Logic, Truth and Description. Essays on Chateaubriand's Logical Forms
Jairo J. da Silva (ed.)

CDD: 160

NEGATION AND NEGATIVE PROPERTIES: REPLY TO RICHARD VALLÉE

OSWALDO CHATEAUBRIAND

Department of Philosophy
Pontifical Catholic University of Rio de Janeiro
Rua Marquês de São Vicente, 225, Gávea
22453-900 RIO DE JANEIRO, RJ
BRAZIL

oswaldo@fil.puc-rio.br

Abstract: I argue in §1 that there is a clear distinction between predicate negation and sentential negation and that sentential negation is a special case of predicate negation operating on the predicate 'is true'. In §2 I reply to Richard's objections to negative properties on the basis of the conception of properties as identity conditions presented in Chapter 12 of *Logical Forms*.

Key-words: Sentential negation. Predicate negation. Negative properties. Plato. Kripke.

Richard is not enthusiastic about negative properties and raises some questions about them and about the relation between sentential negation and predicate negation. I will start with the latter.

1. SENTENTIAL NEGATION AND PREDICATE NEGATION

As I observe several times in my book, it is quite clear that in predicate logic the connectives have a dual function. On the one hand they are used as sentential connectives that combine sentences into more complex sentences, and on the other hand they are used as predicate operators that combine predicates into more complex predicates[1]. Thus in the sentence

(1) $\neg\forall x((\neg Fx \vee Gx) \rightarrow \neg Rx) \vee \neg\exists x(Fx \;\&\; Gx)$

the first and last occurrences of '¬' and the second occurrence of '∨' are sentential operators whereas the second and third occurrences of '¬', the first occurrence of '∨' and the occurrences of '→' and '&' are predicate operators[2]. There is nothing unusual about this, and we prove all sorts of theorems both about the sentential connectives and about the predicate operators. Thus about negation we prove

(2) $p \leftrightarrow \neg\neg p$

in the sentential calculus, using the negation as sentential negation, and we prove

(3) $\forall x(Fx \leftrightarrow \neg\neg Fx)$

in the predicate calculus, using the negation as predicate negation.

Richard begins his argumentation by considering a simple predication of the form '*Fa*' and asking what is the relation between the sentential negation '¬[*Fa*]' of '*Fa*' and the predicate negation '[¬*Fx*](*a*)' of '*Fa*'. In particular, he asks what are the semantic rules for these two negations[3]. He suggests that the semantic rule for sentential negation is (in this case)

---

[1] This is often disguised by talking of predicates as open sentences.

[2] This is assuming that '*F*', '*G*' and '*R*' are "constants", because otherwise *all* the occurrences are predicate operators.

[3] Instead of using Richard's notation 'NEG' for predicate negation I will use the notations in my book in as simple a manner as possible. I trust that the way I made the distinction in the text is quite intelligible.

(SN) If '*Fa*' is true (false), then '¬[*Fa*]' is false (true),

and considers whether the corresponding rule for predicate negation is

(PN) If '*Fa*' is true (false), then '[¬*Fx*](*a*)' is false (true).

He then argues

> If NEG is reiterable, are *Fa* and NEG NEG *Fa* equivalent? Suppose that one says yes. Then semantically NEG cannot be distinguished from sentential negation. Suppose that one says no and contend that we do not have the equivalence. Then one must give the semantics of NEG. (p. 228)

Predicate negation is quite obviously reiterable, as can be seen by such formulas as (3), and if the sentence '*Fa*' is either true or false, then there will be no semantic difference between its sentential negation and its predicate negation. But if '*Fa*' is neither true nor false, then there will be a semantic difference. Thus:

(SN′) If '*Fa*' is neither true nor false, then '¬[*Fa*]' is true.

(PN′) If '*Fa*' is neither true nor false, then '[¬*Fx*](*a*)' is neither true nor false.

This happens because the sentential negation of a sentence *p* means 'it is not the case that *p*' and if *p* is neither true nor false, then it is not true (not the case) and therefore its sentential negation is true. On the other hand, on my interpretation of predicate negation, the predicate negation of a truth-valueless sentence is always truth-valueless. Therefore, the principle of double negation holds for predicate negation but *does not hold* for sentential negation.

This distinction between predicate negation and sentential negation is something that I point out in many places (*e.g.*, pp. 55, 62, 80-1, 113) and I am surprised that Richard does not take it into account – especially because, as I emphasize in pp. 65-6, it is central to my approach to allow truth-valueless sentences.

In any case, Richard seems to be considering only sentences that are either true or false, and for these sentences sentential negation and predicate negation have the same semantic properties – which is basically the semantics or ordinary predicate logic. I believe, therefore, that Richard's considerations about negation rules and quantifier rules do not really apply to the discussion in my book.

Richard discusses some specific examples however, and I find my intuitions to be so at odds with his, that I wonder if I am missing something. Let us consider the example

(4) Quine is not a dentist.

My view is that the most natural interpretation of this statement has the negation as predicate negation. Thus, Richard's option A according to which (4) is true if and only if Quine instantiates the property of not being a dentist is the correct option[4]. I agree however that one *can* interpret the negation sententially as

(5) It is not the case that Quine is a dentist,

which, as I said above, I take to mean the same thing as

(5′) It is not true that Quine is a dentist,

[4] I do not make a distinction between the property of not being a dentist and the property of being a non-dentist.

which is true if it is not true that Quine is a dentist. According to what I said before, (5′) is true because Quine does not instantiate the property of being a dentist, but (5′) could also be true because the statement that Quine is a dentist is neither true nor false. Thus even though in this particular case (4) is true if and only if (5) is true, in pp. 62-3 I point out that this does not hold for the pair

(6) Sherlock Holmes is not a dentist,

which is truth-valueless, and

(7) It is not the case that Sherlock Holmes is a dentist,

which is true.

Richard's option B gives the truth conditions for (4) as either

(8) It is false that ($\exists x\ x$=Quine & x is a dentist),

or

(9) $\exists x$(x=Quine & it is false that x is a dentist).

I find these to be rather incredible *as an account of the truth conditions of* (4)[5], but nothing that I say or do *prevents* me from *using* such sentences. And this applies also to Richard's next example

(10) Peter does not believe that Quine is a dentist.

[5] Richard says (p. 233) that "B is standard, well known and fits logic; A is not standard and is not required by logic." The interpretations B involve a combination of ideas deriving from Russell's theory of descriptions that I discuss and reject in Chapter 3.

Again, I think that the most natural interpretation of this sentence has the negation as predicate negation, but nothing prevents me from using any of the alternatives that Richard provides.

My view of sentential negation and predicate negation is diametrically opposed to Richard's. For whereas he thinks that there is only sentential negation, I think that there is no such thing and that all negations are predicate negations. This is quite clear from my interpretation of sentential negation as negating the truth of a statement. Sentential negations are simply predicate negations that operate on the predicate 'is true'. I discuss this in some detail in connection with propositional logic in pp. 198-204 – and there will be further discussion in Chapter 16.

2. NEGATIVE PROPERTIES

Richard's dislike of negative properties is shared by Frege – as I quote in p. 287 (Note**). I suppose that it all depends on what one means by 'property'. Although I do not propose a worked out theory of the nature of properties in my book, I say in Chapter 12 (p. 421) that the way I think of properties is as *identity conditions*. I refer this idea back to Plato – who always maintained that a form is what *is the same* in all its instances – quoting a passage from Allen where he states "Forms are standards for detecting their instances" (note 5 p. 428). Let us take this as our starting point and consider the property of being a dentist.

If we have identity conditions for being a dentist – which I take it we do, even if they are not *precise* identity conditions – then we also have identity conditions for *not* being a dentist. (It would be rather odd if we could identify dentists but could not identify non-dentists.) Therefore, I hold that there is a property of *being a non-dentist*. This argument generalizes to all properties, in fact. Given *any* property *P* (including relational properties) conceived as a set of identity conditions, these identity conditions determine (by negation) a negative property *not-P*.

Therefore, for any property *P* there is a corresponding negative property *not-P*[6]. And, of course, the iteration of this procedure gives us back the original identity conditions – so that *not-not-P* is just *P*.

Richard asks at the beginning (p. 228) why an object cannot exemplify both *P* and *not-P*. Since we obtain negative properties by "opposition", it seems to be part of the very meaning of 'not' that an object cannot exemplify both *P* and *not-P*. We may also think of this as Aristotle's principle of non-contradiction that is expressed in the predicate calculus by

(11) $\forall x \neg(Fx \,\&\, \neg Fx)$,

and that to my mind is the most basic principle of logic.

At the end Richard claims that negative properties "lack specificity" and that they also "lack causal power" (p. 233). I have already argued that if we have identity conditions for *P* we also have identity conditions for *not-P*. So at least in this sense of 'specificity' they do not lack specificity. As for "causal power", I am always a little weary as to what this is supposed to mean. I gather that according to Richard whereas the property *non-dentist* does not have causal power, the property *dentist* does. What is this causal power of *dentist* supposed to be? On *what* does the property exert its causal power and *how* does it do it? Is the idea that it causes things that have the property to be so? Is the dentistness of dentists *caused* by the property of being a dentist? Or is the idea that it *causes us* to recognize dentists? How does it do that? What is the notion of 'cause' in question? Since I do not know how to answer these questions, I will go back to the idea of properties as identity conditions.

I suppose that one can consider this idea to be unmotivated, but in pp. 422 ff. I give some specific examples of how properties are treated as identity conditions. One example that I have already mentioned is

[6] As I point out in note 20 (p. 72) one has to relativize negative properties to types.

Plato's account of forms[7]. Here is a passage from *Parmenides* 132a that I quote in p. 337:

> I imagine your ground for believing in a single form in each case is this. When it seems to you that a number of things are large, there seems, I suppose, to be a certain single character that is the same when you look at them all; hence you think that largeness is a single thing.

Another example that I give in pp. 422-23 is Kripke's procedure for introducing natural kind terms – which I claim is *exactly* the same as Plato's. Here are two remarks from *Naming and Necessity*:

> The original concept of cat is: *that kind of thing*, where the kind can be identified by paradigmatic instances. (p. 122)
>
> If we imagine a hypothetical (admittedly somewhat artificial) baptism of the substance, we must imagine it picked out by some such 'definition' as, 'Gold is the substance instantiated by the items over there, or at any rate, by almost all of them'. (p. 135)

Although these quotations prove nothing, of course, they seem to me to suggest that my abstract conception of properties that includes negative properties is not as unmotivated as Richard thinks it is.

REFERENCE

KRIPKE, S. *Naming and Necessity.* Cambridge, Mass.: Harvard University Press, 1980.

[7] Plato is supposed to be a thoroughly bad influence however, at least as far as these metaphysical issues are concerned, and appeals to Plato do not seem to cut much ice nowadays.