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THE NATURE OF PROPOSITIONS: REPLY TO JAIRO JOSÉ DA SILVA

OSWALDO CHATEAUBRIAND

Department of Philosophy
Pontifical Catholic University of Rio de Janeiro
Rua Marquês de São Vicente, 225, Gávea
22453-900 RIO DE JANEIRO, RJ
BRAZIL

oswaldo@fil.puc-rio.br

Abstract: In §1 I reply to Jairo's objections to my account of truth and falsity showing that my account of falsity does not imply that false sentences refer to something. In §2 I argue that Jairo's main objection to my account of propositions as abstract properties is based on a misunderstanding concerning the purpose of this account. In §3 I examine Jairo's suggestion that contradictory sentences can be said to describe possible states of affairs.

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I mention in several places that Part I of my book is primarily concerned with ontological issues and not with epistemological or linguistic issues – which are taken up in Part II. So when Jairo criticizes me for developing an ontological notion of proposition and ignoring linguistic, subjective and epistemological issues he is misinterpreting me. I do not deny the importance of these aspects in the analysis of various notions, but I am just not dealing with them in Part I – although even in this volume there is evidence that my position is not what Jairo takes it to be. But let me comment on some of Jairo's specific points and criticisms.

1. TRUTH AND FALSITY

Jairo runs together several different issues in the second paragraph of his critique. It is true, as he says, that I embrace a realist theory of truth and that I claim that true propositions (statements, sentences, beliefs, judgments, etc[1]) identify aspects of reality[2]. I say that this is a natural view and Jairo claims that it is not. "And", he says, "the strongest argument that it is not lies ... in the fact that there are false propositions".

The problem of falsity plays an absolutely central role in my discussion, and it is introduced at the very beginning of Chapter 1 (p. 47). I quite agree that propositions that "intend" to describe reality may fail to do so and I distinguish two cases: the truth-valueless propositions that fail because they do not connect appropriately with reality – by involving non-denoting expressions, for example – and the false propositions that fail because their (predicate) *negation* identifies an aspect of reality. Jairo may not like my solution, but large parts of my book (including the development of the subject-predicate analysis) are an elaboration and defense of it, and Jairo's main objection in p. 142 is based on misconceptions.

He says:

> As for false assertions, Chateaubriand believes that they are "lost bullets", which aimed at something end up hitting a completely different target. By asserting falsely that Theaetetus is flying, he says, I'm in fact somehow identifying the state of affairs <other-than-flying, Theaetetus>, even though I never intended such a thing.

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[1] I maintain that a reasonable account of truth should allow us to talk about truth for any of these things (see the bottom of p. 241, for example, and also my reply to Arno Viero). Since I take propositions to be abstract aspects of reality independent of language and thought – and so characterize them in chapters 1, 11 and 12 – I use 'statement' in what I take to be a sense related to that in which Jairo uses 'proposition'. Thus, in Chapter 1 I use 'statement' until p. 61 and then switch to 'sentence' for the reason given there. In my comments on Jairo's paper I will try to use his 'proposition' and resolve ambiguities as I go along.

[2] I do not see the need for the qualification "*reality itself*", but I will come back to this later.

There are two mistakes here. One is the reference to lost bullets, which I suppose Jairo took from p. 415. What I discuss there is not my own view but Russell's. The other (more serious) mistake is that he attributes to me a view of falsity that is neither my view nor Russell's.

Russell's view – which I discuss in chapters 1, 5 and 12 – is that a false proposition is one which *points away* from a fact. Thus the proposition "Theaetetus is flying" is false, according to Russell, because it points away from the (negative) fact <Other-than-flying, Theaetetus>.

My view – which I introduce in Chapter 1 (p. 57) and discuss throughout the book – is that the proposition "Theaetetus is flying" is false because its *predicate negation* "Theaetetus is not flying" is true; and *this proposition* is true because *it* identifies the fact <Other-than-flying, Theaetetus>.

Therefore, neither in Russell's view nor in mine do false propositions *point to* or *identify* anything.

2. PROPOSITIONS

There is a standard discussion in logic and in the philosophy of language concerning the so-called bearers of truth[3]. The list of candidates is very long, and among them are: propositions, statements, assertions, sentences, thoughts, judgments, beliefs, opinions, utterances, inscriptions. Although the word 'proposition' is a word of ordinary language that can be taken to mean the same as some of the other words that I listed[4], there is a fairly general consensus in logic and philosophy to use the word 'proposition' to refer to *abstract entities*. Propositions in this sense are not supposed to be mental entities like thoughts and beliefs, or linguistic entities like sentences, or concrete entities like

[3] I am referring to the contemporary discussion, although there is also a traditional discussion detailed in Nuchelmans (1973).

[4] The *Concise Oxford Dictionary* gives 'statement' and 'assertion' as the main meanings.

utterances and inscriptions. Some of the objections that are leveled against propositions derive from their abstractness, others derive from their alleged lack of identity criteria, yet others derive from their alleged causal inertness, and so on.

In my book I propose a specific notion of proposition in this abstract sense. My notion of proposition has a certain relation to Frege's *objective* notion of thought, as something independent of actual acts of thinking. Frege's thoughts are senses, and so are my propositions. As I explain briefly in Chapter 1, and in more detail in Chapter 11, a sense for me is an identifying property: something that can be expressed in the form 'is the so-and-so'. To say that a sense identifies something is to say that it is instantiated. Propositions are senses that if instantiated at all, are instantiated by states of affairs. True propositions are those that are instantiated – *i.e.*, that identify a state of affairs. False propositions are those whose predicate negation is instantiated. Other propositions are neither true nor false.

By characterizing propositions in this way I do not mean to deny the existence or importance of *any* of the other truth bearers. I state this at various points, as I mentioned in note 1. Another example may be found on p. 319, where referring to the hierarchy of objects, properties and states of affairs I say:

> ... acceptance of the ontological structure does not deny us the use of any of the interesting work that has been done nominalistically, mentalistically, linguistically, etc. On the contrary, it may help to illuminate such work in various ways without rejecting its specificity.

Jairo attributes to me a number of views about propositions that I do not hold. One is that I consider propositions to be "vectors", or "arrows", or "tags" that "point to", or "point away", or "attach" to aspects of reality. Some of this applies to Russell, but not to me. Jairo might have been misled by my discussion in Chapter 12 where I use some of these terms in connection with a somewhat "generic" discussion of various authors and views.

Jairo's charge that I am led to this view of propositions by "[t]aking the language of science and mathematics as the model for language in general" (p. 142) is also incorrect – but this is something that I will discuss in Chapter 13 ("Language") of Part II.

Jairo also remarks (p. 143) that

> faithful to a philosophical tradition that flees in horror from the simple mention of subjective aspects in logic (for Frege supposedly showed us that this is a sure sign of dreadful psychologism), Chateaubriand ignores completely the problem of the genesis of propositions in *acts* of judgment (in fact his realism precludes any questioning of such a nature).

A mathematician-philosopher who plays a very important role in my book is Brouwer, who is a thoroughly subjectivist thinker. Although Brouwer's role is much more central in the discussions of Part II, I refer to him sympathetically in the introduction and in chapters 9 and 12. In fact, after a discussion of some of Brouwer's views in pp. 21-3, I remark that "I have sympathy for Brouwer's approach and I think that in some respects it is not that far from Frege's" (p. 23). I think that anyone who reads my remarks about Brouwer in those pages (and in pp. 338-39 and 425-26) should realize that I do not "flee in horror" from subjectivism (or from mysticism).

3. POSSIBILITY

In his second paragraph Jairo says that "one may use propositions – *e.g.*, in counterfactuals – to refer *intentionally* to non-actual possibilities". I can agree with this, depending on how the notions of reference and of possibility are interpreted[5]. But it is not clear to me how Jairo interprets either. In p. 142 he talks about 'fictional "realities"', and in the next few pages about '*alternative* realities', 'possible situations', 'possible scenarios', 'possible states of affairs', and so on. I do not know if Jairo intends some

[5] See also §2 of my reply to Frank Sautter.

of these possibilia to be "part" of reality – as Quine's possible fat men in the doorway – or to some notion related to the notion of possible world, or to a linguistic notion of possibility. The latter is suggested by his remarks beginning in p. 144 – and especially by his concluding remarks about possibility in mathematics.

In any case, one of his specific objections to me goes as follows:

> Chateaubriand considers a modal treatment of propositions, but dismisses it right away. The reason is that there are meaningful propositions that are *necessarily* false, like 2+2=5. If they describe a possible state of affairs, as I claimed above, there must exist impossible possibilities! The way out of this dilemma lies in the correct understanding of what "possible" means in this context. Since the material meaningfulness of propositions depends on compatibility of material *types* exclusively, not *instances* of types, it may happen that certain contents are materially compatible, i.e. compatible considered *solely* as elements of certain types, but not in fact compatible as the *particular* elements they are. 2+2, for instance, could have been equal to 5, considered exclusively as numbers (for any two numbers are compatible with respect to equality), but not as the particular numbers they are. 2+2=5 is *a priori* possible (since it is a situation described by a meaningful proposition), but *a posteriori* (*i.e.* after calculation) impossible. If you think all this is nonsense, answer fast: is 153448+93745 = 248193 possible? What about the Riemann hypothesis? (pp. 145-146).

Well, I do think that it is nonsense; or at least very misleadingly stated. What does it mean to say "2+2 ... could have been equal to 5, considered exclusively as numbers ..., but not as the particular numbers they are"? What does it mean to say "any two numbers are compatible with respect to equality"? As far as I can make out Jairo's explanations, these statements mean that equations of the form '$n=m$' make sense – and perhaps that equations of the form '$n+n=m$' also make sense. What has this got to do with *possibility*? This so-called notion of possibility does not seem to me to be a notion of possibility at all, but rather an expression of lack of knowledge. My answer to Jairo's questions is the following: For all I know '153448+93745 = 248193' may be true or it may be false (because I have not carried out the calculation), and for all I know the Riemann

Hypothesis may be true or it may be false (because as far as I know nobody has either proved it or disproved it). Where is the mystery in that?

I continue to think that there is no reasonable sense of 'possibility' and of 'state of affairs' according to which '2+2=5' describes either a *possible* or an *impossible* state of affairs[6]. But let me go back to Jairo's initial remark about counterfactuals[7].

There is a clear sense in which we use counterfactuals with impossible antecedents in mathematics, and that is when we do proofs by *reductio ad absurdum*. In the classical proof of the irrationality of √2, for example, we assume that √2 is a rational number and conclude that an irreducible fraction is reducible. Let us state this subjunctively:

(1) If √2 were a rational number, then there would be a reducible irreducible fraction.

Should we say that the antecedent describes a possible situation? That it describes an impossible situation? What are we doing when we do the *reductio* proof? The standard idea is that we *assume* that √2 is rational and then derive a contradiction from this assumption. Does it follow that by making our assumption we are *describing* a situation in which √2 is rational? Or even that we are *envisioning* such a situation? How does this compare with an everyday subjunctive? Suppose I say:

(2) If Jairo were in my office now, we would be discussing his paper.

In this case it is natural to say that the antecedent describes a possible situation. I could make up a little story about Jairo coming to Rio to see

[6] See also §1 of my reply to Luiz Carlos Pereira.

[7] On counterfactuals more generally see my reply to Claudio Pizzi.

an opera and dropping by to have a chat with me. It is this kind of possibility that gives some bite to the idea that the antecedent describes a possible situation – and this is what is behind Kripke's idea of treating possible worlds as counterfactual situations. Could we do this in the mathematical case? Could we make up a little story as to how √2 came to be a rational number and how an irreducible fraction came to be reducible? I do not think so.

Also, when we start formulating mathematical counterfactuals, we soon become involved in very confused situations. Let me take some simple examples of the kind that Jairo discusses. Consider

(3) If 2+2 had been equal to 5, then 2 would have been equal to 3.

We could justify this on the basis of the law

(4) $\forall n \forall m \forall r(n+r = m \rightarrow n = m-r)$.

But then we could use the law

(5) $\forall n \forall m(n = m \rightarrow n+1 = m+1)$.

to justify

(6) If 2 had been equal to 3, then 3 would have been equal to 4.

If we now use (3), (6) and all the similar counterfactuals that we can get from (5), then by repeated uses of transitivity we can justify

(7) If 2+2 had been equal to 5, then 2 would have been equal to 3,

(8) If 2+2 had been equal to 5, then 3 would have been equal to 4,

and so on. And if we use other arithmetical laws we will soon be led to a completely chaotic situation where *anything* goes[8]. In particular, we could use an ω-inference from the infinitely many counterfactuals of the form

(9) If 2+2 had been equal to 5, then n would have been equal to $n+1$

to conclude

(10) If 2+2 had been equal to 5, then there would have been only one natural number.

Of course, we could also argue that if 2+2 had been equal to 5, then the usual arithmetical laws would no longer hold and none of the counterfactuals that I listed above would be justified. But how do we decide what to maintain and what to change? Thus, it is not so easy to develop a reasonable account of mathematical counterfactuals[9].

There is an interesting class of cases where the situation may not be so clear, however. This is the case of metamathematical counterfactuals.

Consider, for instance:

(11) If it had been provable in *ZF* that $2^{\aleph_0} \neq \aleph_2$, then it would have been provable in *ZF* that $2^{\aleph_0} = \aleph_1$.

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[8] These absurd conclusions are precisely what we seek in proofs by *reductio ad absurdum*. The difference is that whereas in the *reductio* proofs we derive clear absurdities from an assumption that on the face of it is not absurd, in these examples we already start with clear absurdities.

[9] It may not be impossible to develop an account of mathematical counterfactuals, but I do not see how to do it in a coherent way.

If *ZF* is consistent, then both antecedent and consequent are false – and hence impossible. Yet Hajnal's (1956) proof of (the indicative form of) this result seems to be sufficient to sustain the counterfactual. The feeling we have is that since proving is a human activity we can perfectly well imagine a situation in which somebody *proved* in *ZF* that $2^{\aleph_0} \neq \aleph_2$. Given the independence results this alleged proof could not be a *correct* proof, of course, but we could also imagine that it is the independence proofs that contain errors.

Another similar example is:

(12) If Fermat's Last Theorem had been false, it would have been refutable in first-order Peano Arithmetic.

The justification in this case derives from the well-known fact (or law) that all numerical equations can be proved in first-order Peano Arithmetic, and that a counter-example to Fermat's Last Theorem would have been such an equation. This example is interesting in that whereas the consequent is a metamathematical statement the antecedent is simply the assumption of falsity. Consider now:

(13) If there were natural numbers n, m, r and $k > 2$ such that $n^k + m^k = r^k$, this would have been provable in first-order Peano Arithmetic.

It is easier to make up a little story to the effect that *somehow* Fermat's Last Theorem turned out to be false than to make up a little story in which some natural numbers n, m, r and $k > 2$ turn out to be such that $n^k + m^k = r^k$.

What sustains these counterfactuals are not the little stories that we can make up about them however, but rather the fact that we have "independent" *proofs* leading from the antecedents to the consequents.

There are many interesting issues that we can raise concerning these (and other) examples. A particularly interesting case of *actual* subjunctive reasoning in mathematics is provided by some of Brouwer's so-called metamathematical proofs – such as the proof of the Bar Theorem[10] – which I discuss in some of the chapters of Part II[11].

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[10] See, for instance, Brouwer (1954).

[11] In pp. 23-4 of the Introduction I discuss briefly the problem of negation in intuitionistic logic which also has a bearing on these issues.