasi-truth, since the latter can be characterized in a different way, but still preserving all its features, independently of the standard Tarskian type account of truth (Bueno and de Souza 1996). This provides, of course, the strongest argument for the dispensability of full structures (as well as of the Tarskian account) *vis-à-vis* quasi-truth. Therefore, full, A-normal structures are entirely inessential from the formal point of view (although the realist may insist upon them); their use here is only a convenient device.

(2) The second objection raises the problem of whether quasi-truth is a *truth* notion at all. If "quasi" means something like "approximate", the objection goes, what we have is, at most, an *epistemic* notion – but which has nothing to do with truth. After all, truth is radically *non*-epistemic, it is *not* constrained by evidence. In reply, we notice that there is no reason why an *epistemic* notion of truth should not be countenanced. On the contrary, as Putnam indicated with his celebrated model-theoretic argument against metaphysical realism, serious difficulties are faced by a *non*-epistemic account of truth (see Putnam (1976) and (1980)). The fact that quasi-truth is constrained by evidence, rather than being a difficulty, is in fact the strength of this notion, since it allows us to accommodate the dependence of our truth claims on the evidence at our disposal (which is represented by a given partial structure). In this respect, our judgments of quasi-truth reflect the current state of information about the domain we are concerned with.

But there is a further argument to the effect that quasi-truth is a truth notion. In his discussion of the concept of truth in *The Problems of Philosophy*, Russell indicates "three requisites which any theory of truth must fulfill" (Russell 1912, Chapter XII). The first is that not everything that might be true actually is true – and so, Russell notes, truths cannot be identified with facts. The second is that "a world of mere matter, since it would contain no beliefs or statements, would also contain no truth or falsehood". The third is "the truth or falsehood of a belief always depends upon something which lies outside the belief itself". What is striking – and encouraging – about Russell's three requisites is that all of them are satisfied by the notion of quasi-truth.

With regard to the first requisite, it is not the case that if a sentence might be quasi-true, it actually is quasi-true. For example, Aristotelian physics might be quasi-true (given the extant information about physical processes centuries ago), but it is not quasi-true. For it is not compatible with the accepted set of sentences *P* that inform current physical research. With regard to the second requirement, quasi-truth is a "property" of sentences or theories. So, "a world of mere matter", strictly speaking, would contain no quasi-truths. Moving now to the third requisite, the quasi-truth of a belief "depends upon something which lies outside the belief itself". After all, it depends on the relationships among the objects that constitute the domain under consideration, which "lie outside the belief" in question.[4] In this way, according to Russell's requirements, quasi-truth (or pragmatic truth) is really a *truth* notion.

(3) Finally, it may be argued: given that quasi-truth is formulated in terms of Tarski's account of truth-in-a-structure, and since this account does *not* provide an epistemic notion, it is unclear how quasi-truth *can be* epistemic. In reply, notice that, as opposed to Tarski's account, quasi-truth is always *relative* to a given *partial* structure. And given that such a structure represents our information about a given domain, not only our

[4] This point is surely realist sounding. But the divergence between realists and anti-realists depends on the *objects* that constitute the relevant domain of inquiry. An anti-realist like van Fraassen will accept that there are realist components in scientific practice, but they concern *observable* phenomena (van Fraassen (1980)). Realists, in turn, will typically argue that we should also be realists about the *unobservable* aspects of the world (see e.g. the papers in Churchland and Hooker (eds.) (1985)).

judgments of quasi-truth become relative to the extant information, but so does quasi-truth itself. After all, if a sentence α is quasi-true it will remain forever as such (given that if later on we discover that α is indeed true, it will still be quasi-true). Moreover, α's quasi-truth depends upon the partial structure we consider. In a *different* partial structure, α might not be quasi-true. This indicates the sense in which quasi-truth is an epistemic notion: it is relative to the information we have, that is, relative to a partial structure.

Having presented the formalism of quasi-truth, we shall now put forward a new interpretation of it, stressing the connection with the coherence theory of truth.

3. THE COHERENCE THEORY OF TRUTH AND ITS REFORMULATION

There are several versions of the so-called coherence theory of truth. And it is no trivial task to provide an exegesis of the writings of defenders of this theory, such as Bradley (1914) and Bosanquet (1885). In what follows, we have no intention to provide such exegesis. We will rather put forward a version of the coherence theory inspired by certain ideas from Neurath (1932-1933) (for a brief discussion, see Gillies 1993, pp. 122-124). According to Neurath, truth has some "social" features. We are born in a setting that imposes upon us certain truths; that is, we are bound to accept certain propositions or to have certain beliefs because they force themselves upon us, either from our daily life or from science. We then try to extend such "first truths" in order to obtain maximally consistent belief systems.

However, faced with epistemological difficulties, it might be necessary to revise our belief system, including those propositions expressing "immediate experiences". In Neurath's view, even protocol sentences are open to criticism and might be expelled from the body of true propositions. In his celebrated words, "We are like sailors who must rebuild their ship on the open sea, never able to dismantle it in dry-dock and to reconstruct it there out of the best materials" (Neurath (1932-1933), p. 201).

The process of expanding and contracting a belief system is described by Neurath as a process of truth evaluation in the following terms.

> In unified science we try to construct a non-contradictory system of protocol sentences and non-protocol sentences (including laws). When a new sentence is presented to us we compare it with the system at our disposal, and determine whether or not it conflicts with that system. If the sentence does conflict with the system, we may discard it as useless (or false), as, for instance, would be done with "In Africa lions sing only in major scales". One may, on the other hand, *accept* the sentence and so change the system so that it remains consistent even after the adjunction of the new sentence. The sentence would then be called "true".
>
> The fate of being discarded may befall even a protocol sentence. No sentence enjoys the *noli mi tangere* which Carnap ordains for protocol sentences. (Neurath 1932-1933, p. 203)

In this way, in Neurath's view, by expanding and contracting our belief system, we try to adjudicate the truth of the resulting system. However, in this process, there is no privileged, unquestionable body of sentences (see, more recently, Fuhrmann (1997)). Every sentence is open to revision. As a result, a holism is introduced, since the expansion or the contraction of a belief system depends on the properties of the system as a whole. The aim is to provide *maximally consistent* systems.

The requirement of maximal consistency derives from the need for having a belief system as broad and comprehensive as possible. Bradley, for instance, is very clear about this. Discussing the issue of the test of truth, he stresses that we should take *both* coherence and comprehensiveness as basic. As he points out:

> The test [of truth] which I advocate is the idea of a whole of knowledge as wide and as consistent as may be. In speaking of system I mean always the union of these two aspects, and this is the sense and the only sense in which I am defending coherence. If we separate coherence [...] from comprehensiveness, then I agree that neither of these aspects of system will work by itself. [...] All that I can do here is to point out that both of

> the above aspects are for me inseparably included in the idea of system, and that coherence apart from comprehensiveness is not for me the test of truth or reality. (Bradley 1914, pp. 202-203)

In other words, it is not enough to have a coherent system; the latter should also be comprehensive. In our view, one way of accommodating the comprehensiveness of a system is by stressing its *maximal* consistency. This becomes a crucial requirement. After all, if a system Σ is maximally consistent, it will lose its consistency if we add a further sentence to Σ. In this way, by emphasizing maximal consistency, we can represent the idea of a system being "as wide and as consistent as may be". It goes without saying that the comprehensiveness also meshes nicely with the holistic component mentioned above, since it is a *system* as a whole that is supposed to be comprehensive.

But what should be said about the notion of *coherence*? It's often the case that coherence theorists don't provide an account of what characterizes the coherence of a belief system. The coherence is only taken to be *more* than mere consistency. In our view, besides consistency, coherence involves *pragmatic* factors, such as simplicity, tractability, close interrelationship between the various parts of the conceptual system under consideration etc. (The point is not new, but it is worth stressing.) We grant that there is some vagueness with such pragmatic factors (the notions of simplicity and tractability, for instance, depend on the context we consider). However, the notion of coherence is similarly vague: under what conditions can we claim that a system is *coherent*? One of the advantages of stressing pragmatic factors in the understanding of coherence is that we make the latter more manageable (and less metaphysical). For despite the vagueness of simplicity and tractability, given a particular context we are typically able to determine whether a theory is simpler and more tractable than another. In this way, we are able to determine if such a theory (supposed consistent) is also coherent.

Can we provide a formal framework that accommodates, at least in part, some of these intuitions? Here is an attempt. Similarly to what happens with Tarski's theory of truth and with pragmatic truth (or quasi-truth), as conceived by da Costa *et al.* (see Mikenberg, da Costa and Chuaqui (1986), and da Costa, Bueno and French (1998)), truth depends on the language. Let us assume, for the sake of simplicity, that we are dealing with a first-order language with identity. We shall not make any changes in the logical laws and rules, which are those provided by classical logic. (In future works, however, we shall explore the consequences of changing the underlying logic.)

Let Δ be a given domain of knowledge; in principle, it could be the whole universe. We shall denote by P the set of basic sentences, which are taken to be true *ab initio*. On the other hand, we shall assume that some atomic propositions are given which partially determine some predicates and relations of the language in question, connecting some of its constants. We shall then consider the maximally consistent sets of sentences that contain the partially defined relations, or more precisely, the atomic sentences above, and the elements of P.

We shall call a *basic coherence structure* the following linguistic construction:

$$A = \langle D, R_i, P \rangle_{i \in I},$$

where D is the set of constants of the language under consideration, say L, R_i are the atomic sentences which "define" the relations R_i, $i \in I$, and P is the set of basic sentences.

A maximally consistent set of sentences that contains R_i, $i \in I$, and P is called an *A-normal set of sentences.* We shall assume that there are such sets, since it is not difficult to establish conditions for their existence in A. We say that a sentence α of L is *coherently true in* A *according to an* A*-normal set of sentences* Σ if $\alpha \in \Sigma$. If $\alpha \notin \Sigma$ we say that α is *coherently false in* A *according to an* A*-normal set of sentences* Σ. Moreover, α is *coherently true in* A if there is an $\mathcal{A}$-normal set of sentences Σ such that $\alpha \in \Sigma$. If there is no such a set, we say that α is *coherently false in* A.[5]

If the set Σ has the above property of coherence, we can say that α is coherently true, strictly speaking, in $\mathcal{A}$ according to Σ. We can even claim that there is only one maximally consistent set which is coherent. In this case, the truth as coherence is unique. The existence of such a set could be guaranteed by a *function of coherence* that selects, among the various maximally consistent extensions, that which better capture certain components of the coherence. (In future works, we shall study the properties of this function.)

The analogy between this characterization of truth as coherence and pragmatic truth (as defined in Mikenberg, da Costa and Chuaqui (1986)) is clear: the former is a syntactic version of the latter, and the latter is the semantic counterpart of the former. In both cases, truth is characterized in terms of structures: pragmatic truth is truth in a pragmatic structure; truth as coherence is truth in a basic coherence structure. Moreover, in both cases, an auxiliary "full" structure is introduced: an *A-normal structure*, which extends the partial information about $\mathcal{A}$ to full information, in the case of pragmatic truth; and an $\mathcal{A}$-

[5] In this characterization of "coherence truth", we have assumed classical logic. But there is no reason why we shouldn't extend the account into a non-classical setting. For example, we could assume that the underlying logic is paraconsistent. (Roughly speaking, a logic is paraconsistent if it can be the underlying logic of inconsistent but non-trivial theories. A theory T is non-trivial if there is a sentence of the language in question which is not a consequence of T; for details, see da Costa, Béziau and Bueno (1995).) In a paraconsistent setting, instead of requiring the existence of maximally consistent sets of sentences, it is enough to demand that there are sets of sentences that are maximally *non-trivial*. The latter sets may even be inconsistent. Once the underlying logic is paraconsistent, what matters is the lack of triviality.

normal set, which is maximally consistent, in the case of the coherence account of truth.[6]

This suggests that there is a certain "equivalence" between the coherence theory (as put forward above) and the pragmatic theory of truth, since they have the "same structure".[7] Now, as presented by Mikenberg, da Costa and Chuaqui, the pragmatic theory encompasses the correspondence theory, since the accepted sentences P are taken to be true in the *correspondence* sense. Thus, we can say that the three major interpretations of the notion of truth (the correspondence, the coherence and the pragmatic accounts) can be put together in the present formal framework. Each of them captures a distinct aspect of truth: syntactic features are accommodated by a coherence theory; semantic aspects by pragmatic truth;[8] and a strong relationship between language and the world by the correspondence theory. This may help to explain why these three interpretations have been so widespread in philosophical discussions about truth.[9]

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[6] The outcome of the formal similarities between pragmatic truth and the coherence theory is that the mathematical results that have been established for pragmatic truth (see, e.g., da Costa, Bueno and French (1998)) can be easily adapted to the present account of the coherence theory.

[7] Of course, in terms of the "content", the two theories are different since one is syntactic the other is semantic.

[8] As noted above, pragmatic factors are involved in both the coherence and the pragmatic theories of truth.

[9] Haack provides an interesting combination of the foundationalist and the coherentist views, articulating what she calls "foundherentism" (see Haack (1993)). Haack's proposal focus on epistemological issues, in particular, on the theory of justification. Her goal is to provide an "explication of epistemic justification" that makes room for "the relevance of experience to empirical justification", as the foundationalist would have it, but which also allows "pervasive mutual support among beliefs", as the coherentist emphasizes (Haack (1993), p. 73). Instead of focusing on theories of justification, we emphasize here (i) issues from *theories of truth* and, in particular, (ii) the *formal core* that unifies the three best known conceptions of truth.

4. APPLICATION: COHERENTISM AND THE PHILOSOPHY OF MATHEMATICS

Having provided a new interpretation of quasi-truth, in terms of a "syntactic" notion of coherence, we shall now explore one application of the resulting formal framework. The crucial difference between pragmatic truth and the coherence theory of truth, as formulated here, is that the domain D of a basic coherence structure $A = \langle D, R_i, P\rangle_{i\in I}$ is made of *linguistic* entities (constant terms of the language), and so are the remaining components of A: atomic *formulas* R_i and a set of accepted *sentences* P. As opposed to this, the components of a partial structure (or a pragmatic structure) are *not* linguistic: they are particular *objects* of a domain of inquiry and partial *relations* defined over them (disregarding, of course, the set P of accepted sentences). And it is this difference between the two notions which makes truth as coherence a particularly interesting concept. In what follows, we shall explore the usefulness of the coherence theory in a context where syntactic notions are particularly strong: in the development of a nominalist view in the philosophy of mathematics.

Nominalism (about mathematics) is the view according to which there are no mathematical entities (such as sets, functions and numbers).[10] One of the difficulties faced by the proposal is to show that mathematics can be reformulated without ontological commitment to mathematical objects. This is no easy task. In the contemporary scene, several proposals have been developed to try to accomplish this task – with limited degrees of success. We shall briefly consider two of them: Field's mathematical fictionalism and Hellman's modal-structuralism.

In a series of provocative works, Field has argued that there are no mathematical objects (therefore, all existential mathematical assertions are

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[10] Of course, there are other kinds of nominalism depending on the domain of inquiry under consideration. But typically the nominalist tries to avoid ontological commitment to abstract entities. Depending on the domain in question such entities may include universals, propositions, mathematical objects etc. Here we are only concerned with nominalism *about mathematics*.

false). However, the success of applied mathematics can be explained in terms of its *conservativeness*. A mathematical theory is conservative if it is consistent with every internally consistent physical theory (see Field (1980) and (1989)).

The main idea is that mathematics, despite being false, is *useful*, since it helps us in shortening our derivations, both in science and in mathematics itself. In order to support this claim, Field provides examples in geometry – considering Hilbert's axiomatization of geometry, without the use of real numbers – and in classical mechanics – putting forward a nominalist formulation of Newtonian gravitational theory (see Field (1980)). As he points out, an important consequence of the conservativeness of mathematics is that it does not yield new empirical results. For if a mathematical theory M is conservative, given a nominalist assertion A (i.e. an assertion in which there is no quantification over mathematical objects) and a body B of such assertions, A is a consequence of B + M only if A is a consequence of B alone. In other words, if M is conservative, its use does not bring about any new nominalist consequence (see Field (1980) and (1989)).

But, as Field points out, in order for this account to work, we need first to have nominalist versions of scientific theories; that is, formulations of those theories which do not quantify over mathematical entities. After all, as we saw, the conservativeness of mathematics concerns only *nominalist* bodies of assertions. It is therefore crucial for Field's program to provide nominalist restatements of particular scientific theories. In his 1980 book, he presented a detailed nominalist reformulation of Newtonian gravitational theory, and left open the possibility of extending his nominalization technique to other scientific theories.

One of the major difficulties that have been raised with regard to Field's program comes in at this point. In nominalizing Newtonian theory, Field was committed to space-time points as, roughly speaking, surrogates for real numbers. In this way, in order for his proposal to be nominalist about mathematical objects, it has to be realist about space-time. The problem here, as Malament (1982) pointed out, is that it is far from clear how this strategy can be extended to theories distinct from Newtonian gravitational theory, such as quantum mechanics. And it is only when we are able to nominalize more domains of science, including quantum mechanics, that the prospects of a Field-type nominalism will increase. This point explains why Field's proposal, despite its interest and importance, has been received with so much skepticism (for additional critical remarks, see Chihara (1990)).

Hellman provided a different nominalization strategy. Elaborating on a suggestion of Putnam's (see Putnam (1967)), Hellman argued that mathematics can be nominalized by invoking appropriate modal operators (see Hellman (1989) and (1996)). The main idea is that, instead of being committed to actual mathematical objects, we can decrease our ontological commitment by quantifying only over *possible* mathematical structures.

Two steps are then taken. The first is to present an appropriate translation scheme in terms of which each ordinary mathematical statement S is taken as elliptical for a hypothetical statement; namely, that S *would hold* in a structure of the appropriate kind (the translation is run in a modal second-order language). For example, if we are considering number-theoretic statements, such as those articulated in Peano arithmetic (PA, for short), the structures we are concerned with are "progressions" or "ω-sequences" satisfying PA's axioms. In this case, each particular statement S is to be (roughly) translated as

$$\Box\forall X\,(X \text{ is an } \omega\text{-sequence which satisfies PA's axioms} \rightarrow S \text{ holds in } X).$$

This is the *hypothetical component* of the modal-structural interpretation (for a detailed analysis and a more precise formulation, see Hellman (1989), pp. 16-24). The *categorical component* constitutes the second step (see (1989), pp. 24-33). The idea is to assume that the structures of the appropriate kind are logically possible. In that case, we have that

$$\Diamond \exists X \text{ (}X\text{ is an } \omega\text{-sequence satisfying PA's axioms).}$$

Following this approach, truth preserving translations of mathematical statements can be presented without ontological costs, given that only the *possibility* of the structures in question is assumed. Moreover, as Hellman stresses, the modal operators introduced in the translation are taken as *primitive* (Hellman (1989)).

The main problem with this move derives from the introduction of these operators. As Shapiro argued, since they are taken by Hellman as primitive, and given Hellman's rejection of the set-theoretical framework, it is unclear why the modal-structural interpretation is entitled to the traditional, model-theoretic explications of these operators. After all, these explications are articulated in set theory (see Shapiro (1993), p. 467). Furthermore, in order to account for mathematical knowledge – and this is one of the issues in Hellman's agenda (see his (1989), pp. 3-4) – instead of an epistemology of abstract objects, we need an epistemology of *possible* abstract objects. However, no reason is given why the latter is more tractable than the former (see Shapiro (1993), p. 466).

Since nominalism certainly has interesting features (the fact that it allows the partial reduction of ontological commitment is certainly among the most attractive of them), one wonders whether a new formulation of nominalism can be presented which does not face the predicaments found in Field's or in Hellman's proposals. In other words, what we need is a formulation of nominalism that does not have the limitations of Field's account (and thus can be extended to science as a whole), and does not rely on modal operators, which brought so many problems to Hellman's view.

In what follows, using the coherence theory of truth indicated above, we shall give a first step towards this end, sketching a nominalist view along the required lines. It is certainly not the only proposal to meet these requirements (nominalism after all has many faces), but it is worth spelling it out. After outlining the proposal in section 5, we shall discuss some of its features and limitations in section 6.

5. SYNTACTIC MODELS OF SET THEORY

The main idea of this nominalist view is that mathematics doesn't have to be true to be good: it only has to be *coherently true*. As characterized above, quasi-truth is strictly weaker than truth, and so is the coherence account of truth. After all, a *false* sentence may be included in a maximally consistent set of sentences. So, similarly to Field's approach, truth is not taken here as an aim of mathematics; something weaker is: *truth as coherence*. Having this aim in mind, what we have to do is to construct a model of set theory (in which most classical mathematics can be formulated) in *syntactic* terms. As we shall see, the model will be constructed with the language of set theory itself. So it is denumerable, and we won't incur any ontological commitment beyond that level. But before discussing the features of the model, let us spell out the overall strategy.

First, let us introduce some terminology. ZF is the system of Zermelo-Fraenkel set theory, and ZFC is ZF with the axiom of choice (for details of this system, see Shoenfield (1967)). Let *L*(ZF) be the language of ZF, and let us suppose that we add to *L*(ZF) Hilbert's ε symbol (see Leisering (1969), da Costa (1980), and Bourbaki (1968)). The ε symbol generates terms by biding variables of formulas, and it satisfies the following postulates, with obvious restrictions:[11]

[11] See again Leisering (1969), da Costa (1980), and Bourbaki (1968). We suppose, of course, that the concepts of formula, bound variable etc. have been appropriately extended.

(ε_1) $\varepsilon x\, A(x) = \varepsilon y\, A(y)$
(ε_2) $\forall x\, (A(x) \leftrightarrow B(x)) \rightarrow \varepsilon x\, A(x) = \varepsilon x\, B(x)$
(ε_3) $\exists x\, A(x) \rightarrow \exists x\, (x = \varepsilon x\, A(x) \wedge A(x))$
(ε_4) $\neg\exists x\, A(x) \rightarrow \varepsilon x\, A(x) = \varepsilon x\, (x \neq x)$

Let us denote by ZF_ε the system ZF having as its underlying logic the classical first-order predicate calculus with the ε symbol and the postulates above. The specific axioms and axiom schemes of ZF_ε are the same as ZF's, that is, the ε symbol does not occur in them. The following proposition can then be proved without difficulty using the second ε-theorem (see Leisering (1969), p. 79) and the results found in Shoenfield (1967), Chapter 9:

Theorem 1. If ZF is consistent, then ZF_ε, ZF_ε + AC, ZF_ε + GCH, and ZF_ε + V=L are also consistent. Moreover, ZF_ε + ¬AC, ZF_ε + AC + ¬GCH, and ZF_ε + V≠L are also consistent. (AC = axiom of choice, GCH = generalized continuum hypothesis, and V=L = Gödel's constructibility axiom.)

The proof of this theorem is constructive. Furthermore, all the results obtained by (syntactic) forcing and by Gödel's theory of constructible sets hold for ZF_ε (an exposition of syntactic forcing and Gödel's theory can be found in Shoenfield 1967, Chapter 9). Analogous remarks are also true for the results considered below.

Let ZF^ε be the system ZF_ε when the ε symbol can occur in ZF's specific axioms. Thus, we have the following results:

Theorem 2. If ZF is consistent, so is ZF^ε.

Theorem 3. The axiom of choice can be proved in ZF^ε.

Theorem 4. GCH is independent of ZF^ε.

Let T be the set of all closed terms (without free variables) of ZF^{ε} (or of ZF_{ε}). We can then introduce a denumerable (syntactic) model of ZF^{ε} (supposing it is consistent). In order to do so, we need the following definitions.

Definition 1. If τ_1 and τ_2 belong to T, then $\tau_1 \approx \tau_2 \leftrightarrow \vdash_{ZF\varepsilon} \tau_1 = \tau_2$.

Definition 2. If τ is a closed term of T, $\tau^* = \{x: \vdash_{ZF\varepsilon} \tau = x\}$.

Theorem 5. The relation $\approx$ is an equivalence relation over T.

Definition 3. M is the following structure for $L(ZF^{\varepsilon})$, in the sense of Shoenfield (1967) and da Costa (1980), with evident adaptations:

(1) The universe of M is $T/\approx$.

(2) $\tau_1^* = \tau_2^* \leftrightarrow \vdash_{ZF\varepsilon} \tau_1 = \tau_2$.

(3) $\tau_1^* \in \tau_2^* \leftrightarrow \vdash_{ZF\varepsilon} \tau_1 \in \tau_2$.

(4) To the ε symbol it corresponds a choice function (see Leisering (1967) and da Costa (1980)).

Theorem 6. If ZF^{ε} is consistent, then M is a model of ZF^{ε}. Moreover, if A is a sentence of $L(ZF^{\varepsilon})$, then

$$M \models A \text{ if and only if } \vdash_{ZF\varepsilon} A.$$

The last result is also valid for ZF^{ε}, ZF^{ε} + GCH, ZF^{ε} + ¬GCH, ZF^{ε} + V=L etc. In particular, the model M of ZF^{ε} + V=L is related to Cohen's minimal model (see Cohen (1966)).

If we add to ZF^{ε} new axioms that preserve the consistency of the resulting systems, we have "syntactic" models of these new systems. It should be noticed that the construction could be performed without ε. Of course, given the equiconsistency results between ZF, ZF^{ε} and ZF_{ε}, the introduction of ε does not bring any problems (and it simplifies the exposition). Moreover, by the second ε-theorem, ZF_ε is a conservative extension of ZF (without ε). The obtained model is also a model of ZF.[12]

6. A NEW FORM OF NOMINALISM?

The previous results show that there are denumerable, "syntactic" models of set theory (that is, constructed with the very language of this theory) in which all classical mathematics can be formulated. Indeed, the model *M* constructed above contains almost everything that we want from classical mathematics. Moreover, items (2) and (3) of Definition 3 provide an interesting "interpretation" of equality and membership. In this way, we have a "linguistic", "syntactic" formulation of mathematics, which can be taken as the basis for a new kind of nominalism.

The crucial feature of this proposal derives from the model *M*. Since it is formulated with the language of set theory, and given that this language is denumerable, all that is required is the existence of denumerably many objects. (After all, there are denumerably many objects in *M*'s domain.) These objects, as we saw, are equivalence classes of terms in the language of set theory. And *if* these objects are nominalistically acceptable (a big *if* as we will see[13]), it is then possible to interpret

12 Some comments on the literature about this issue are in order. (1) Apparently, the first author who treated systematically the issue of syntactic models of set theory was Ilse Novak (see Novak, 1951). (2) For the use of Hilbert's ε-symbol in the construction of models, see Guillaume (1964) (see also Guillaume, 1960). (3) The main text about the ε-symbol is Hilbert and Bernays (1934/39). An excellent book about the ε-symbol and its applications, in which the ε-theorems are presented in an extremely clear way, is Leisering (1969) (see also da Costa (1980)). (Leisering's book has a mistake in the proofs of the ε-theorems; but this was corrected in Flannagan (1975).) For a development of set theory with Hilbert's ε-symbol, see Bourbaki (1968), Chapters 1 and 2 (Bourbaki represents the ε-symbol by τ).

13 We discuss below the issue as to whether equivalence classes of terms are nominalistically acceptable.

mathematics nominalistically only assuming denumerably many objects. In particular, given that we require at most denumerably many objects, we incur far less commitments than Field does. After all, in a clear way, there are "more" non-denumerable objects than denumerable ones.

But what have we established with our interpretation? It may be argued that we haven't established more than the relative consistency of ZF. Of course, the model suggested was "syntactic" (in the sense that it was formulated using ZF's language). But still, does this show that ZF is *true*? Clearly, this is not the case. And certainly the nominalist is not willing to claim that a mathematical theory has to be true to be good (see Field (1980) and (1989)). If the theory is *coherently true*, in the sense indicated above, it should be enough. And the point of constructing the syntactic model M is exactly to highlight that classical mathematics is coherently true in that sense.

In other words, if we are able to claim that mathematics is *consistent* (or, at least, relatively so), this is enough for the nominalist needs (supposing that other pragmatic factors, such as simplicity and tractability, are also satisfied). But notice that, as opposed to Hellman's approach, we do *not* postulate a primitive notion of modality to support this claim. Rather, we construct a model of set theory, syntactically.[14]

As a result, the present proposal is different from Field's, since it does not quantity over non-denumerably many entities, and it also differs from Hellman's, given that no primitive modal operator is countenanced.[15]

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[14] It is worth noting that if we add to ZFC convenient postulates, for instance guaranteeing the existence of inaccessible cardinals, of compact cardinals, of measurable cardinals etc., if ZFC remains consistent, M still exists and is *denumerable*. However, "within" M there are monstrous cardinals and sets. In a certain sense, the denumerable "captures" *everything*.

[15] According to *radical* nominalists, in nominalizing mathematics we should provide concrete physical objects as surrogates for mathematical entities. (Roughly speaking, Field is a radical nominalist in this sense, since he "replaces" space-time regions by real numbers as part of his nominalization strategy.)

But someone may argue that this proposal suffers a serious difficulty. The construction of the model of set theory *M* involves certain "abstractions". For instance, the objects of *M* are equivalence classes, which are *abstract objects*. Therefore, the present account is not completely nominalist.[16] Moreover, even the *symbols* of the language of set theory are ultimately equivalence classes, characterized by the similarity between them. In this case, even if those symbols were *physical* objects, they still would depend upon a similarity relation. For example, the symbols "s" and "S" are similar, and it is crucial for the development of the syntax of set theory to have such a relation between the symbols, since it will then be used to define similarity between terms and formulas of the language. But this similarity relation is *abstract*. Finally, our approach depends upon the notion of denumerability; but this notion is defined with reference to *natural numbers*.

In reply, we first notice that the use of platonist resources in devising a nominalist proposal can be justified as *a reductio* of platonism (see Field (1980)). In other words, we are showing how, by using platonist techniques, we can avoid ontological commitment to those very entities countenanced by platonism. Secondly, and more importantly, we could adopt a *constructivist* attitude towards the symbols of the language, stressing our capacity of *construction* of mathematical objects (*à la* Brouwer). The similarity relation that is needed to define the relevant equivalence classes is the product of *our own* constructions. *We* are the ones who determine the similarity, and in this sense the equivalence classes can be seen as the

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Although we are *not radical* nominalists, we think the syntactic model presented above can be useful for those who are. All they have to do is to "reinterpret" the construction of the model *M*, assigning to each object in *M*'s domain a given physical object. All that is required to accomplish this is the existence of denumerably many concrete objects.

[16] We recall that nominalism is the doctrine according to which there are *no* abstract objects.

result of our conceptual constructions (see da Costa (1997)).[17] Thus, if the nominalist adopts a constructivist metalanguage, he or she is able to use the present reformulation of the coherence theory of truth as the basis for a nominalistic reconstruction of mathematics. After all, the symbols needed to develop the approach can be regained in terms of the constructivist metalanguage.[18]

As we saw, what was needed was a nominalization strategy that does not face the two main problems faced by the previous forms of nominalism. In this context, the advantages of the present view are twofold:

(1) It provides a nominalization strategy of mathematics which is not limited to some physical theories (as Field's proposal is), since it applies to classical mathematics as a whole, and in particular to the mathematics required to formulate current theories in physics. So, the present proposal provides a nominalization strategy for mathematics directly. There is no need for focusing only on applied mathematics as Field does.

(2) The present view does not require modal operators, as Hellman's account does. The (relative) consistency of mathematics is not taken as primitive, but is articulated by the construction of appropriate "syntactic" models. As opposed to Hellman's approach, the present view takes mathematical claims at face value: there is no translation scheme in the nominalization of mathematics. Mathematicians' talk of *consistency* can be taken seriously. But, roughly speaking, the notion of consistency is taken here in its "syntactic" form, not in its semantic version.

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[17] This is a further aspect in which the version of nominalism suggested here assigns a role to *pragmatics*.

[18] It is worth noting that the present proposal also makes room for Bourbaki's reconstruction of mathematics (see, e.g., Bourbaki (1968)). Thus, the whole of classical mathematics can be regained in a nominalist setting, in terms of the syntactic reformulation of quasi-truth and the nominalization strategy presented.

In this way, by adopting a syntactic interpretation of quasi-truth, new steps towards the nominalization of mathematics are taken. And if what we said here is nearly right, an argument for the usefulness of the coherence theory of truth is also presented.

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