CDD: 005.131

TRAPPED ION QUANTUM COMPUTING AND THE PRINCIPLES OF LOGIC

ALFREDO PEREIRA JR

Adjunct Professor
Instituto de Biociências, UNESP
Campus de Botucatu
18618-000 BOTUCATU, SP
BRAZIL

apj@ibb.unesp.br

ROBERSON SARAIVA POLLI

Research Associate
Instituto de Biociências, UNESP
Campus de Botucatu
18618-000 BOTUCATU, SP
BRAZIL

Abstract: An experimental realization of quantum computers is composed of two or more calcium ions trapped in a magnetic quadripole. Information is transferred to and read from the ions by means of structured lasers that interact with the ions' vibration pattern, causing changes of energy distribution in their electronic structure. Departing from an initial state when the ions are cooled, the use of lasers modifies the internal state of one ion that is entangled with the others, then changing the collective states. In such quantum computers, some of the physically possible electronic states are avoided or not taken into consideration, to force the system to work as a binary device. In this essay, we discuss the dynamics that the ions could spontaneously display and its possible implications for the principles of computational logics.

Key-words: Principles of Logic. Quantum Computation. Electronic Structure. Calcium Ion.

INTRODUCTION

In the classical Turing machine paradigm, computations are carried by means of symbolic operations implemented in physical devices, with the usage of a binary language. In this context, the property of *multiple realizability* of such machines (i.e., the possibility of implementing the computations in a variety of physically different systems; see Bickle 1998) is heavily dependent on the fact that a large variety of physical systems is susceptible of displaying binary states, e.g. being magnetized or not magnetized, transmitting or not transmitting an electrical current, reflecting or not reflecting light.

McCullough and Pitts (1943) conceived the neuron as a binary device, with two kinds of states relevant for computational processes: "firing" and "not-firing". This simplification of neurobiological reality was useful for the development of the first models of neural networks, but does not correspond to current knowledge in neurobiology. Neuroscience evolved to understand that besides receiving and sending signals neurons perform important bio-molecular functions. Neuronal firing occurs when the membrane electric potential reaches a threshold and transmits a signal along the axon. However, under the threshold value there are important functions controlled by neuronal excitation, as the activation of membrane receptors and voltage-gated ion channels, triggering a cascade of events in the cell (for a review, see Pereira Jr. and Lungarzo 2005).

A "naturalistic" paradigm of computation, focusing on how computational processes spontaneously occur in biological, molecular and quantum systems, questions the binary assumption present in traditional approaches to computation – including the neural network approaches derived from the McCullough and Pitts modeling. In natural systems, the computations are determined by their respective physical and biological properties, instead of being determined by a "program" artificially introduced by the engineer.

Together with the developing field of Non-Classical Logics, such naturalistic efforts suggest that the three Aristotelian Principles of Classical Logic, instead of being absolutely necessary, are pragmatic choices that helped logicians and computer scientists to achieve their historical goals.

In this essay we discuss the logical principles implicit in the linguistic description of the computations carried by the calcium ion. This ion is important for computational as well as biological purposes (the biological importance will not be discussed here; see Jaiswal 2001; Carafoli 2002). The computational importance derives from the fact that the ion is sufficiently complex to carry information in its electronic structure, while being sufficiently simple to be experimentally manipulated.

One of the most successful and promising experimental realizations of quantum computers is the Ion-Trap Quantum computer (ITQC), using the calcium cation (Cirac and Zoller 2000; Kielpinski et al. 2002). This kind of quantum computer is composed of two or more calcium ions trapped in a magnetic quadripole. Information is encoded in the computer and read from it by means of structured lasers. The lasers interact with the external vibrational activity of the ion, then changing its internal state (i.e. the distribution of energy in the electronic structure). Departing from an initial state when all the ions are cooled to their ground state, the lasers modify one ion that is entangled with the others, then also changing their internal states.

In such artificial quantum computers, some of the physically possible transitions of electronic states of the ions are avoided or not considered, to make the system work as a binary device. In this essay, we discuss the dynamics that the ions would spontaneously display when not constrained to work as a binary device, and its possible implications for the principles of computational logics.

1. SOME PHILOSOPHICAL ASSUMPTIONS

The principles of classical computation converge with the three principles of Aristotelian Logic: the Principle of Identity (PI), the Principle of Non-Contradiction (PNC) and the Principle of the Excluded Middle (PEM).

How are such classical principles of logic expressed in the classical computational context? Since this issue is controversial in the literature, here we make some philosophical assumptions that condition the analysis presented in the essay.

It is important to note that we do not propose a *realist* approach to classical or quantum logic, but on the contrary our analysis is restricted to the *language* used to describe such phenomena. The computational language – including its syntactic and semantic aspects – is meant to be or not to be binary, and to follow or not to follow the classical principles of logic. This approach differs fundamentally from the framework adopted by realistic views of quantum logic, as discussed below.

In our approach, we first assume that in the classical context PI implies that the values of the properties (symbols, predicates) *do not change* with time or under any operation undergone by the system.

Second, we assume that PEM implies that in a binary choice *only two outcomes are permitted*, e.g. or 0 or 1. Any other possibility (e.g. 1.5) is forbidden.

Third, we assume that PNC rules out any mix of 0 and 1. Since there is no other admitted possibility besides 0 and 1 (from the previous assumption of PEM), then 0 is *semantically* equivalent to the *negation* of 1, and vice-versa. Therefore any mix of 0 and 1 is a contradiction.

Quantum Physics (QP) has inspired new developments in Logic, particularly by triggering the field of Quantum Logics.

The main feature of QP that has historically attracted the attention of philosophers is the phenomenon of superposition, a well-known property of isolated quantum particles. In a superposed state, a particle has a probability of being in state A, B, or any combination of A and B.

In the linguistic description of this case, according to our above assumptions, there is a violation of PNC, since the particle is referred to as possibly being in states A and non-A at the same time. In the final section of the essay we return to the discussion of the relation of superposition and PNC.

Does the violation of PEM occur? The answer to this question is more difficult, since the discussion in the philosophy of QT often assumes a binary linguistic framework.

In the Copenhagen interpretation of QT, PEM is violated only theoretically, but not for practical purposes. This interpretation introduced the concept of "collapse" or "reduction" of the wavefunction, stating that the particle collapses into A or B upon measurement, i.e., upon the interaction with the measuring apparatus (for some theoreticians, the collapse requires an interaction with the observer's non-physical mind, but we will not discuss this issue).

In other interpretations of QT (those that do not accept the idea of the collapse), the violation of PEM leads – following a realist interpretation

– to unorthodox views of physical reality, as the ontology of a simultaneously existing plurality of universes ("many-worlds" interpretation) and/or hidden variables (for a historical and systematic account of these debates, see Jammer 1974; Bub 1989).

In the realist approach to quantum logic, the focus of analysis is not the language used to describe the phenomena, but *the referents* of the mathematical formalisms used in the theory. Bain (2005), for instance, understands that as long as QT uses the formalism of vectors and operators in Hilbert space, while classical physics refers to states and functions in phase space, the *structure of quantum properties* is different from classical properties.

This approach leads to a different understanding of superposition, which is conceived not as a violation of PNC (as we interpreted it) but as a violation of PI. In QT, the value of a property corresponds to a Hilbert subspace that has a *non-distributive* structure, implying that properties are *disjunctive* (i.e., they have a value of the kind <a1 or a2 or a3…>). Therefore, properties do not always have the *same* value, a statement that corresponds to a violation of PI. As a consequence, some approaches to quantum logics focus on issues related to non-identity, while in this essay we focus on the possibility of violation of the two other Aristotelian principles.

2. SOME PROPERTIES OF THE CALCIUM ION

The informational capacity of Ca^{++} is derived from its electronic structure. Calcium ions are endowed with a structure able to carry information both in external vibratory states and in the flexible arrangements of the electronic structure. The biological importance of this ion derives from the capacity of interacting with proteins like a hormone (see e.g. Loewenstein 1999). The calcium atom has 20 electrons, having an orbital distribution of 2-8-8-2 (see Figure 1). The calcium cation (Ca^{++}) loses the electrons of the outer orbital, being attracted to molecules with negative charge. While attracted, it can also carry information that is encoded in the flexible electronic structure.

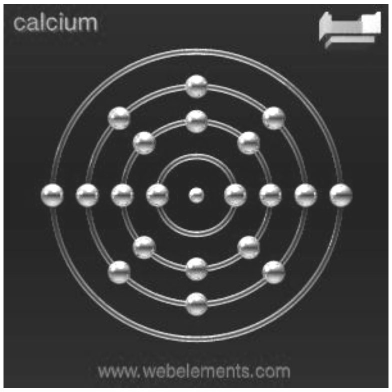

Figure 1. Illustration of the Electronic Shell of Calcium

Discussing the informational capacity of Ca^{++}, Loewenstein (1999) notes that "crystallographers have known for some time that calcium is a cut above the rest of the small inorganic ion crowd: it has a flexible crystal field – bond distances and angles are adjustable, with coordination numbers that can vary from six to ten – and it has higher ionization energies. The adaptable coordination sphere permits a wide variety of cooperative packings, giving the ion an advantage in the cross-linking of crystal structures" (pp. 236-237).

The calcium ion has been used in experimental realizations of quantum computing, following the model of Ion-Trap Quantum Computers (ITQC). An ITQC is composed of two or more ions linearly trapped in a magnetic quadripole. Information is transferred to and read from the ions by means of structured lasers that interact with the ions' external vibrational pattern, causing changes of energy levels in their internal electronic structure. Departing from an initial state when the ions are cooled, the use of lasers modifies the internal state of one ion that is correlated with the others, then changing the collective states. In order to explain these operations, we will make a brief review of Quantum Theory for the nonspecialized reader (therefore, physicists are invited to skip the following section).

3. BASICS OF QUANTUM THEORY

In this section we present some basic notions of quantum theory necessary for the discussion of quantum computing in relation to the principles of computational logics.

The electrons in an atom organize themselves in levels and sublevels. There are 7 levels indicated by 1, 2, 3, 4, 5, 6, and 7 (also known as *primary quantum numbers*) and 4 sublevels: S, D, P and F. Each level allows a number of sublevels and each sublevel allows a number of electrons. For example, the S sublevel allows up to 2 electrons and the P sublevel, 6. The levels or sublevels show the region where the electron might be found.

The levels and sublevels have specific energy values in crescent order:

$$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 6s^2 4f^{14} 5d^{10} 6p^6 7s^2 5f^{14} 6d^{10}$$

The electrons in each atom fill the lowest energy sublevels. The calcium atom, with 20 electrons, has the configuration:

1S, 2S, 2P, 3S, 3P, **4S**,

where the last value, **4S**, refers to the *atom valence level.*

When a particle – like the electron – possesses an internal state called "spin" (this term is an analogy with rotational movement), it develops a magnetic momentum. Boson particles like the photon have only spin values +1 or –1. Fermion particles like the electrons, protons and neutrons, have spin values +1/2 ("up") and -1/2 ("down").

In an atom, the total spin values of the electrons are spontaneously adjusted to the total spin values of the nucleus, eventually allowing different combinations. The relation of this spin balance with the electronic structure is shown in Figure 2.

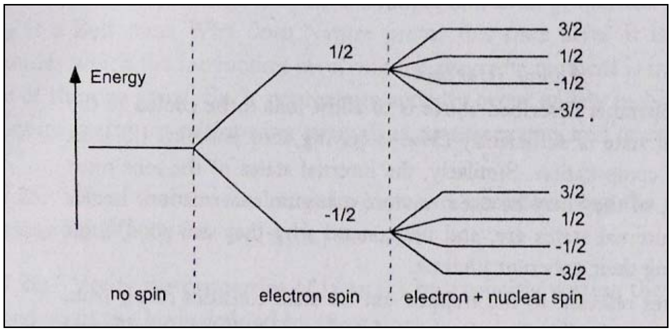

Figure 2: Nuclear and Electronic Spins (adapted from Nielsen and Chuang 2000)

All kinds of physical systems display vibrations, mainly because of thermal agitation. In the case of atoms, the spin values also contribute to the vibratory patterns. When the atoms are cooled, the spin-related vibrations are dominant over the thermal patterns. In this situation, the specific pattern related to each distribution of spin sublevels in the electronic structure of the atom is called a *vibrational state*.

Vibrational states are denoted by:

$${}^{2m_s+1}L_{mj}$$

where: $m_j = m_s + m_l$;

L is the sublayer in the electronic structure of the atom;

m_s is the number of the electronic spin;

m_l is the number of the nuclear spin;

m_j is the number of the total spin (electronic + nuclear).

The calcium ion has $m_s=+1/2$ and m_j goes from +1/2 to +5/2.

4. TRAPPED CALCIUM ION QUANTUM COMPUTING

Given the evidence about Ca^{++} as an information carrier, knowledge about its electronic structure arrangements, the dynamics of the transitions and associated electromagnetic frequencies becomes important to understand how it processes information.

Electrons can pass from an energy sublevel to another one, once receiving the right amount of energy. In ITQC, this energy is provided by the laser, which, like light, has small packets named *photons* with energy:

$$E = h\nu ,$$

where h is Planck's constant and ν is the frequency.

We use $\nu = \frac{c}{\lambda}$, where c is the light velocity and λ is the wave length, and then obtain:

$$E = \frac{hc}{\lambda}$$

The laser is a light pattern, having a definite wavelength that can deliver a definite amount of energy to the trapped ion. In order to reach a more energetic vibrational state, for each kind of transition it is necessary for the ion to absorb a specific amount of energy. For a $4^2S_{1/2} \rightarrow 4^2P_{1/2}$ transition of the calcium ion it is necessary a pulse with 396,847 nm, and for a $4^2S_{1/2} \rightarrow 3^2D_{3/2}$ transition, a pulse with 732,389 nm (Figure 3).

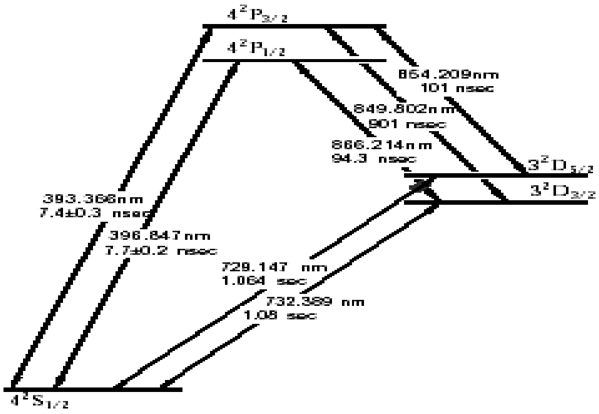

Figure 3 – Energy levels in the electronic structure of calcium, and associated laser frequencies necessary to change the distribution (adapted from Hughes *et al. 1998). Five vibrational states are depicted, being one of the S, two of the P and two of the D kinds.*

If an ion is in the *ground state* $4^2S_{1/2}$ (Figure 3) and receives a 396,847 nm pulse, it will change to state $4^2P_{1/2}$. As this last state is more energetic (and therefore unstable, with a short life around 7 nanoseconds), the ion will spontaneously return to the ground state, emitting an electromagnetic (photonic) radiation, or will change to state $3^2D_{3/2}$ without a photonic emission. Therefore, this last state is known as the "dark state". Such a difference of transitions, one with and the other without the emission of photons, is crucial for the usage of this system as a quantum computing device.

The ion takes around 7 nanoseconds for the transition $4^2P_{1/2} \rightarrow 4^2S_{1/2}$, while taking around one second for the transition $3^2D_{3/2} \rightarrow 4^2S_{1/2}$. Therefore, the $3^2D_{3/2}$ state is useful for computational purposes, since it lasts for a time duration that is sufficient for the realization of logical operations.

Considering the above physical properties of the ion, *we can attribute* the value $|0\rangle$ to the ground state, and the value $|1\rangle$ to the dark state (or vice-versa). If the first convention is adopted, it is possible to know that an excited ion is in state $|0\rangle$ when it emits light, and in state $|1\rangle$ when it does not emit light. Since the transition times to these states are extremely short, they are not considered in the attribution of states. It is important to note that such an attribution of binary states is a useful convention that allows for quantum systems to operate with a binary logic similar to the language of classical computers, and even to execute all classical logic gates (see Schmidt-Kaler et al. 2003).

The single calcium ion, with the above described electronic structure and respective transitions, can be considered as a quantum computer with the capacity of 1 *qubit* (for an exposition of the meaning of the *quantum bit*, see the next section). The input of information to the computer is made by laser pulses that remove the ion from the ground state, while the reading of information is made by means of the detection of the emission – or not – of light.

This single ion model can be extended to more powerful quantum computers, using two or more ions (see Kielpinski et al. 2002). In this case, the number of *qubits* processed by the quantum system increase, proportionally to the number of ions. For these multi-ion computers, the property of *quantum entanglement* is essential for the communication between the ions.

Entanglement is a property of the quantum world that is central to the operation of multi-particle quantum computers. Two states are entangled when the measurement of one gives a result corresponding to the result obtained from an independent measurement of the other. Two qubits are entangled when the measurement of one (of a binary) state (e.g., 0) gives the opposite value of the other (i.e., 1), in spite of the spatial distance between the particles. For this reason, entanglement is also considered to be a kind of *non-local communication* that challenges the classical worldview. The particles composing a quantum computer system communicate by means of entanglement.

In the operation of a multi-ion quantum computer, a message is encoded in one ion that is entangled with the others. In this condition, the state obtained in one of these ions is found in all the others. This property makes quantum computers more powerful than the classical ones, since the reading of one ion – i.e., one binary choice – affords the knowledge of the state of the other ions.

In the Los Alamos experiment (Hughes et al. 1998) the ions were cooled to a temperature near the absolute zero, in order that the contribution of the spin to vibratory states could be greater than thermal effects. However, it is not necessary to cool the ions; recently, new modalities of ion-trap quantum computing have been proposed (Milburn et al. 2000; Schneider et al. 2004) to perform quantum computation with "hot" calcium ions. This modality of ITQC, if feasible, makes possible an approximation of artificial modeling with the biological roles of calcium cations.

5. A DISCUSSION OF LOGICAL PRINCIPLES INVOLVED IN ITQC

In this section we discuss the fundamental concepts involved in the abovementioned quantum computational operations, and their relation with the principles of computational logics.

The initial efforts to develop quantum computing have been based on the computational language of classical computing, which has been partly modified to cover the new features presented by quantum computers. The classical bit was extended to the concept of a quantum bit or "qubit". Some examples of qubits are the magnetic moment of the nucleus of an atom, the vibrational state of an atom or a polarized light beam.

Mathematically, qubits are described as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle ,$$

where: $|\psi\rangle$ is a generic qubit;
α^2 is the probability of obtaining 0;
β^2 is the probability of obtaining 1;
$|0\rangle$ is a qubit in the 0 state;
$|1\rangle$ is a qubit in the 1 state.

This definition involves the concept of a *superposition*. The qubit is a superposition of states 1 and 0, according to their respective probabilities. In the quantum universe, there are not definite states like in the classical world, but only probabilities; the states 0 and 1 coexist in the probabilistic realm.

The interpretation of the concept of superposition is an important aspect of quantum logic. According to Bain (2005), a qubit displaying the superposition of two states – "hard" and "soft", "a) Can't be said to be *hard*; b) Can't be said to be *soft*; c) Can't be said to be both *hard* and *soft*, and d) Can't be said to be neither *hard* nor *soft*".

However, exact as this definition may be in the description of the reality denoted by quantum theory (i.e., the *reference* of the theoretical language), it fails to note that *in a specific sense* the concept of superposition effectively violates the PNC. In a superposition of states, which are linguistically denoted by 0 and 1, the quantum particle has the probability α^2 of being in state 0, while having *at the same time* the probability β^2 of being in state 1. Therefore, it has the *possibility* of being in states 0 and 1 at the same time, although the macroscopic observer cannot tell with certainty if the system is actually in both states at the same time.

It is necessary for the computational language *to be able to express* this possible situation. In this case, if the PEM is assumed, the negation of one state is semantically equivalent to the other allowed state, and therefore the quantum system violates the PNC.

Does the linguistic description of the quantum system formed by an ITQC really obey the PEM? As above shown, the calcium ion has *five or more possible distributions* of energy in its flexible electronic structure; however, only two kinds of state were considered in the computational language that describes them. The ion could be considered to reach and remain in other energy distributions besides the chosen two, as long as:

a) an appropriate quantity of energy is provided and sustained;

b) an adequate technology is used to distinguish and manipulate the different additional states, and

c) the description of the system is made by means of a computational language that obeys the rules of many-valued paraconsistent logics.

In conclusion, we claim that in the context of ITQC the computational possibilities imply a violation of PEM, and then also the violation of PNC. For practical purposes, the researchers have chosen a computational model close to the classical computer, which coexists with an ontology that violates the PI (but not the other principles). In this context, some possibilities allowed by nature are not being considered. The use of a different set of logical principles and rules could allow the construction of more interesting, flexible and adaptive quantum computers, getting closer to a naturalistic paradigm of computation.

Acknowledgement: CNPq (Alfredo P. Jr.)

REFERENCES

BAIN, J. (2005) "Modal Interpretations and Quantum Logic".
URL = http://ls.poly.edu/~jbain/philqm/philqmlectures/11.Modal&QL.pdf

BICKLE, J. "Multiple Realizability". *The Stanford Encyclopedia of Philosophy* (Fall 2002 Edition), Edward N. Zalta (ed.)
URL = http://plato.stanford.edu/entries/multiple-realizability/

BUB, J. "The Philosophy of Quantum Mechanics". *British Journal for the Philosophy of Science,* 40(2), pp. 191-211, 1989.

CARAFOLI, E. "Calcium Signaling: a tale for all seasons". *Proc. Natl. Acad. Sci.*, 99(3), pp. 1115-1122, 2002.

CIRAC, J.I. and ZOLLER, P. "A Scalable Quantum Computer with Ions in an Array of Microtraps". *Nature*, 404, pp. 579-581, 2000.

HUGHES, R.J. et al. "The Los Alamos Trapped Ion Quantum Experiment". *Fortschritte der Physik*, 46, pp. 329-361, 1998.

JAISWAL, J.K. "Calcium - How and Why?". *J. Biosci.*, 26(3), pp. 357-363, 2001.

JAMMER, M. *The Philosophy of Quantum Mechanics: the interpretations of quantum mechanics in historical perspective*. New York: John Wiley and Sons, 1974.

KIELPINSKI, D., MONROE, C. and WINELAND, D.J. "Architecture for a Large-Scale Ion-Trap Quantum Computer". *Nature*, 417, pp. 709-711, 2002.

LOEWENSTEIN, W.R. *The Touchstone of Life: Molecular Information, Cell Communication and the Foundations of Life*. New York: Oxford University Press, 1999.

MCCULLOUGH, W.S. and PITTS, E. "A Logical Calculus of the Ideas Immanent in Nervous Activity". *Bulletin of Mathematical Biophysics*, 5, pp. 115-133, 1943.

NIELSEN, M.A. and CHUANG, I.L. *Quantum Computation and Quantum Information*. Cambridge: Cambridge University Press, 2000.

PEREIRA JR., A. and LUNGARZO, C. "A Framework for the Computational Approach to Cellular Metabolism Supporting Neuronal Activity". *International Journal of Computational Cognition*, 3(3), pp. 87-92, 2005.

MILBURN, G.J., SCHNEIDER, S, and JAMES, D.F.V. "Ion Trap Quantum Computing With Warm Ions". *Fortschritte der Physik*, 48, pp. 801-810, 2000.

SCHMIDT-KALER, F. et al. "Realization of the Cirac-Zoller Controlled-NOT Quantum Gate". *Nature*, 422, pp. 408-411, 2003.

SCHNEIDER, S., JAMES, D.F.V. and MILBURN, G.J. "Method of Quantum Computation with 'Hot' Trapped Ions". *ArXiv:quant-ph/9808012 v1 7 Aug 1998*, 2004.