BOOLE ON REFERENCE AND UNIVERSES OF DISCOURSE: REPLY TO JOHN CORCORAN

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Abstract: In §1 I examine Boole's "principle of wholistic reference" in relation to Frege's postulation of truth-values as referents for sentences. I also consider in this connection Frege's interpretation of quantification and his view that functions and concepts (of objects) must be defined for all objects. I then present my own contrasting views on the reference of sentences. In §2 I discuss Boole's introduction of the notion of universe of discourse and consider whether one of the issues implicit in John's paper is a confrontation between absolute interpretations of logic and relativistic interpretations of logic. I conclude with a brief examination of Tarski's views on this issue.

Key-words: Wholism. Truth-values. Quantification. Universe of discourse. Boole. Frege. Gödel. Tarski.

Although John's comments refer to my book in a somewhat indirect way, there are some significant issues between us. I will divide my discussion in two parts, the first relating primarily to the question of wholistic reference and the second to the question of universes of discourse.

1. WHOLISTIC REFERENCE

When I said in pp. 52-3 that for Frege the connection of statements (thoughts, sentences) with reality must be interpreted "globally", I was referring to his thesis about the truth-values the True and the False. If it makes any sense at all to say that all statements refer to one or another of two specific objects, then it seems to me that these "objects" must in some way "represent" very general aspects of reality. The obvious candidate for the True is to represent something like reality as a whole. The problem is what to do with the False, which cannot be coherently interpreted as representing unreality – at least not in any straightforward sense. After coming back to this issue several times in the book, I suggest in Chapter 12 (p. 417) that the True represents something like an "acceptance" by reality of the combination of some of its (reality's) aspects in accordance with their articulation in the thought, and that the False represents a "rejection" of the same. This is consistent with Frege's idea that while thoughts do not refer to specific aspects of reality, in judging the truth of a thought one divides the truth-values into "parts". I even suggest in Chapter 2 (p. 79) that with this idea Frege is getting close to a conception of states of affairs even if not attributing any kind of ontological specificity to them.

What John calls 'the principle of wholistic reference' is the view that "the universe of discourse is the ultimate subject" that he quotes from Boole in p. 163-164. This can be interpreted in relation to Frege's postulation of truth-values, but there is a more definite connection with Frege's views about quantification.

Frege held that the interpretation of quantification must be completely unrestricted so that quantifiers over objects quantify over *all* objects, quantifiers over functions quantify over *all* functions, and so on. Moreover, in the same vein Frege maintained that a function of objects must be defined for all objects, so that the addition function, for instance, must be defined for the sun and the moon as well as for 3

and 5[1]. For Frege the universe of discourse is absolute and he would not have agreed to the idea of restricted universes of discourse. If one wants to restrict one's discourse to human beings, for instance, one must relativize all quantification to humans by means of the concept (or predicate) 'is human'.

As long as the universe of discourse is interpreted absolutely Frege might agree to the principle of wholistic reference as formulated in terms of quantification, but I do not know whether this had any connection with Frege's idea of postulating the True and the False as the referents of sentences and thoughts.

My own view is "nearly diametrically opposed to the principle of wholistic reference", as John says in p. 166. I find it incredible that a statement such as

(1) Plato is mortal

should refer to the whole of reality rather than to a very specific and well defined aspect of it. But let us consider this in more detail.

Take the quantified version suggested in John's final remarks:

(2) Every object that is Plato is mortal.

Am I referring to every object in (2)? Are quantifiers denoting terms, as Russell used to say? When we say 'everybody' or 'somebody' are we referring to everybody? Although these puzzling questions were supposedly put to rest by the proper use of quantification, I do not think that the ordinary use of quantification is unambiguous in this respect.

In accordance with the subject-predicate analysis and the Fregean view of quantification as predication that I use in my book there are several interpretations of (2). Let us consider the following two:

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[1] See the quotation from Frege in p. 210 (note 17).

(3) $[\forall y(y = x \rightarrow y \text{ is mortal})](\text{Plato})$

(4) $[\forall y Zy]([y = \text{Plato} \rightarrow y \text{ is mortal}](y))$.

According to (3) we are predicating the property

(5) $[\forall y(y = x \rightarrow y \text{ is mortal})](x)$

of Plato, and according to (4) we are predicating the property

(6) $[\forall y Zy](Z)$

of the property

(7) $[y = \text{Plato} \rightarrow y \text{ is mortal}](y)$.

It seems reasonably clear to me that the property (5) – which we can read informally as 'whatever is x is mortal' – is just the property of (x) being mortal[2]. From which I would conclude that (2) as interpreted by (3) *does not* refer to every object.

Let us consider (4) now. Here we are predicating the level 2 property Universality of the level 1 property (7) – which can also be expressed as the property of not being Plato or being mortal. The level 2 property Universality is a property that applies to a level 1 property if the latter applies to all objects, so by predicating (6) of (7) we are saying that (7) applies to all objects, and it seems reasonable to say that we are referring to all objects by means of our predication.

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[2] Consider the conditions of applicability of these properties. Would you be tempted to examine *every* object in order to find out that if it is Plato, then it is mortal?

The moral that I draw is that whereas the natural interpretation (3) of (2) refers to the specific fact of Plato being mortal (as does (1)), there are other ways – *e.g.*, (4) – of construing (2) according to which we are predicating of a property that it applies to all objects. There are also other construals; as for instance

$$(8)\ [[\forall y(y = x \rightarrow Zy)](Z, x)]([y\text{ is mortal}](y), \text{Plato}),$$

which says that the level 2 relation Application relates the level 1 property of being mortal and Plato. Although it might seem that (8) says the same thing as (1), the distinction between them is the same as the distinction between set-theoretic facts of the forms

$$(1^{e})\ a \in B$$

$$(8^{e})\ \langle a, B \rangle \in \{\langle x, Y \rangle\colon x \text{ is an individual \& } Y \text{ is a set of individuals} \\ \& \ x \in Y\}.$$

Although it is true that one of these cannot obtain without the other obtaining, in my view they are not the *same* fact.

One can argue that all the construals that I gave for (2) are logically equivalent to each other and to (1), and therefore that they should refer to the same state of affairs, but one of the main claims that I defend is that neither c-logical equivalence nor tv-logical equivalence guarantee that sentences that are equivalent in these senses refer to the same state of affairs[3].

2. UNIVERSES OF DISCOURSE

John considers Boole's (1847) introduction of the symbol '1' for *the* universe of discourse a "momentous innovation" that marked "a milestone in logic". Further, he considers Boole's (1854) relativistic use

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[3] See pp. 81, 143-146, 192-195.

of '1' for *a* universe of discourse as "part of an even more revolutionary sea change in logic". One of the reasons for John's assessment is that "[i]n this way Boole opens up room for one and the same language to be used in many different interpretations in a way that had never been done before" (pp. 162). But it *had* been done before for the language of algebra, and this is what Boole mentions as an inspiration at the beginning of the introduction of (1847). I quite agree that Boole's was the first systematic mathematization of logic – as I say in p. 14 of my own Introduction. I also agree about the importance of Boole's algebraic work for the development of logic – although I mainly discuss and emphasize Boole's distinction between primary propositions and secondary propositions[4], which John does not.

The issue between John and me in relation to Boole may have to do in part with the interpretation of logic. I interpret logic absolutely as a science, or theory, about some fundamental aspects of reality. This is how Frege and Russell viewed logic, and it is also clearly the way in which Gödel viewed logic[5]. The main aim of my book is to defend a metaphysical view of logic according to which logic is both an ontological theory of properties – and especially a theory of logical properties – and an epistemological theory of deduction. In particular, while not denying the importance of the mathematical work in model theory, proof theory, recursion theory and set theory, I think that the generalized tendency to view logic as being fundamentally a study of formal languages that can be interpreted this way or that is misguided. This said, let me go back to Boole's universes of discourse.

I do not attach as much importance as does John to the notion of universe of discourse as such[6]. I agree that Boole's algebraic treatment of

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[4] See pp. 199-204.

[5] See my brief discussion of Gödel's views in §3 of my reply to Frank Sautter.

[6] John says that Boole's use of 'the universe' in (1847) corresponds to the logical notion of individual (or thing or object). But when Boole introduces

logic was a fundamental development that led to very important work done in the late Nineteenth Century and in the early Twentieth Century. This work helped set up the basis for the later development of model theory begun in the early 50's by Tarski, A. Robinson, and their students and associates. We can say, therefore, that Boole's algebraic work eventually led to a momentous innovation in logic. The question in my mind, however, is whether John thinks – or implicitly claims – that the metaphysical conception of logic inspired by Frege, Russell and Gödel, that I defend in my book, is a wrong conception of logic? In other words, does he agree with Boole that "we ought no longer to associate Logic and Metaphysics"?[7]

Tarski may have agreed with Boole's claim, because for him the word 'metaphysics' had undesirable connotations. But even though Tarski was a nominalist and was not metaphysically oriented in any of the ways in which Frege, Russell and Gödel were, I believe that an

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this notion he says: "Let us employ the symbol 1, or unity, to represent the Universe, and let us understand it as comprehending *every conceivable class of objects whether actually existing or not*" (Boole, 1847, p. 15 my italics) One wonders which class of "things" this actually is.

[7] The passage is the following:

> Let it be granted that the problem which has baffled the efforts of ages, is not a hopeless one; that the "science of a real existence," and "the research of causes," "that kernel" for which "Philosophy is still militant," do not transcend the limits of the human intellect. I am then compelled to assert, that according to this view of the nature of Philosophy, *Logic forms no part of it*. On the principle of a true classification, we ought no longer to associate Logic and Metaphysics, but Logic and Mathematics. (Boole, 1847, p. 13)

Evidently Boole's statement makes a lot of sense in the historical context, and I (as well as all the logicians whom I mentioned before) agree with Boole on the association between logic and mathematics. The question is whether this *excludes* the connection between logic and metaphysics.

absolute view of logic should also be attributed to him, as I will elaborate in my concluding remarks.

In pp. 14-5 of my book I give the misleading impression that Tarski was primarily responsible for the elimination of meaning from logic and for the development of the relativistic conception of logic inspired by model theory. I want to qualify this in some important respects. It is correct to say – as I do toward the end of p. 15 – that Tarski's work on truth made possible the elimination of meaning from logic, but it is incorrect to refer to this as the "Tarskian view of logic" in which "the notion of meaning largely disappears", as I do at the beginning of the next paragraph. It would be much more accurate here to substitute 'Quinean' for 'Tarskian' and to drop the 'largely'. Similarly, it is correct to say that Tarski's work on truth and consequence and his later development of model theory led to a certain mathematical relativism in logic, but the implicit suggestion that this might have been Tarski's view seems to me incorrect.

In many of his papers Tarski consistently refers to logic as a science. And we get an explicit statement of this view of logic – while at the same time declining to discuss the question 'What is Logic?' –even as late as his conference "What are Logical Notions?":

> I take logic to be a science, a system of true sentences, and the sentences contain terms denoting certain notions, logical notions. (Tarski, 1986, p. 145) [8]

This view of logic is also clearly present in Tarski's book *Introduction to Logic and to the Methodology of Deductive Sciences*. Nowhere in this book is there the smallest hint of a relativistic interpretation of logic. When introducing the notion of universe of discourse, for example, Tarski says:

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[8] This is also (part of) my own view. There is some discussion of Tarski (1986) in §1 of my reply to Frank Sautter.

> Instead of using the general logical concept of individual within a particular mathematical theory, it is sometimes more convenient to specify exactly what is considered an individual thing within the framework of this theory; the class of all those things will be denoted again by "V" and will be called the UNIVERSE OF DISCOURSE of the theory. In arithmetic, for instance, it is the class of all numbers which forms the universe of discourse. (Tarski, 1965, p. 73)

In the Preface of his book Tarski says about logic:

> the word 'logic' is used ... in the present book ... as the name of a discipline which analyzes the meaning of the concepts common to all the sciences, and establishes the general laws governing the concepts. (Tarski, 1965, p. xiii)

Although Tarski's interpretation of 'concept' was quite different from Gödel's, the previous remark as well as the entire development of Tarski's book suggests to me that he would have agreed with Gödel's opening remarks in the Russell paper:

> Mathematical logic, which is nothing else but a precise and complete formulation of formal logic, has two quite different aspects. On the one hand, it is a section of Mathematics treating of classes, relations, combinations of symbols, etc., instead of numbers, functions, geometric figures, etc. On the other hand, it is a science prior to all others, which contains the ideas and principles underlying all sciences. (Gödel, 1944, p. 125)

But perhaps Boole would also have agreed with this.

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