

WHOLISTIC REFERENCE, TRUTH-VALUES, UNIVERSES OF DISCOURSE, AND FORMAL ONTOLOGY: TRÉPLICA TO OSWALDO CHATEAUBRIAND

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Abstract: In its strongest unqualified form, the principle of wholistic reference is that in any given discourse, each proposition refers to the whole universe of that discourse, regardless of how limited the referents of its non-logical or content terms. According to this principle every proposition of number theory, even an equation such as “ $5 + 7 = 12$ ”, refers not only to the individual numbers that it happens to mention but to the whole universe of numbers. This principle, its history, and its relevance to some of Oswaldo Chateaubriand’s work are discussed in my 2004 paper “The Principle of Wholistic Reference” in *Essays on Chateaubriand’s “Logical Forms”*. In Chateaubriand’s réplica (reply), which is printed with my paper, he raised several important additional issues including the three I focus on in this tréplica (reply to his reply): truth-values, universes of discourse, and formal ontology. This paper is self-contained: it is not necessary to have read the above-mentioned works.

Key-words: Boole. Chateaubriand. Church. Frege. Gödel. Tarski. Wholistic. Refers. Denotes. Sentence. Proposition. Truth-value. Universe of discourse. Ontology. Epistemology.

The principle of wholistic reference (PWR) was first put forth by George Boole in 1847 when he espoused a monistic fixed-universe viewpoint similar to the one Frege and Russell espoused throughout their careers. Later, Boole elaborated PWR in 1854 from the pluralistic multiple-universes perspective. In 2003 when I first discovered this

principle in Boole's work, it brought to mind several passages in Oswald Chateaubriand's 2001 book *Logical Forms*, an engaging work I first read in manuscript in the late 1980s and which I had read and reread in print. At the time I made a mental note to write something exploring connections of forms of the principle of wholistic reference to Oswald's views, to the views he attributes to Frege, and to contemporary interpretations of standard first-order logic. The mental note gave rise to my short paper "The Principle of Wholistic Reference" in *Essays on Chateaubriand's Logical Forms*.

My Introduction to the 2003 Prometheus Books edition of Boole's *Laws of Thought* reported the following on page xxi without using any expression like 'Principle of Wholistic Reference'.

[Boole wrote]¹: "this universe of discourse is in the strictest sense the ultimate subject of the discourse" (Boole 1854, 42). ... For Boole, not only was each proposition about the universe of discourse, but each of the terms of an equation contained the concept of the universe of discourse and, moreover, each expression of either term of an equation contains an expression referring to the universe of discourse. For example, Aristotle's "Every square is a rectangle", considered as a proposition of a general theory of geometry, would be treated by Boole as "Being an entity that is square is being an entity that is rectangular that is square" where the word 'entity' expresses the universe of geometrical discourse. [In symbols, where 1 is the universe: $(s\ 1) = (s\ (r\ 1))$.] This aspect of Boole's semantics, or theory of propositions, does not seem to have been explicitly noted by Boole scholars. This strange and fascinating view that the universe of discourse is the ultimate subject of every proposition is foreshadowed in Boole's earlier work (1847, 15, 16).

When I revisited the topic, I coined the expression 'principle of holistic reference' (Corcoran 2003b, 275). Later, trying to avoid unwanted connotations and associations, I added the 'w'. (This paragraph amplifies remarks on pages 164-166 of "The Principle of Wholistic

¹ In this article square brackets are used to indicate my explanatory interpolations to quoted texts, not words in the original text quoted.

Reference” (Corcoran 2004).) The basic ingredients of the principle, perhaps even forms of it, are found in earlier recent writings by me and by others, for example in a 1999 encyclopedia entry “Universe of Discourse” (Audi 1999, 941) and in a 1999 article (Sagüillo 1999).

Boole’s principle of wholistic reference is entwined with his logical theory of the logical forms² of propositions and with his epistemological theory concerning how we grasp or come to know propositions before we can assert them or even make a judgment of truth or falsity. According to the strongest form of the logical theory, in a logically perfect language, every sentence contains a symbol that denotes the universe of discourse. In Boole’s implementation the symbol ‘1’ was used but Boole was clear about the fact that choice of characters is arbitrary. According to the epistemological theory – which his detractors are quick to pejoratively call psychological³ – the first step in grasping a proposition is to conceive of the universe of discourse. This corresponds to the practical advice to get clear about the range of one’s variables before formulating assertions or even hypotheses. Even if this epistemological belief is false, it seems to suggest good practical advice that will save time and prevent confusion and errors. Despite being intrigued by its acceptable implications and by the important issues it brings into focus, I have never found an unqualified form of this principle to be among my personal beliefs. This paragraph amplifies

² Readers of Oswaldo’s *Logical Forms* will be quick to note that its sense of the ambiguous expression ‘logical form’ is closer to Tarski’s concept “logical notion” than to the sense of ‘logical form’ used here and elsewhere in my writings (Audi 1999, 511-12). In fact, I have never used this two-word expression in Oswaldo’s sense or in any closely resembling sense.

³ Ironically, Boole is also credited with freeing logic from the psychological. Kneale and Kneale (1962/1988, 407) wrote that one of Boole’s “chief titles to fame” was that he brought about the revival of logic as an independent science and that his work “showed clearly by example that logic could be studied profitably without any reference to the processes of our minds”. Boole never confused propositions per se with mental judgments.

remarks on pages 164-166 of “The Principle of Wholistic Reference” (Corcoran 2004).

As I was writing my contribution to *Essays* I did not know whether Oswaldo would agree with my interpretation of the passages where I took him to be implying that Frege accepted a version of the principle of wholistic reference, nor was I confident that Oswaldo would concur in my tentative speculation that indeed Frege did accept such a principle. Thus it was gratifying to read Oswaldo’s implicit acceptance of my interpretation (Chateaubriand 2004, 174). And it was reassuring to read his guarded concurrence with my speculation (Chateaubriand 2004, 175):

As long as the universe of discourse is interpreted absolutely Frege might agree to the principle of wholistic reference as formulated in terms of quantification, but I do not know whether this had any connection with Frege’s idea of postulating [the truth-values] the True and the False as the referents of sentences and thoughts.

Oswaldo’s discussion brought in more issues, three of which I would like to address briefly: truth-values, universes of discourse, and formal ontology. It is my feeling that, despite Frege’s brilliance and despite his generally beneficial influence on subsequent developments, he held mistaken beliefs concerning these issues. It is also my feeling that Oswaldo’s brief remarks about them deserve to be amplified.

In all of my post-1990 writings – with small exceptions – I tried to follow Church’s terminology in regard to the words ‘express’, ‘connote’, ‘denote’, ‘name’, ‘designate’, ‘refer’ and their cognates (1956a, 1956b, esp. 1995, 69). Unless explicitly indicated otherwise, I assume, as does Church, that the object-language is fully interpreted and that it is being used in an unambiguous way. A proper name expresses or connotes its sense or connotation, if any, and it names, denotes, or designates, the entity, if any, of which it is a name, i.e., its denotation or designation. According to Church, the verb ‘refer’ is not properly used for the name relation; that is, he thinks it should not be used as a

synonym for ‘denote’, ‘name’, or ‘designate’. Rather it takes a range of subjects much wider than the class of proper names. Any expression or any sense that is about something, that has a subject-matter, refers to that which it is about, to its referent or referents. Whatever refers in this broad sense is about what it refers to even in cases where it would be incorrect to say that it names.

Church (1995, 69)⁴ says he is “allowing the verb *to designate* and the noun *designation* as occasional alternative terminology [for ‘to denote’ and ‘denotation’, respectively], but certainly not *to refer* and *reference*, which are so contrary to standard English usage ... as to make them repellent.” Oswaldo agrees with Church on this point of terminology (personal communication, hereafter abbreviated ‘per. comm.’).

Of course, in order to avoid confusion, it is important to recognize that the use of ‘refer’ as a synonym for ‘denote’, which Church (justifiably in my opinion) regarded as a misuse, has become so common, especially in philosophy, that it must now be regarded as normal by all but some holdout purists. Thus I allow two senses of ‘refer’: a narrow sense and a broad sense. In the *narrow* sense disapproved of by Church, the verb ‘refer’ is simply a synonym of ‘denote’ (‘name’ or ‘designate’); only proper names refer. In this narrow sense it is incoherent to say that a text, a science, a proposition, a sense or anything other than a proper name refers. However, in the *broad* sense it has the deliberately vague and general sense indicated above: number theory refers to numbers, this article refers to the principle of wholistic reference and the principle of wholistic reference refers to propositions and universes of discourse.

⁴ Church presented this paper at the University of Buffalo in 1990 on the occasion of conferral of the *Doctor Honoris Causa* for his contributions to logic. In his informal remarks he was much more explicit about the deliberations leading up to his choice of words and about philosophical and mathematical importance of choosing a terminological framework that respects the connotations and associations of ordinary English usage. He gave me a handwritten ms. and asked me to arrange for its publication: the result is Church 1995.

There is no precise boundary of what can coherently be said to refer. In the narrow sense the expression ‘the successor of zero’ refers to one and to nothing else; in the broad sense it refers to zero, to one and to the successor function⁵.

Oswaldo recognized the possible impropriety of using the word ‘refer’ in the narrow sense he alternates it with ‘denotes’ and ‘designates’ to lessen the risk of misunderstanding (Chateaubriand 2001, 86). For Frege, as for most of us, there was never a question of whether a true or false sentence refers in the broad sense. His question – answered affirmatively with his hypostatization of truth-values – was whether true or false sentences refer in the narrow sense, whether they name, denote, or designate anything. As far as I know, Frege was not only the first person to answer this question affirmatively, he was the first person to ask it. But, as Da Silva points out in his contribution to *Essays*, Husserl’s views deserve attention (Da Silva 1999, esp. 367-9).

I regard the word ‘refer’ as one of the many words which, through wide misuse, have come to accumulate one or more additional senses, thus becoming ambiguous or increasing its ambiguity. The noun ‘Indian’ can be taken as typical or paradigmatic of such words: it originally denoted people from India but, through misuse commonly blamed on Columbus, it came to have an additional meaning thereby introducing an ambiguity that had not previously been present. Examples familiar to logicians include ‘implication’, ‘model’ and ‘variable’⁶.

⁵ The authoritative 2000 *Merriam-Webster’s Collegiate Dictionary* lists four meanings of the verb ‘refer’ but by omission it excludes the sense that Church says is “so contrary to standard English usage”. Thus, it would seem to confirm Church’s skill as a lexicographer. But, I would be guilty of writing a half-truth if I were to fail to report that on the same page, in the entry for ‘referent’, the same dictionary uses the word ‘refer’ as a synonym for ‘denote’.

⁶ G. E. Moore (1922/1948, 296) observed: “Mr. Russell in the *Principles of Mathematics* calls [it] ‘material implication,’ and he and Dr. Whitehead in *Principia Mathematica* call [it] simply ‘implication.’ Why logicians should have ...

In my discussions of the principle of wholistic reference I use the word ‘refer’ and its cognates in their respective broad senses.

1. TRUTH-VALUES

Oswaldo mentioned “Frege’s idea of postulating [the truth-values] the True and the False as the referents of sentences and thoughts”. The word ‘postulating’ admits of a range of meanings going from its epistemic use in traditional geometry – where it means something like “recognizing as real or true” – to its use by the American Postulate Theorists⁷ and others, where postulating something carries no epistemic or ontic connotation except perhaps a negative one by implication⁸. Oswaldo seems to be using it in the sense found in traditional geometry – both in the above-quoted passage (2004, 175) and on page 315 of *Logical Forms*. When the American Postulate Theorists postulate something they are implying that it does not exist or at least that it probably does not. They use the word to avoid “ontological commitment”. Alonzo Church alluded to something like this in a clarifying footnote to a passage in his discussion of truth-values. The clarification is quoted below. But first I quote the passage (Church 1956, 25):

chosen to use the word ‘implies’ as a name for a relation, for which it never is used by any one else, I do not know.”

⁷ For an overview of the American Postulate Theorists, see Scanlan (1991).

⁸ Between these two extremes a range of other uses can be identified. Consider a statement to the effect that the proposition “other minds exist” is a postulate. In the first sense the implication is that “other minds exist” is known to be true by the speaker. In the second sense the implication is that “other minds exist” is not even believed to be true. But in an intermediate sense used by C. I. Lewis and others, the implication is that “other minds exist” is believed but not known to be true. He wrote (1970, 301): “We ... believe in other minds ..., but we can not know that such exist. This belief is a postulate.”

Therefore, with Frege, we postulate two abstract objects called truth-values, one of them being *truth* and the other *falsehood*. And we declare all true sentences to denote the truth-value truth, and all false sentences to denote the truth-value falsehood. In alternative phraseology, we shall also speak of a sentence as having the truth-value truth (if it is true) or having the truth-value falsehood (if it is false).

I think that Gödel is closer to the truth about Frege's views when he writes (1944, 129):

Frege actually drew this conclusion [that all true sentences have the same signification]; and he meant it in an almost metaphysical sense, reminding one somewhat of the Eleatic doctrine of the "One". "The True" – according to Frege's view – is analyzed by us in different ways in different propositions, "the True" being the name he uses for the common signification of all true propositions [Frege 1892b, 35].

Oswaldo is fully aware of the force of Gödel's view; it is explicitly mentioned at least twice and quoted in full in *Logical Forms* (2001, 146, 158, 416, 427).

In the clarifying footnote attached to the word 'postulate' in the above passage Church wrote:

To Frege, as a thoroughgoing Platonic realist, our use of the word 'postulate' here would not be acceptable. It would represent his position better to say that there are two such things as *truth* and *falsehood* (*das Wahre* and *das Falsche*).

Oswaldo's potentially misleading choice of the word⁹ 'postulating' is not the only problematic aspect of his discussion of truth-values in

⁹ Perhaps people using the verb 'postulate' would do their readers the courtesy of explaining what they take the act of postulating to be: how the act is to be performed, what is to be accomplished by performing it, what its criteria of success and failure may be, and what alternative methods there may be for reaching the same goals.

connection with the principle of wholistic reference, or even with his passage quoted above. The phrase ‘Frege’s idea of postulating [the truth-values] the True and the False as the referents of sentences and thoughts’ implies that Frege took *sentences* to denote or name truth-values, which is true. But it also implies that Frege took *thoughts* to denote or name truth-values or to be about truth-values, which is far from true. For Frege sentences are what Church (1956b) calls propositions in the traditional sense, that is, *concrete* propositions, “a judgement expressed in words” (1956, 26). Concrete propositions are abstract propositions *expressed* in words or symbols in a particular way, whereas a thought is an abstract proposition. A sentence or concrete proposition has a “wording”, it contains words or symbols (1892b, 34), but a thought or abstract proposition does not. For Frege sentences – which have wordings – *express* thoughts, but they *name* truth-values (1892b, 31). Frege said that sentences contain thoughts (1892b, 32) emphasizing that a thought is intrinsic to a sentence expressing it. I do not know whether Frege ever explicitly used the word ‘determines’ this way, but he held that the thought expressed by a given sentence determines which truth-value, if any, the sentence denotes. Chateaubriand (2001, 83, 146) refers to “Frege’s principle that sense determines reference” and to “Frege’s principle that sense determines denotation”. After the first noted passage Oswaldo makes some truly subtle points about the use of the verb ‘determines’ in English, but his insightful discussion begs the question of whether Frege ever explicitly said that sense determines reference. It also implicitly suggests the question of whether Frege ever said – however obliquely – that although sense determines reference, a person can not determine the reference from the sense alone, for example, that even though the sense of a sentence determines its truth-value, a person can not judge what that truth-value is simply from the sense.

For Frege, the sentence is a composite of wording and meaning, an expression combined with a thought – much as the human being is sometimes regarded as a composite of “body and soul”, a material entity

and a mental entity. The sentence is to its wording and its thought, respectively, as the human being is to its “body” and its “soul”. Since about 1980 I have not used the word ‘sentence’ except for what Frege might call the wording of a sentence. I use the word ‘proposition’ for the other part, the “abstract proposition”, something having no intrinsic wording, but perhaps expressible in various wordings in various languages.

Church’s choice of *truth* and *falsehood* as substitutes for Frege’s *das Wahre* and *das Falsche* suggests what should be obvious, namely, that he takes Frege’s truth-values, which were unprecedented and unanticipated in the history of logic, to be not much more than hypostasizations or reifications of the properties of being true and being false. This suggestion becomes even more probable in view of the fact that Church does not mention any of the unexplained and puzzling things Frege says about the nature of truth-values. We have just seen Gödel’s citation of the strange idea that for Frege the truth-value truth is “analyzed by us in different ways in different propositions”. Can we get anything out of this?

The expression ‘to analyze the truth-value truth’ seems puzzling, perhaps incoherent. But if we look at the page from “On Sense and Reference” that Gödel cites (Frege 1892b, 35), not only do we find what Gödel attributes to Frege, we also find passages that seem to imply a form of the principle of wholistic reference not considered by Boole or by any one else, as far as I know. There Frege implies that the referent of a part of a sentence is part of the referent of the sentence. Thus if the referent of any true sentence is *truth*, then anything referred to by any part of a true sentence is part of the truth-value *truth*. Likewise, *mutatis mutandis*, for false sentences. Since anything referred to by part of a true sentence is referred to by part of a false sentence and conversely¹⁰, it would seem to follow that absolutely everything referred to by any part

¹⁰ Every sentence is part of its own negation, which has the opposite truth-value.

of any sentence is part of each of the truth-values. And thus that absolutely everything referred to by a part of any sentence is referred to by each and every sentence. Thus, every sentence refers to everything referred to by any part¹¹ of any other sentence. This goes far beyond the idea that all sentences having the same truth-value refer to that truth-value. Surely it deserves to be regarded as a principle of wholistic reference.

Another challenging example is one of the first things Frege wrote about truth-values: “By the truth-value of a sentence I understand the circumstance that it is true or false” (Frege 1892b, 34, quoted Chateaubriand 2001, 79). Here Frege’s translators seem to have coined a new meaning for the word ‘circumstance’ according to which the circumstance that ‘Aristotle was Greek’ is true is the same as the circumstance that ‘The medians of a triangle intersect at a point’ is true. The Frege interpreters I have read – at least those who do not simply ignore this inconvenient passage – have offered sometimes ingenious construals of it which fit neatly into Frege’s other views. But, instead of solving the mystery of Frege’s strange word choice they suggest a new mystery: why Frege did not choose their formulation instead of the bizarre expression translated ‘circumstance that it [a sentence] is true’. What could he have had in mind that made this expression seem appropriate? Unfortunately, these suggestions just hide what Frege could have meant by the sentence just quoted. Frege’s understanding of the nature of truth-values might have been clearer to us if he had never written that sentence.

Oswaldo seems to be coming close to the hypostatization or reification interpretation himself when he says (*loc. cit.*) that “the circumstance that a sentence is true or false seems to be a characteristic

¹¹ No responsible discussion of this passage will omit mentioning the notorious ambiguity of the relational noun ‘part’ and the relational verb ‘is a part of’. In several senses of ‘part’ Rio is part of Brazil. But can we be sure that there is no sense of ‘part’ in which Brazil is part of Rio? I have often said that Buffalo is part of me.

or property of a sentence, not an object”. Incidentally, I find Oswaldo’s remark to be implausible as it stands, and to contradict Frege’s generally clear view that “the singular definite article always indicates an object, whereas the indefinite article accompanies a concept-word [predicate]” (Frege 1892a, 45). But before I am confident that I have interpreted Oswaldo correctly, I need to reread his criticisms of Frege’s treatment of the definite article (2001, Chapters 3 and 11). To be perfectly clear, the issue here is Oswaldo’s interpretation of Frege and not his own view of truth and falsity, which he takes to be properties, not objects in the range of individual (first-order) variables (per. comm.).

Frege’s view that sentences name truth-values, combined with his view that truth-values are objects in the universe over which the object variables range, seems to imply that sentences refer to truth-values in the narrow sense and in the broad sense. It never ceases to amaze me that Frege shows no signs of hesitation in stating and using these views even though in the entire history of logic they seem to be unprecedented. This is surely a tribute to Frege’s supreme confidence in himself and his low opinion of his predecessors.

It is relevant to note that philosophically sensitive logicians I have consulted are uniformly uncomfortable about the use of “truth-values” outside of narrowly technical conventional contexts. Oswaldo discusses a closely related point in *Logical Forms* (2001, 203, 204, 215). Tarski, for example, studiously and conspicuously avoided the word ‘truth-value’ for most of his life. In his last work *A Formalization of Set Theory Without Variables* written with S. Givant and published posthumously in 1987, Tarski breaks his pattern of avoiding ‘truth-value’ – but he waits until page 165 of the 271 pages of text. With one possible exception, I know of no living logician who accepts Frege’s views that sentences name truth-values and that truth-values are among the existent objects over which the object variables range. I do not accept either view. Like most logicians I know, I regard truth-values as convenient

fictions, and I take their names to be empty idioms, ways of speaking, not words that denote objects.

2. UNIVERSES OF DISCOURSE

The pluralistic principle of multiple universes of discourse (PMU) in one of its strongest forms, perhaps first suggested in 1846 by Augustus De Morgan¹² (Audi 1999, 941), is that every cognitive discourse, including every established scientific discourse, presupposes its own limited subject-matter, or universe of discourse. Citing *Posterior Analytics* 76b10, Sagüillo (1999, 268) suggested that it may go back to Aristotle. M. Mulhern (per. comm.) agrees, adding that this idea is also in *Metaphysics* 1025b5-8:

Every science which is ratiocinative or at all involves reasoning deals with causes and principles, exact or indeterminate; but all these sciences mark off some particular being—some genus, and inquire into this, but not into being simply or *qua* being.

This principle in some form or other has been a cornerstone of my thinking and teaching since the early 1960s. Over these more than forty years I have taught or written about number theory, pure set theory, group theory, string theory, geometry and other deductive sciences. In each case I have always started by articulating this principle and discussing the relevant universe of discourse or, in technical settings, the range of the individual variables. For example the first two sentences of my 1973 article “Gaps between logical theory and mathematical practice” are:

The view of mathematics adopted here can be called neutral platonism. It understands mathematics to be a class of sciences each having its own subject-matter or universe of discourse.

¹² De Morgan’s technical expression was not ‘universe of discourse’ but simply ‘universe’ although in most contexts of actual use he adds a qualification such as ‘of an assertion’, ‘of an argument’, and ‘of a proposition’ (De Morgan 1846, Section I; De Morgan 1966, xxv, 2).

I have found this principle to be liberating and clarifying, a kind of logical Ockham's razor, neatly avoiding the confused and confusing need to discuss "the universe" of all individuals, which I have never understood or felt comfortable with. From my student days I have felt that the universe of *all* objects or individuals was another vague but convenient fiction which could not carry any literal ontological or foundational weight¹³.

In formalizing a science using first-order logic, I have followed the same paradigm used by Hilbert, Veblen, Zermelo, Gödel and others. In each particular case this involves identifying a restricted universe of discourse and taking it to be the range of the first-order variables. The case of Hilbert's geometry with three sorts of variables, for points, lines, and planes, is especially interesting. It continues to astound me that a person as intelligent as Frege thought that this was impossible: he thought that people who follow the pluralistic restricted-universe paradigm are somehow mistaken. Oswaldo states the Frege view with admirable accuracy, clarity, directness and brevity (Chateaubriand 2004, 174-5).

Frege held that the interpretation of quantification must be completely unrestricted so that quantifiers over objects quantify over *all* objects, quantifiers over functions quantify over *all* functions, and so on. Moreover, in the same vein Frege maintained that a function of objects must be defined for all objects, so that the addition function, for instance, must be defined for the sun and the moon as well as for 3 and 5. For Frege the universe of discourse is absolute and he would not have agreed to the idea of restricted universes of discourse. If one wants to restrict one's discourse to human beings, for instance, one must relativize

¹³ It has fascinated me for years to reflect on the irony that the incorrigibly vague and obscure expression 'the universe of all objects', or a variant near synonym thereof, occurs so often in the writing of people who are repulsed by vagueness and obscurity and who are conscientiously striving for exactness and clarity. Exactly what is found among the "objects" in this "universe": pains, sunsets, marriages, judgments? Is this another irony like fighting fire with fire? Or is it a symptom of "denial" or a lapse in self-knowledge and self-awareness?

all quantification to humans by means of the concept (or predicate) 'is human'.

Why *must* the quantifiers be total? What is the force of this *must*? Is Hilbert's geometry somehow bogus because it uses limited universes? Is Tarski somehow mistaken in the famous truth-definition paper when he takes his limited universe of discourse, the range of his object-language variables, to be exhausted by, in his words, "classes of individuals" (Tarski 1956, 169)?

The view that any function applicable to objects *must* be applicable to all objects seems equally gratuitous and groundless. The view that the expression 'the sun plus five' is coherent and denotes an object seems bizarre, where 'plus' is used in the same sense as in 'one plus five is six'. Oswaldo, by his failure to emphasize, or even express, any doubts about these views, may seem to accept them. But the opposite is the case. In fact, he wrote (per. comm.): "My whole book is against this kind of wholistic view".

His statement about his difference with me concerning universes of discourse is also open to misinterpretation. He wrote (Chateaubriand 2004, 178): "I do not attach as much importance as does John [Corcoran] to the notion of universe of discourse as such". This suggests that he does not think that the notion of universe of discourse as such is very important. But again, the opposite is the case (per. comm.). What he thinks unimportant is De Morgan's 1846 discovery of it, De Morgan's emphasis on it (De Morgan 1847) and Boole's 1854 introduction of it into mathematical logic. His view is based on his belief that these ideas are "so obvious" (per. comm.). I disagree here for two reasons. First, I think that these ideas are not so obvious: Frege, Russell, and Lewis seem to have missed them. Second, even if they are obvious, I think that in many cases stating and emphasizing the obvious can be very important. Being important and being obvious (trivial, elementary) are not mutually exclusive extrinsic attributes of concepts or of propositions.

3. FORMAL ONTOLOGY

The last three pages of Oswaldo's "Reply" (Chateaubriand 2004, 178-181) contain a useful description of what has been called since the early 1990s 'logic as formal ontology' – a side of logic that is centered on logical truth and that supplements the traditional "logic as formal epistemology", which instead is centered on logical consequence. In the course of his description he refers to several logicians, including Gödel and Tarski, who approve of this non-traditional side of logic despite its obvious kinship with the much maligned branch of philosophy known as metaphysics. In the article mentioned on p. 144 above comparing Aristotle and Boole I wrote (Corcoran 2003b, 262):

In *Prior Analytics* Aristotle addressed the two central problems of logic as formal epistemology: how to show that a given conclusion follows from given premises that formally imply it and how to show that a given conclusion does not follow from given premises that do not formally imply it. Using other equally traditional terminology, Aristotle's problems were how to establish validity and how to establish invalidity of an arbitrary argument, no matter how many premises or how complicated its propositions.

It is clear from reading Boole's logician predecessors that nothing systematic in formal ontology had been attempted. I explain in the 2003 article that Boole had a clear conception of logic as formal ontology, not well developed to be sure, *and* that he saw its introduction as broadening the scope of the science of logic. The second to last paragraph of the Conclusion (Corcoran 2003b, 286) reads:

If we divide logic into formal epistemology and formal ontology as has been done above, then we can give credit where due by saying that Aristotle was the founder of logic as formal epistemology and that Boole was the founder of logic as formal ontology. Aristotle laid down the groundwork for a science of determining validity and invalidity of arguments. Boole laid down the groundwork for a science of formal laws of being, in Tarski's words "general laws governing the concepts common to all sciences" (1941/1994, xii) or "the most general laws of thinkables", to use the words that Kneale and Kneale (1962/1988, 407) applied to what Boole called "laws of thought".

Earlier I had referred to the same passage from Tarski's 1973 Buffalo lecture "What are logical notions?" (Tarski 1986)¹⁴ that Oswaldo quoted (Chateaubriand 2004, 180). Tarski's words are well worth repeating.

I take logic to be a science, a system of true sentences, and the sentences contain terms denoting certain notions, logical notions.

In this article Tarski was concerned exclusively with logic as formal ontology. Many of his books and articles are focused on the other side of logic. See for example Tarski's *Logic, Semantics, Metamathematics*, especially articles II, III, V, and, of course, XVI "On the Concept of Logical Consequence".

Before Boole the history of logic was the work of a series of what we may call monists, logicians who recognized only one side of logic, namely, formal epistemology. Boole was the first full-fledged dualist who saw both sides of logic. After Boole most logicians have been dualists but there have also been monists, and of two sorts: reactionaries who refused to accept logic's formal-ontology side and radicals who refused to accept logic's traditional formal-epistemology side. Passages that admit of interpretation as leaning toward a reactionary epistemological monism that leaves no room for logic's formal-ontology side are easy to find. For example, in his comprehensive *Introduction to Mathematical Logic* (1987, 1), Mendelson wrote:

¹⁴ The only presentation of this paper in the Americas was at the 1973 Conference on the Nature of Logic sponsored by the Buffalo Logic Colloquium at which Oswaldo was an invited speaker. Alfred Tarski and Hilary Putnam were the featured speakers. Afterwards I wrote an account of the lecture and I urged Tarski to have the paper prepared for publication. He said then and several times later that he wanted me to edit it but he never gave me the typescript until 1982 when I was involved in other projects (Tarski 1986, 144).

The truth or falsity of the particular premisses and conclusions is of no concern to logicians. They want to know only whether the premisses imply the conclusion.

Likewise, passages that admit of interpretation as suggesting a radical ontological monism that leaves no room for logic's traditional formal-epistemology side are easy to find. For example, in his *Philosophy of Logic* (1986, vii), Quine defines logic as being "the systematic study of the logical truths". Earlier, in the Introduction¹⁵ to his 1950 *Methods of Logic*, Quine had said:

Logic, like any science, has as its business the pursuit of truth. What are true are certain statements; and the pursuit of truth is the endeavor to sort out the true statements from the others, which are false.

Moreover, some, but by no means all, of Frege's interpreters, Michael Dummett for one, thought that Frege was among the radical monists. Some of these interpreters thought that Frege was right about his monism. I never did. But some, like Dummett, were bitterly critical of Frege in this regard. According to Dummett (1973, 432-3):

The founders of modern mathematical logic, Frege, and, after him, Russell, had formalized logical systems on the quite misleading analogy of an axiomatized theory: namely, by ... axiomatically stipulating the validity [logical truth or tautologousness] of formulas of certain forms. In such formalization, attention is concentrated on the postulation of logical truths and the derivation of further logical truths from them. This was quite deliberate on Frege's part: in this respect ... Frege's new approach to logic was retrograde. He characterized logic by saying that, while all sciences have truth as their goal, in logic truth is not merely the goal but the object of study. The traditional answer to the question what is the subject matter of logic is, however, that it is not truth, but inference, or, more properly, the relation of logical consequence. This was the received opinion ... until ... Frege and it is, surely, the correct view.

¹⁵ This important document is reprinted in Hughes 1993, which is still in print. Sadly, Quine 1950 has been out of print for some years.

Dummett does not leave this topic without adding, a few sentences later:

It remains that the representation of logic as concerned with the characteristic of sentences, truth, rather than of transitions from sentences to sentences, had highly deleterious effects both in logic and in philosophy.

I would agree fully with Dummett here, if the words ‘concerned only with’ were substituted for the words ‘concerned with’ and ‘rather than also of’ were to replace ‘rather than of’. This brings us to Oswaldo’s interesting inquiry (Chateaubriand 2004, 174-5):

The question in my mind, however, is whether John [Corcoran] thinks – or implicitly claims – that the metaphysical conception of logic inspired by Frege, Russell and Gödel, that I defend in my book, is a wrong conception of logic?

The answer is the same as Dummett would give: No, by itself this view of logic merely emphasizes one indispensable aspect of logical research. However, I think that any monistic one-sided view of logic is a gross distortion – regardless of whether it excludes traditional formal epistemology or whether it excludes modern formal ontology. It is clear from many of my writings – unfortunately not all – that I am a logical dualist like Boole, and like Church and Tarski for that matter.

Oswaldo’s phrase “the metaphysical conception of logic inspired by Frege, Russell and Gödel” may be read by many as presupposing that logicians’ interest in the formal-ontology side of logic starts with Frege. The fact is that one of Boole’s proudest achievements was to bring into logic the formal ontology that had been regarded as a domain of philosophy. This is what Boole had in mind in the passage on page 13 of the 1847 work quoted in full by Oswaldo (Chateaubriand 2004, 179):

Let it be granted that the problem which has baffled the efforts of ages, is not a hopeless one; that the “science of a real existence,” and “the re-

search of causes,” “that kernel” for which “Philosophy is still militant,” do not transcend the limits of the human intellect. I am then compelled to assert, that according to this view of the nature of Philosophy, *Logic forms no part of it*. On the principle of a true classification, we ought no longer to associate Logic and Metaphysics, but Logic and Mathematics. (Boole 1847, p. 13)

On the topic of overlooking Boole, a reading of selected passages may seem to warrant the conclusion that Boole is open to the same charge Dummett levels at Frege and Russell, namely, misconstruing logic on the model of an axiomatic theory of the Euclidean sort. A more thorough reading of Boole reveals a balanced approach that never overlooks concern to deduce conclusions from arbitrary premises not just from “logical axioms”; Boole never limited logic to formal ontology.

What I am calling formal ontology cannot be limited to or identified with what is called ontology by philosophers, because the same logicians who seek the most general laws common to all sciences also want to know how this knowledge comes about. Thus formal ontology contains an epistemological dimension. Boole is accused of doing psychology when a more nuanced interpretation finds that he was engaged with epistemic issues concerning formal ontology. Boole was no more an advocate of psychologistic logic than Frege. Likewise, what I am calling formal epistemology cannot be limited to what is called epistemology by philosophers, because the same people who seek to understand how we determine whether or not a conclusion follows from given premises also want to know the ontological presuppositions of these processes. Thus formal epistemology contains an ontological dimension.

It is important to be clear that the one-monism/one-pluralism issue of one unrestricted universe versus multiple restricted universes of discourse, where historically there are two opposing views – for example Boole 1847 and 1848 versus Boole 1854 – is entirely independent of the two-monisms/one-dualism issue of the character of logic. In regard to

the latter, historically there are three opposing views: the monism of exclusive formal epistemology often attributed to Aristotle, the dualism of formal epistemology complemented with formal ontology which Boole originated, and the monism of exclusive formal ontology attributed by Dummett to Frege. Moreover, these two issues are also independent of the issue between the “methodologists” – who follow Tarski in holding that the methodology of deductive sciences necessarily conducted in a metalanguage is a legitimate science – and the “anti-methodologists” – who distrust semantics and who hold that metalanguage is literally nonsense (Tarski 1956/1983, xv-xxv, esp. xx), a view often attributed to Wittgenstein and even to Frege.

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