THE MANY SENSES OF COMPLETENESS[*]

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In this paper I study the variants of the notion of completeness Husserl presented in "Ideen I" and two lectures he gave in Göttingen in 1901. Introduced primarily in connection with the problem of imaginary numbers, this notion found eventually a place in the answer Husserl provided for the philosophically more important problem of the logico-epistemological foundation of formal knowledge in science. I also try to explain why Husserl said that there was an evident correlation between his and Hilbert's notion of completeness introduced in connection with the axiomatisation of geometry and the theory of real numbers when, as many commentators have already observed, these two notions are independent. I show in this paper that if a system of axioms is complete in Husserl's sense, then its formal domain, the manifold of formal objects it determines, does not admit any extension. This is precisely the idea behind Hilbert's notion of completeness in question. Therefore, the correlation Husserl noted indeed exists. But, in order to see it, we must consider the formal domain determined by a formal theory, not its models.

Widely neglected and often misinterpreted, Husserl's investigations into the theory of axiomatic systems were born out of his struggle to provide sound logico-epistemological foundations and justification for purely symbolic reasoning. Since at least 1890, that is even before

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the publication of his *Philosophy of Arithmetic* (*PA*; 1891), Husserl had been hard at work trying to solve the following problems that imaginary (i.e. negative, rational, irrational and complex) numbers create for a theory of numbers: what sort of entities are imaginary numbers, and how can they provide knowledge about numbers proper (i.e. non-negative integers)? Important as they are, these questions were nonetheless only the starting point of a much broader and philosophically important problem: how can a calculus be justified on both logical and epistemological grounds? What sort of knowledge, if any, does a purely formal theory (formal mathematics in particular) provide?

A solution to this problem, on which Husserl had worked for most of the last decade of the Nineteenth Century, was presented in the *Logical Investigations* (*LU*; 1900-01). But first Husserl had to abandon completely his earlier psychologistic approach to logic and arithmetic, reevaluate and reshape the analytic-synthetic dichotomy, distinguish between formal and material *a priori* knowledge, and totally remodel the domains of formal logic in order to accommodate symbolic reasoning within the realms of science. In *LU*, purely symbolic reasoning embodied in formal axiomatic systems is justified to the extent that it provides an articulate body of analytic truths, i.e. formal knowledge of possible (i.e. not necessarily actually existing, but logically conceivable) realms of objects.

One of my concerns here will be the insights into different notions of the completeness of axiomatic systems that Husserl acquired in providing what he considered adequate logical and epistemological foundations for symbolic reasoning in science. Husserl thought that some sort of completeness was a necessary condition for formal systems to be extendable by imaginary numbers, or to be the formal molds of possible theories proper, i.e. theories of domains of determinate objects.

Here I will be concerned principally with Husserl's treatment of the concept of completeness in *Ideen I* (1913) and the Göttingen lec-

tures of 1901. In *Ideen I* Husserl presented two different although equivalent versions of the notion of completeness for *interpreted* axiomatic systems. One of these is semantic in character, the other is syntactic, which shows that instead of confusing semantics and syntax, as some commentators believe, Husserl was in fact clearly distinguishing them. In the Göttingen talks Husserl also presented *two* notions of completeness, for purely formal systems in this case, that are the exact equivalent for formal systems of the notions of *Ideen I*. However, depending on how the concept of formal manifold is defined, the two notions presented in the Göttingen lectures may not be equivalent. Husserl used his concepts of completeness to find, in particular, a solution for the problem of imaginary numbers, and, in general, an answer to the broader question concerning the foundations of symbolic reasoning in science.

Another question that will concern me here is the relation between Husserl's notions of completeness for axiomatic systems and Hilbert's notion of completeness related to his axiom of completeness for geometry and the theory of the real numbers[1]. Is there any connection between the two? In a footnote to §72 of *Ideen I*, Husserl says that there is. However, many commentators think that Husserl is wrong in this belief, that Hilbert's axiom of completeness simply selects a *maximal model* for the (consistent) system of axioms it is added to, whereas the notion of syntactic completeness (which is the notion of completeness that Husserl found germane to Hilbert's axiom) only implies that a given syntactically complete theory *itself* cannot be consistently extended by the adjunction of new axioms. Nothing is stated with respect to its *models*.

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[1] This question is, of course, related to the still open problem of the mutual influences between Husserl and Hilbert after Husserl moved to Göttingen in 1901.

Of course, these commentators are right on this point. But Husserl, I will argue, does not have in mind the models of a formal theory, as we understand this notion today, but only the *formal domain* determined by a formal theory. As I will show below, there is indeed a close connection between a formal theory and its formal domain, for example, the maximality of one, if syntactically complete ("absolutely definite", in Husserl's terminology), implies the maximality of the other. And it is precisely an idea of maximality (of a domain described by a theory) that is behind Hilbert's axiom of completeness. Therefore, Husserl is quite right when he sees a "close relation" between his notion of definiteness and the notion of completeness related to Hilbert's axiom. One of the goals of this paper is to make clear why he is right.

A fundamental notion we will have to deal with here is that of a formal domain of (formal) objects determined by a formal theory. Husserl does not seem willing to consider, as Hilbert did, theories from a purely syntactic perspective. For him, formal theories, albeit not theories *of* given determinate domains of objects, are nonetheless theories that determine a form of domain. It is as if formal theories preserved the "intentional drive" (so to speak) of the theories proper from which they are obtained by formal abstraction.

For Husserl the concept of completeness (or definiteness-*Definitheit* in the original German) for either purely formal or interpreted systems depends on the notion of the objective domains these systems describe. Obviously, an interpreted theory has a domain of objects the theory is *about*. So, in order to extend the notion of completeness for non-interpreted systems, Husserl had to answer the question: what is the domain of a purely formal theory? However, before expounding his answer to this question, let me first introduce Husserl's notion of definiteness as presented in *Ideen I*, §72.

1. DEFINITENESS

In §72 of *Ideen I* Husserl presents the following notion of completeness for a given domain of determinate mathematical objects:

> [a definite manifold] has the following distinctive feature, that a *finite number of concepts and propositions* – to be drawn as occasion requires from the essential nature of the domain under consideration – *determines completely and unambiguously on lines of pure logical necessity the totality of all possible formations in the domain*, so that *in principle*, therefore, *nothing further* remains *open* in it.

What Husserl seems to have in mind is the following: a domain (or, as we would say today, a mathematical structure) is a *definite* or *mathematical manifold* when the set of all assertions that are true in it is finitely axiomatisable[2]. A definite manifold is, for Husserl, one that can be completely mastered by a finite theory written in a language "extracted" from the domain itself. He says:

> A manifold of this type has the distinctive property of being "*mathematically, exhaustively definable*". The "definition" lies in the system of axiomatic concepts and axioms, and the "mathematically-exhaustive" herein that the defining assertions in relation to the manifold imply the greatest conceivable prejudgment – nothing further is left undetermined (*Ideen I*, §72).

Husserl also presents what he considers to be an equivalent formulation of this definition:

> Every proposition constructed out of the designed axiomatic concepts, and in accordance with any logical form whatsoever, is either a pure formal implication of the axioms, or formally derivable from these as the opposite of what they imply, that is, formally contradicting the axi-

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[2] The requirement of finiteness was not, for Husserl, a restriction, for he could not conceive of a theory with an infinite number of axioms. For him the finiteness of theories was, I believe, a *sine qua non* epistemological condition.

oms; the contradictory opposite would then be a formal implication of
the axioms (*Ideen I*, §72).

That is, if a domain is a definite manifold, then there is a finite
theory T such that any assertion that makes sense in this domain is
either a consequence of T or is in contradiction with it. Another way of
stating this second version of the concept of definite manifold is the
following: a mathematical domain D is *definite* when there is a language
L strong enough to allow us to say whatever we want to say about D,
in which we can write a finite theory T consisting of true assertions
about D, such that any assertion A of L is either a logical consequence
of T or is in contradiction with it (i.e. the negation of A is a conse-
quence of T). In other words, D is definite when there is a finite, *syntac-
tically complete* theory T consisting of assertions true in D.

The first version of the definition clearly implies the second, for
if A is any assertion of L, A is either true or false in D. In the first case,
A is a consequence of the theory T that finitely axiomatises the set of
all assertions of L that are true in D, in the second case, the negation of
A is true in D, so it is a logical consequence of T.

Let us check the converse. Let T be syntactically complete finite
theory with respect to the set of all assertions of L (all assertions that
"make sense" in D). Now, in order to show that T is a finite axiomati-
sation of the set S of all assertions of L true in D, we must show that
any element of S is a consequence of T, and conversely. The axioms of
T are, of course, elements of S (T is chosen, after all, as an appropriate
set of true assertions in D). In order for the consequences of T to be
also in S, we must presuppose that all the logical (i.e. syntactic) conse-
quences of assertions true in D are also true in D. Husserl appears to
take this for granted. In order to show the converse, let A be true in D.
According to the second version of the notion of a definite manifold,
either A or its negation is a consequence of T. In the first case we are

done. In the second case, according to the above presupposition, the negation of *A* must be true in *D*, which is a contradiction.

Hence, the equivalence of Husserl's two formulations of the concept of definiteness for manifolds depends on the fact that "formal implications" of true assertions are themselves true assertions, i.e. the correctness of the notion of formal implication, which Husserl implicitly takes for granted.

For Husserl, the notion of a definite manifold embodies a mathematical ideal: the submission of a potentially infinite realm of knowledge by a finite theory which can, alone, master, through pure logical necessity, this entire realm. A definite manifold is one in which "true" and "formal implication of the axioms", and "false" and "formally contradicting the axioms" are equivalent, that is, it is a context in which the notions of truth and falsity have purely formal equivalents.

Having defined the concept of definiteness for mathematical manifolds, Husserl can, still in the same paragraph, extend this notion to interpreted theories. For him an (interpreted) finite theory[3] is definite just when it is the theory of a manifold that is definite in the above sense. He writes:

> I also refer to a system of axioms which on pure analytic lines "exhaustively defines" a manifold in the way described as a *definite system of axioms* (*Ideen I*, §72) .

Since Husserl had just introduced the notion of definiteness for manifolds in two different, but equivalent ways, it is clear that we can imagine two possible ways of defining definiteness for interpreted theories. The first introduces definiteness *semantically*: a finite theory *T* is definite when it is the complete theory of a certain mathematical

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[3] An interpreted theory is, for Husserl, one that describes a certain specific domain, or, as we would say today, a theory with only one model worthy of attention, its *intended* model.

domain, its intended model (i.e. if there is a language *L* in which a domain *D* can be described such that *T* is a finite axiomatisation of the set of all assertions of *L* that are true in *D*). Definiteness, in this version, means *semantic* completeness in the following sense. An interpreted theory *T* is semantically complete, with respect to a certain *particular* model *D* of *T*, when any true assertion *about D* is a theorem of *T*.

Another version is purely syntactic, an interpreted theory is definite when it is a *syntactically* complete theory, in the sense that it decides any assertion written in its language (again, any assertion that "makes sense" in the domain it describes). These two versions are equivalent, for the same reasons that the two versions of definiteness for manifolds are equivalent.

But now the problems begin. After introducing these definitions, in the same paragraph, Husserl makes two claims: (1) that the notion of definiteness for interpreted theories just given can be extended to non-interpreted, purely formal axiomatic systems:

> The definitions remain as a system even when we leave the material specification of the manifold fully undetermined, thus making a generalization of the formalizing type. The system of axioms is thereby transformed into a system of axiomatic forms, the manifold into a form of manifold, and the discipline relating to the manifold into a form of discipline (*Ideen I*, §72).

And (2) that the notion of definiteness for non-interpreted theories has a "close relation" to the axiom of completeness that Hilbert introduced in the axiomatisation of geometry and the theory of real numbers:

> The close relation of the concept of definiteness to the "Axiom of Completeness" introduced by D. Hilbert for the Foundations of Arithmetic will be apparent without further remark on my part to every mathematician (*Ideen I*, §72, n. 6).

For Husserl, a "generalization of the formalizing type", or formal abstraction, amounts to the complete elimination of any determinate objective reference of the theory under formalization. That is, the transformation of a theory proper into a mere form of a theory. What then, in this case, plays the role of the domain described by the theory? More puzzling still is the connection Husserl sees between his and Hilbert's notions of completeness.

Hilbert's axiom of completeness is only a way of selecting among all the possible realizations of a theory (its models) that which is maximal with respect to inclusion (i.e. complete in a certain sense). The fact that *T* is a definite system of axioms does not imply that it is complete in Hilbert's sense (it is enough to take any finite axiomatisation of an infinite domain by a complete elementary theory; this theory is definite but does not satisfy Hilbert's axiom of completeness – just apply the Löwenheim-Skolem theorem). Also, a theory may be Hilbert complete but not definite (take Hilbert's own axiomatisation of the theory of real numbers; this axiomatisation includes Hilbert's axiom of completeness, but it is not definite). So, it might seem quite clear that Husserl is mistaken on this point, as has indeed been claimed by many commentators (for instance, Bachelard (1968), pp. 59-63; Cavaillès (1947), pp.70-72; Sebestik (1997), p.135).

But was he really? Firstly let us notice that Husserl makes the connection between his notion of completeness and Hilbert's only *after* he indicates (admittedly in a extremely vague way) that the notion of definiteness can be extended to non-interpreted, purely formal axiomatic systems. So, it seems that the connection he has in mind is between the definiteness of non-interpreted theories and the inextensibility of their formal domains, *not* their material instantiations (their models).

So, in order to accomplish the extension of the notion of completeness to non-interpreted theories and see the connection he alludes

to in this famous footnote, we must give some attention to the notion of a formal domain, to which we turn now.

2. FORMAL DOMAINS

In order to accomplish the extension of the concept of definiteness to non-interpreted theories, we must turn to a couple of talks which Husserl gave in Göttingen in 1901[4] (*Doppelvorträge*, *DV*; 1901) at Hilbert's invitation. Husserl probably had these talks in mind when he indicated in *Ideen I* the possibility of extending the notion of definiteness to purely formal theories.

The problem is that the notion of definiteness for interpreted theories explicitly mentions the manifolds these theories describe. But, we may think, non-interpreted theories do not have intended interpretations. Husserl, however, would not concur. For him, purely formal theories also have intended domains, i.e. domains they describe. But these are not domains of given determinate objects. Rather, they are domains of indeterminate objects only formally determined by their theories. Husserl called these domains *formal* manifolds, and the forms of objects they contain, *formal* objects. So, a non-interpreted theory describes a formal manifold, or equivalently, a formal manifold is the objective correlate (the denotation, we may say) of a purely formal theory.

If we transfer the notions of definiteness for interpreted theories to non-interpreted theories by simply substituting formal manifolds for intended interpretations, we get the following. A finite non-interpreted theory is definite when: (a) it is the complete theory of its formal manifold or, equivalently, it decides any assertion referring to its formal manifold; (b) it is syntactically complete. Depending on how formal manifolds are characterized, these two versions may not be equivalent.

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[4] Whose drafts are published in *Husserliana*, XII, pp. 430-51.

To complicate matters, Husserl has two non-equivalent characterizations of the notion of formal manifold: one is presented in *LU*, and is the one he uses, to the best of my knowledge, everywhere else except in the Göttingen talks (*DV*), the other is precisely the notion presented in *DV*.

Let us take a look at the *DV* version first. As Husserl told his audience in Göttingen, a formal manifold is the collection of all formal objects that a formal theory somehow identifies. If a formal theory proves that a unique object exists satisfying a certain property, then a formal object exists in the formal manifold determined by this theory satisfying this property, and conversely. One fundamental question that can be asked with respect to a formal domain concerns the conditions for a formal object (the elements of formal domains) to belong to it. In *DV* Husserl has a clear-cut answer to this question: a formal object, that is, an indeterminate "something", characterized by a certain property $F(x)$, belongs to the formal domain determined by a formal theory T provided that the formula $F(x)$ belongs to the language of T and, moreover, T proves "there is a unique x such that $F(x)$", and conversely. In other words, a formal object belongs to the formal domain of T if, and only if, it can be formally defined in T.

A formal object is simply a uniquely determined "something" that, according to a theory, exists satisfying a certain property that can be expressed in the language of the theory in question. Or, in other words, a "something" that, although completely indeterminate with respect to its nature, is determined by the theory with respect to its *form*. "It" is the only "something" that has the formal structure expressed by a certain formula, its defining formula. The formal domain of a formal theory is simply the totality of such formal objects.

Unfortunately Husserl did not tell his audience in Göttingen what it would be for assertions of a formal language to refer to a formal manifold. It would seem that any assertion of the language of a formal theory should refer to the formal manifold determined by this

theory. But Husserl made it clear that he thought that the assertions referring *exclusively* to a formal manifold constituted a subset of the set of all assertions that are expressible in the language of the theory in question[5].

So, and this is the only thing that interest us here, for a formal theory to be definite, according to the first sense above (sense (a)), given the notion of formal manifold of the Göttingen talks, it suffices that it be syntactically complete with respect to a subset of the assertions expressible in the language of the theory, precisely those assertions that refer to its formal manifold. Hence, a formal theory can be definite with respect to its formal domain, but not be syntactically complete, although any syntactically complete theory will be, *a fortiori*, definite with respect to its formal domain.

This means that the two versions of definiteness for non-interpreted systems mentioned earlier are non-equivalent if we take the notion of formal manifold given in the Göttingen talks. Indeed, the text of these talks make it clear that Husserl recognizes that there are two non-equivalent notions of definiteness for formal theories. One he calls *relative definiteness*, for theories that are definite only with respect to their formal domains. The other he calls *absolute definiteness*, for theories that are simply syntactically complete. So, the two equivalent versions of definiteness for interpreted theories of *Ideen I* give us, with the concept of formal manifold of the Göttingen talks, the non-equivalent notions of relative and absolute definiteness Husserl presented to the Göttingen audiences.

As we have seen, in *DV* Husserl clearly states a necessary and sufficient condition for a formal object to belong to a formal domain, but he is not as clear with respect to the conditions for an assertion of the language of a formal theory to refer to its formal domain. In *LU*,

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however, the situation seems to be reversed. Husserl seems to accept that *any* assertion of the language of a theory refers to its formal domain, but he is silent with respect to the conditions for a formal object to belong to a formal domain. In *LU* a formal manifold is simply defined as the objective correlate of a formal theory. Its objects remain completely indeterminate with respect to matter and are determined exclusively with respect to form by the relations and operations to which they are submitted. These operations and relations themselves are also indeterminate with respect to their content, only their formal properties are determined by the basic formal laws which are taken as valid for them (the formal axioms of the theory in question).

Given a formal theory *T*, written in a certain formal language *L*, considering now the presentation of the notion of a formal domain of *LU*, we still have the following two questions to answer: (1) when does a sentence of *L* refer to the formal domain of *T*? (2) when does a formal object belong to the formal domain of *T*? Since in *LU* Husserl does not make any distinction between a relative and an absolute notion of completeness, I believe we are safe in supposing that in it he does not distinguish between assertions referring *exclusively* to the formal domain of *T* and assertions of *L* in general. So, I claim, in *LU* Husserl implicitly accepts that *any* sentence of *L* refers to the formal domain of *T*.

Now, since the Göttingen talks were delivered *after* the ideas of *LU* were already established, I also believe that we can safely suppose that Husserl did not consider the conditions explicitly stated in *DV* for a formal object to belong to a formal domain (or something close to them) to be inconsistent with the characterization of formal domains given in *LU*. Therefore, I propose to give to the question (2) above the same answer Husserl gives to it in *DV*: a formal object belongs to the formal domain of *T* if, and only if, it can be formally defined in *T*.

Considering now that *any* assertion expressible in the language of a formal theory is supposed to refer to its formal domain, to say that a

theory is definite with respect to its formal domain is simply to say that it is syntactically complete. Therefore, taking into consideration the notion of formal manifold of *LU*, the two notions of definiteness for non-interpreted systems presented above, more than just being equivalent, actually coincide. Hence, a purely formal system is definite just when it is syntactically complete.

Let us turn now to the question of the relation of Husserl's notion of definiteness to Hilbert's axiom of completeness.

3. HUSSERL'S NOTION OF DEFINITENESS AND HILBERT'S AXIOM OF COMPLETENESS

In order to see the connection between the notion of definiteness and the idea of inextensibility that presides over Hilbert's axiom of completeness we must answer the following question: how can a formal manifold be extended? No matter which presentation of the notion of formal manifold we choose, that of *DV* or *LU*, a *new* formal object can only be adjoined to a formal manifold if the theory of this manifold is enlarged by new axioms. This is due to the fact that new formal objects can only be added to a formal manifold if new formal expressions, which require these new objects to exist, can be proved. And this requires an enlargement of the system by the addition of new axioms.

Now the connection between Husserl's notion of definiteness and Hilbert's axiom of completeness is evident, as Husserl said it was. Since a formal manifold can be extended if, and only if, its axiom system can also be extended, the formal domain of a definite formal theory is inextensible. Unless, of course, we enlarge the language of this theory in order to define and refer to new formal objects, and redefine operations in order to take care of these new entities, as we will see in what follows.

4. DEFINITENESS, IMAGINARY ELEMENTS AND THE VINDI-
CATION OF SYMBOLIC REASONING

As we have said before, it was the problem of imaginary numbers that pushed Husserl into an investigation of the theory of formal systems. So, in the end, what was his answer to this problem? In the Göttingen talks he says that an imaginary entity is, from the perspective of a formal axiomatic system, an entity that does not belong to the formal domain of this system. Moreover, imaginary entities can be adjoined to a formal domain as convenient, although dispensable auxiliary devices, provided that the theory of this manifold is definite with respect to it[6].

If we consider the notion of definiteness for theories that Husserl seems to consider definitive (i.e. syntactical completeness), imaginary objects can be adjoined to a formal domain provided the theory of this domain is syntactically complete. As mentioned earlier, an extension of a formal domain can only be accomplished by extending its theory. So, the introduction of imaginary objects into a formal domain requires the enlargement of the language used to describe it with new symbols (symbols for new operations, new variables for the imaginary entities, etc). New axioms will then be introduced in order to define the imaginary entities and extend to them the operations defined previously only for the elements of the narrower domain. The narrower axiomatic system, and *a fortiori* its enlargement, remain definite only with respect to the assertions of the narrower language, and so they are in a sense only relatively definite (as Husserl stated in DV, the solution of the problem of imaginary entities requires only a restricted notion of definiteness).

Regardless of its technical merits, Husserl seems to have attached only restricted philosophical relevance to this solution of the

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[6] See my "Husserl's Two Notions of Completeness" (2000).

problem of imaginary numbers[7], for the Göttingen talks are the only place, as far as I know, where he discusses it in any detail. The reason seems clear. Husserl had already found in *LU* a solution of the broader question concerning the logico-epistemological justification of symbolic reasoning in general: symbolic thought is scientifically justified to the extent that it provides formal *a priori* knowledge. But definiteness still has a role to play here. Since formal theories are seen as theories of possible objective domains, definiteness is required of formal theories as an ideal of completion. Non-definite theories do not give us everything that can be known *a priori* of possible domains of objects exclusively with respect to their form.

There is in Husserl a clear connection between a syntactic and a semantic notion of completeness: to a definite formal theory corresponds a formal manifold that cannot be extended by the expressive powers of the language of this theory alone. Or, in other words, formal domains of definite formal theories are maximal with respect to inclusion, provided we do not extend the language of the theory in question by adjoining to it new symbols.

Of course, it takes some good will to accept that a *formal* domain is a *semantic* counterpart of a formal theory. The fact is that, for Husserl, it is not possible to conceive a theory, even a purely formal theory, independently of a domain, even if only a formal domain, this theory refers to. Let us see why.

5. FORMAL ONTOLOGY[8]

For Husserl, the reference of a sentence is a state-of-affairs, a *fact*. Names, symbols for relations, operations and similar syntactic

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[7] Which, nonetheless, maintains its mathematical relevance, as Husserl stressed on many occasions.

[8] See my "Husserl's Conception of Logic" (1999).

components of sentences are also each supposed to refer to a component of a state-of-affairs. In the process of formalization, terms, relation and operation symbols are devoid of any pre-determinate reference whatsoever. So, we may ask, what does the formalized sentence refer to?

Any state-of-affairs can be seen as composed of a matter and a form. Its matter is constituted by the specific objects, relations and operations that are present in this state-of-affairs, its (logical) form is given by *how* these components are related in this particular state-of-affairs. For example, the form of the state-of-affairs given by $2 < 5$ is expressed by $x\,R\,y$, where R is a relation-denoting variable and x and y are object-denoting variables. This form can be described in the following way: two indeterminate objects standing in an indeterminate binary relation.

We can say, and this is the answer to the question above, that, for Husserl, formal sentences denote *forms* of states-of-affairs. Hence, a formal theory, viewed simply as a collection of sentence forms, denotes the *form* of an objective domain, or, in other words, a domain characterized exclusively with respect to its form. This form is characterized as follows: we select a language having symbols for indeterminate objects (object-variables), operations (operation-variables), relations (relation-variables), properties (property-variables), and the like, and express in this language whatever formal properties are considered valid for whatever entities interpret the symbols of this language. For instance, the commutativity of a particular operation, the reflexivity of a certain particular relation, etc. The collection of these formal expressions constitute a formal theory, whose objective correlate, its reference, is a formal domain (or, equivalently, a form of domain). Precisely the kind of formal domains we discussed above. My point here is only that, for Husserl, the introduction of formal domains as correlates of formal theories is a straightforward consequence of his ideas concerning the denotation of sentences. Sentences denote states-of-affairs, theories

proper – collections of sentences – denote determinate domains of objects, formal theories denote formal domains (or, equivalently, forms of theories denote forms of domains).

For Husserl, this perspective opens regions in the domain of logic that are not usually considered to belong to it. In general logic is viewed as concerned only with statements (or propositions, or any other truth-bearer we may prefer), how they are properly formed and can be composed in order to form complex statements (logical grammar), how some statements can be derived from others (the theory of deduction), etc. In short, logic is supposed to be concerned, in one way or another, as Frege wanted it to be, with truth. This is why it is usually thought to be focused exclusively on truth-bearers. But for Husserl logic is *also* a theory of objects, provided that these objects are considered exclusively as the matter on which logical forms are imprinted. Or better, for Husserl, logic is also a theory of logical forms themselves. This is what he called *formal ontology*.

Formal ontology must be concerned, among other things, with the study of particular formal domains, their properties, how they are related to other formal domains, and the like. Of course, this is not a new science, but just what formal mathematics has been doing at least since Grassmann and Riemann, the mathematicians who introduced the idea of the mathematical study of formal manifolds. So, for Husserl, formal ontology includes the whole of formal mathematics.

Of course, there must be a close relation between formal theories and their domains, since whatever can be said about a formal domain must be derivable in the theory to which this domain is a correlate. In general, Husserl sees a close connection between the logic of statements and formal ontology. We do not need to enter into the details of this connection here, all I want to stress is that to any metalogical assertion of the logic of statements corresponds a similar assertion of formal ontology. Logical rules, laws and principles are always twofold. For Husserl, they are supposed to refer, by a shift of perspec-

tive, to both statements and states-of-affairs, or else formal theories and formal manifolds. So, it is to be expected that a property of theories such as syntactic completeness will necessarily translate into a property of formal domains. In this case the property of being maximal with respect to the expressive powers of the language in question. And precisely *this* is the property that Husserl considered germane to the idea of completeness behind Hilbert's axiom of completeness.

Hence, from the right perspective of the necessary correlation between formal theories and their domains, we can see why Husserl claimed that there is a "close relation" between his notion of definiteness and Hilbert's notion of completeness related to the axiom of completeness.

6. CONCLUDING

I believe that by showing that Husserl's notions of completeness and the different versions in which he presents them correspond to careful distinctions he made between semantics and syntax, theories and manifolds, interpreted theories and formal theories, the mathematical problem concerning imaginary entities and the philosophical problem concerning symbolic reasoning, I have shown that Husserl's notions and his remarks concerning their relation to Hilbert's axiom of completeness are free of the sort of confusion that some commentators have attributed them.

Although the presentation of the notion of definiteness in §72 of *Ideen I* can be read as either semantic or syntactic completeness, for Husserl introduces it first for *interpreted* theories, he also considers its extension to purely *formal*, non-interpreted theories. In this case the notion of definiteness corresponds to *syntactic* completeness. Since to any formal theory there corresponds a formal domain that inherits from it certain properties, the property of being maximal with respect to the inclusion of new (formal) axioms (i.e. syntactic completeness),

for a formal theory, translates, to its formal domain, into the property of inextensibility that Husserl thought as essentially the same Hilbert tried to capture with his axiom of completeness.

I also believe my analysis helps to bring out at least some of the reasons Husserl had for abandoning his project for a second volume of *Philosophy of Arithmetic* and presenting the logico-epistemological theories he put forward in *LU*.

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