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LOGIC AND MODALITY: REPLY TO FRANK SAUTTER

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Abstract: In §1 I examine the connections between my account of logical properties and Tarski's account of logical notions. In §2 I briefly present some of my views on modality and the basis for my claim that there are intensional as well as extensional relations between properties. In §3 I compare my views on the nature of logic and of mathematics with Gödel's views.

Key-words: Tarski. Logical notion. Logical property. Modality. Gödel.

Frank gives an excellent summary of my views on the nature of logic and raises three main questions. (1) What is the relation between my view of logical properties and Tarski's view of logical notions? (2) What are my views on modality and possible world semantics and how do I justify my claim that there are non-extensional relations between properties? (3) What is the relation between my views and Gödel's logicist program? I will discuss these in turn[1].

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[1] There are many complex issues involved in this discussion and although I cannot deal with them in detail in the context of this reply, there will be a more systematic discussion of some of the issues in Chapter 19.

1. TARSKI ON LOGICAL NOTIONS

Tarski's view of logical notions presented in his posthumously published paper "What are Logical Notions?"[2] is that the logical notions are those that are "invariant under all possible one-one transformations of the world onto itself" (Tarski, 1986, p. 149). This is a very interesting idea and there has been a fair amount of discussion of it in the literature. At the end of the lecture Tarski distinguishes two methods of constructing set theory: the type-theoretic method of *Principia Mathematica* and the first-order method of the formulations of set theory by Zermelo, von Neumann, and others. About the former he says:

> Using the method of *Principia mathematica*, set theory is simply a part of logic. The method can be roughly described in the following way: we have a fundamental universe of discourse, the universe of individuals, and then we construct out of this universe of individuals certain notions, classes, relations, classes of classes, classes of relations, and so on. However, only the basic universe, the universe of individuals, is fundamental. A transformation is defined on the universe of individuals, and this transformation induces transformations on classes of individuals, relations between individuals, and so on. ... When we speak of transformations of the 'world' onto itself we mean only transformations of the basic universe of discourse, or the universe of individuals ... Using this method it is clear that the membership relation is certainly a logical relation. It occurs in several types, for individuals are elements of classes of individuals, classes of individuals are elements of classes of classes of individuals, and so on. And by the very definition of an induced transformation it is invariant under every transformation of the world onto itself. (Tarski 1986, p. 152)

Let me make some remarks about this and compare it with what I do in Chapter 9.

One basic difference with the ontology that I adopt is that my ontology is an ontology of properties rather than an ontology of sets –

[2] I was fortunate to be a participant at the Conference on the Nature of Logic held in honor of Tarski at the State University of New York at Buffalo in 1973 where he gave this lecture on logical notions.

which is how Tarski is interpreting the ontology of *Principia Mathematica*. This is not essential though, because Tarski's idea could be applied just as well to my ontology of properties and states of affairs. And, in fact, using my criterion of universality and omnipresence throughout the hierarchy we seem to get the same classification of properties into logical and non-logical[3]. Tarski's criterion in terms of one-one transformations is sharper than mine, and it is also mathematically more interesting because of the connection he draws with Klein's Erlangen Program, but in my mind it raises a problem concerning the universe of individuals.

I hold that the question of what properties are logical properties should be completely independent of which individuals happen to exist in the world and of what sort of things these individuals are[4]. Even if there were no individuals at all, there would still be logical properties, and they would be exactly the same properties as the logical properties in a universe that contains individuals. Here is a place where it makes a difference that my ontology is an intensional ontology of properties and Tarski's is an extensional ontology of sets. In any case, it seems to me that the characterization of logical properties should be *intrinsic* and not depend on *contingencies*. Of course, we could use Tarski's characterization in terms of *possible universes* of individuals[5], or in terms of models, and this might be compatible with my characterization.

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[3] It is interesting to note in this connection that also Gödel seems to characterize logical properties in terms of universality. Wang says: "For Gödel, logic deals with *formal* – in the sense of universally applicable – concepts. From this perspective the concepts of *number*, *set* and *concept* are all formal concepts". (Wang 1996, p. 267)

[4] At the end of the passage that I quoted above Tarski says that the universe of individuals may be taken to be the universe of physical objects.

[5] *I.e.*, we could say that a property P (of a certain type) is a logical property if and only if in any possible universe of individuals P is invariant with respect to every one-one transformation of the universe onto itself.

From the point of view of universality, the level 1 unary logical properties are Existence and Nonexistence, which correspond to the universe of individuals and to the empty set in Tarski's characterization. The binary logical relations are Identity, Diversity, the Universal relation and its complement the Self-Difference relation, just as for Tarski. Moreover, since I hold that there are relations of arbitrarily high (finite and infinite) arity, there will be infinitely many Identity and pairwise Diversity relations, as well as mixed Identity-Diversity relations. An example of the latter is the ternary relation that holds between three individuals if and only if the first and second are identical and are different from the third. These are the level 1 logical properties.

At level 2 we get again Existence and Nonexistence (of level 2) and *all* the cardinality properties. We also get again the binary relations Identity, Diversity, Universal, Self-Difference as well as (like Tarski) the Aristotelian binary relations of Subordination, Nonsubordination, Exclusion and Nonexclusion, and all the complex properties that we can define by means of the usual logical notions. Among these we get binary relations between level 1 properties and individuals, and in particular the Application relation and its converse the Instantiation relation – where the latter corresponds to the membership relation for sets.

And so on for all higher levels, where my classification generally coincides with Tarski's. But differences will appear because the specific nature of the universe of individuals will introduce limitations for Tarski's classification. For suppose that the universe of individuals is of cardinality κ (finite or infinite). If λ is larger than κ, then the pairwise set-theoretic diversity relation of cardinality λ will be empty – because there does not exist any λ-sequence of pairwise different individuals[6]. Therefore the pairwise Diversity relations of cardinality greater than κ

[6] I am talking here of a level 1 relation. If one goes up *enough* along the hierarchy of (set) levels, then one will find such a pairwise diversity relation for entities of level lower than it.

will not have a set-theoretic counterpart. One way to avoid this is to postulate the Axiom of Units that I suggest in Chapter 9 (pp. 315-17), which ensures that one never runs out of individuals – *i.e.*, that there are as many individuals as there are sets.

2. MODALITY AND POSSIBLE WORLD SEMANTICS

Frank suggests that modal logic "aroused suspicion" in Gödel because he did not think that there is "any clear philosophy in the models for modal logic" (p. 101). I think that this may be a misinterpretation[7]. The problem is not with modal logic but with the attempted accounts of modal logic. I make some remarks along these lines on pp. 355-56 saying that while I agree that the technical work in possible world semantics has brought a fair amount of light to issues in modal logic, I do not think that this work gives a philosophical account of the basic notions of modal logic. Rather than *explaining* the notions of necessity and possibility, the notion of possible world *presupposes* them. If we have a good account of possibility, then we may have an account of the notion of possible world. But possible world semantics as such is not such an account[8].

In those same pages I also complained that the usual accounts of properties in terms of possible worlds do not give us any clear insight into the nature of properties because these accounts are basically extensional – since a property is supposed to be a function that assigns an extension to each possible world. Although this idea actually meshes

[7] The small paragraph where Gödel makes this remark goes as follows: "When I entered the field of logic, there were 50 percent philosophy and 50 percent mathematics. There are now 99 percent mathematics and only 1 percent philosophy; even the 1 percent is bad philosophy. I doubt whether there is really any clear philosophy in the models for modal logic." (Wang 1996, p. 82)

[8] Although we could take a specific system of possible world semantics to be something like an "axiomatic" or "implicit" account of modality.

quite well with the idea of properties as identity conditions that I suggest in p. 421[9], it does not involve an intrinsic (or intensional) characterization of properties, but characterizes them in terms of the *totality* of their instances (in all possible worlds). What I attribute to Plato and to Kripke is an intrinsic characterization of a property in terms of the *nature* of its instances.

I think that the notions of necessity and possibility may have to be taken as primitive notions that cannot be explained in more basic terms. We have various kinds of intuitions about these notions however, and they help to give some bite to our pronouncements about them. Some of these intuitions are based on our ability to conceive, or imagine, or describe certain situations that we then take to be possible. Other intuitions have a more formal character and derive from our understanding of the meaning of the notions in question, and this is what we try to codify into systems of modal logic (both syntactic and semantic).

We are often very sloppy when we argue in terms of conceivability or imaginability. Thus people used to say that they can easily imagine a world in which Sherlock Holmes exists; or a world in which unicorns exist; and so on. One of the great merits of Kripke's work in *Naming and Necessity* was to show that such claims are based on misconceptions[10]. But we might still say that we can imagine a spaceship that travels *faster than* the speed of light, or *at* the speed of light, or *close to* the speed of light. But can we really? *What* are we imagining when we imagine a spaceship traveling faster than the speed of light? Is imagining (or conceiving) a spaceship traveling faster than the speed of light different than imagining (or conceiving) a spaceship traveling at 20,000 kilometers an hour? In which way? I am not saying that we cannot conceive or imagine or describe various kinds of situations, but that this

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[9] See also §2 of my reply to Richard Vallée.

[10] My views on modality were deeply influenced by Kripke's book – much more so than by possible world semantics as such.

conceiving or imagining or describing should involve more than simply *assuming* that such a situation is the case. Which is also not to deny that within the framework of a given theory the possibility of assuming consistently that something is the case may be good enough as a mark of possibility.

So even though I think that there are various ways in which we can back up our intuitions of possibility and of necessity, and that there is some insightful formal work on modal logic, including possible world semantics, I do not think that this amounts to an analysis of the fundamental modal notions.

With respect to the claim in p. 72 (note 18) that there are non-extensional relations between properties, what I had in mind is something quite simple. When we say that all men are mortal, for instance, we normally mean the extensional subordination of the property of being human to the property of being mortal – *i.e.*, that all humans die. It seems to me however, that there is a stronger connection between these properties in that it is in the very nature of humans to die. We don't just *happen* to die, but it is an aspect of our biological nature that we *must* die. Whether this is right or wrong does not matter; the point is that if there is such a connection, then it is a necessary connection[11].

3. GÖDEL'S VIEWS ON LOGIC AND MATHEMATICS

As I mention in the Preface, Gödel was a major influence in the development of my views. When I wrote my book I had read his published works and the account of his views in Wang *From Mathematics to Philosophy*. I am particularly sympathetic to his view that logic is a theory of concepts, which is essentially the view that I defend in my book in terms of properties. If a theory of concepts not involving type distinctions can be developed in a *natural* and *consistent* way along some of

[11] I do not really know what to say about Frank's question concerning the "realm of ought" since I have not thought about this subject in any detail.

the directions that Gödel suggests, then I might choose it over a typed ontology. I just do not see at this point how it could be done.

With respect to mathematics the situation may be a little different. I understand quite well what Gödel means by saying that mathematics is essentially a theory of extensions – or set theory. The problem is that I have a certain difficulty accepting that mathematical entities are *objects*, or that extensions (or sets) – in the sense in which they are used in mathematics – are objects. This is one of the reasons for my speculations in chapters 9 and 10 about extensions, sets and structures. As I discuss in my reply to Abel Casanave, I think that mathematics is also fundamentally a theory of properties (concepts), but it is primarily a theory of *structural* properties. Logic, on the other hand, is a *general* theory of properties as such, as well as a specific theory of logical properties–or of formal concepts, in Gödel's terminology.

This characterization of logic would seem to agree with Gödel's, and maybe he would also agree with the characterization of mathematics as a theory of structural properties. Where Gödel would disagree is with my characterization of the hierarchy of properties. From the reports in Wang (1996), Gödel thought that a hierarchical approach to properties (or concepts) is a way of avoiding the fundamental problems. Wang quotes him as saying:

> Even though we do not have a developed theory of concepts, we know enough about concepts to know that we can have also something like a hierarchy of concepts (or also of classes) which resembles the hierarchy of sets and contains it as a segment. But such a hierarchy is derivative from and peripheral to the theory of concepts; it also occupies a quite different position; for example, it cannot satisfy the condition of including the concept of *concept* which applies to itself or the universe of all classes that belong to themselves. To take such a hierarchy as the theory of concepts is an example of trying to eliminate the intensional paradoxes in an arbitrary manner.[12] (Wang 1996, p. 278, 8.6.20)

[12] Notice, by the way, that whereas in the quotations from Tarski 'class' is used synonymously with 'set', Gödel uses 'class' in the sense of 'proper class'.

As it is clear from the Russell paper and from Wang (1996), Gödel placed a lot of emphasis on the "intensional paradoxes, of which the most important is that of the concept of not applying to itself", and for him a theory of concepts should give a solution to it and not merely escape it. In the Russell paper Gödel suggests that

> It might ... be possible to assume every concept to be significant everywhere except for certain "singular points" or "limiting points," so that the [intensional] paradoxes would appear as something analogous to dividing by zero. (Gödel 1944, p. 150)

But both in this paper and in the reports in Wang (1996, p. 268) Gödel maintains that no such theory is at hand.

In this connection we could consider whether there really is a property *property* – or a concept *concept* – as well as a property *non-self-applicable*. Is there a property *object*, for instance? In my discussion at the beginning of Chapter 9 (p. 322, note 1) I mentioned that Frege characterized objects as non-functions. In terms of identity conditions we may ask: What are the identity conditions for being an object? And what are the identity conditions for being a property? (Cf. my discussion in pp. 421-25.) At the end of Chapter 9 (pp. 319-20) I introduced the notion of 'notion' – which I actually took from Gödel (1944, pp. 137-38) – as a way of talking in this very general way. Also my idea of cumulativity for properties was a way of trying to make up for these lacks. I gather from various remarks in Wang (1996) that none of this would be satisfactory for Gödel. But Wang also quotes the following remark:

> The general concept of *concept* is an *Idea* [in the Kantian sense]. The intensional paradoxes are related to questions about Ideas. Ideas are more fundamental than concepts. The theory of types is only natural between the first and the second level; it is not natural at higher levels. Laying the foundations deep cannot be extensive. (Wang 1996, p. 268)

This suggests a distinction between concepts and ideas (about which we find nothing in Gödel's published work) that may perhaps have a similar role to the one that I am attributing to the notion of 'notion'. Evidently there will be many problems in finding the correct interpretation of Gödel's views, especially since he did not develop them systematically.

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