HUSSERL, FREGE AND "THE PARADOX"

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In letters that Husserl and Frege exchanged during late 1906 and early 1907, when it is thought that Frege abandoned his attempts to solve Russell's paradox, Husserl expressed his views about the "paradox". Studied here are three deep-rooted differences between their approaches to pure logic present beneath the surface in these letters. These differences concern Husserl's ideas about avoiding paradoxical consequences by shunning three potentially paradox producing practices. Specifically, he saw the need for: 1) correctly drawing the line between meaning analyses and logical analyses; 2) an epistemology of pure logic; 3) a subtler understanding of the semantics of statements than Frege proposed. This study is part of a project to lend insight into the questions that Russell's paradox raises for logic and epistemology once signaled by Gödel, an admirer of Husserl's work.

Careful readers of Gottlob Frege's philosophical and mathematical correspondence are bound to notice the intriguing fact that in letters that he and Edmund Husserl exchanged during late 1906 and early 1907 Husserl "expressed his views about the 'paradox'" and that by this he "might have meant Russell's paradox".[1] Conscientious schol-

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[1] It is important to note here that according to Scholz Husserl wrote about 'the paradox' and not 'the paradoxes'. The note to the English edition informs readers that one of Husserl's letters dealt with 'the paradoxes', possibly Russell's paradox and that another contained remarks on the 'the paradoxes' (Frege (1980b), p. 70 n.). However, in the German editions the word "paradox" is in the singular with the definite article (Frege (1976), pp. 101-07; Frege (1980a), pp. 44-46). The English edition of Frege's correspondence was

ars will also further note that those letters were exchanged during the very year in which it is now thought that Frege definitively abandoned his attempts to solve what, because of the preeminent role that Bertrand Russell played in publicizing Frege's errors and in bringing the point home to him, is known as Russell's paradox (Dummett (1981), pp. 21-22; Dummett (1991), pp. 5-6; Hill (1995), p. 110).

This merits elucidation. However, even the most diligent scholar might be forgiven for believing that there could be little to say on the matter. It is only through some suggestive notes once penned by the German logician Heinrich Scholz about the contents of the letters that we even know that Husserl ever wrote to Frege about a paradox that might have been Russell's paradox. Although copies of two letters that Frege wrote to Husserl between October 30 and December 9, 1906 have been retrieved and published, the three letters that Husserl is known to have written to Frege between November 10, 1906 and January 13, 1907 seem irretrievably lost.[2]

Nevertheless, philosophers need not resign themselves to the idea that Scholz's intriguing remarks are all that can possibly be known about the matter. Even though we may never know exactly what Husserl wrote to Frege at that juncture, that does not mean that nothing significant can be ascertained about the content of that obscure interchange at that important time between those two giants of twentieth century philosophy. Facts can be gathered that allow us to combine what Frege wrote in those letters with his cogitations on Russell's para-

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abridged by Brian McGuinness and contains a number of really unpardonable errors (Hill (1995)). One wonders why readers of English were only offered an abridged edition in the first place.

[2] Frege (1980b), pp. 66-71; Scholz (1936a), pp. 379-80; Veraart (1976), p. 104). In addition, there seems to be no sign of a copy that Husserl asked Scholz to send him in 1936, or of Scholz's letters to Husserl at that time. This appears to have gone unnoticed. I wish to thank Dr. Sebastian Luft of the Husserl Archives in Leuven, Belgium for inquiring into this for me.

dox and to connect the result with the ideas of Husserl, whose struggle to come to terms with paradoxical consequences courted when reasoning with symbols played a significant role in the development of the ideas that went into the making of the *Logical Investigations*.

Here I study three paradox courting practices that Frege adopted, but Husserl made a point of eschewing. I do so in support of the presupposition that Husserl makes in the *Logical Investigations* that "one does not wish to be satisfied with developing pure logic merely after the fashion of our mathematical disciplines as an expanding system of propositions with naive objective validity, but that one strives for philosophical clarity with respect to these propositions, i.e., for insight into the essence of the ways of knowing involved in the implementation and ideally possible applications of such propositions and into the accompanying essentially constituting conferrals of sense and objective forms of validity"[3].

1. THE LETTERS AND FREGE'S *NACHLASS*

As a first step in our study of Husserl, Frege and 'the paradox', we need to look at the exchange that took place between him and Frege within the broader context of Frege's ill-starred *Nachlass*. A close study of Frege's epistolary exchanges and other writings of his *Nachlass* that have survived and of what we can know about what did not survive actually proves more revealing than one might expect (Hill (1995)).

Of particular interest here is the unfortunate fact that once one begins looking for information on Frege's ideas about Russell's paradox, the causes of it and potential remedies for it, one finds that practically all the lost letters and many of the other lost writings were written after Russell's discovery of the paradox and broadly concerned Frege's views on what the problem was and why he found the solutions that his contemporaries were proposing unacceptable. These subjects were:

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[3] Introduction to Volume II, §1, my translation.

the paradoxes of set theory and possible solutions to them; extensionality and classes; the Basic Law V that he blamed for the downfall of his project; the differences between concepts and objects; identity; and Frege's opinion of the work of his contemporaries. Moreover, since Frege published so little after Russell's discovery, that never published material was one of the only sources of evidence about the evolution of Frege's ideas on these important matters.[4] However, as I have said, all is not lost and much can still to be pieced together from the available remnants.

2. THE YEAR 1906

For instance, Michael Dummett has argued that Frege's "posthumous papers allow us with high probability to date almost exactly Frege's disillusionment over the attempt, to which he had devoted his life, to derive arithmetic from logic" (Dummett (1981) p. 21). During 1906, Dummett has observed, Frege had begun to write "On Schoenflies: *Die logischen Paradoxien der Mengenlehre*" (Frege (1979), pp. 176-83), an article on the paradoxes of set theory and the inadequacy of the solutions proposed for them by Schoenflies in a January 1906 article and by Korselt in a March-April 1906 article.

"The article was never completed and never submitted", Dummett explains, "but his plan for it contains an item showing clearly that, when he drew it up, he still believed in his solution to the contradiction: 'Russell's contradiction cannot be eliminated in Schoenflies's way. Concepts which coincide in their extension although this extension falls under the one but not under the other'" (Dummett (1991), p. 5). In that surviving plan for the article Frege plainly states that set theory is "in ruins" (Frege (1979), p. 176).

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[4] I documented this thoroughly in the "Smart Bombs" section of "Frege's Letters"(Hill (1995), pp. 106-15).

An incomplete draft of the article has also survived. In it Frege discusses Russell's paradox and problems with extensions and Basic Law V. At one point, Frege alludes to the shock that the law had sustained from Russell's paradox, but suggests that readers put their doubts temporarily aside and carry out the operation that the problematic law would prescribe (Frege (1979), pp. 181-82). "Tantalizingly little of the article survives", Dummett writes, concluding that it "very probably represents the very moment at which Frege came to realize that the attempt was hopeless" (Dummett (1981), p. 22). Dummett next draws attention to "a tiny fragment, dated 5 August 1906" called "What may I regard as the outcome of my work?" that has survived too. Frege's answer to his own question begins with the affirmation that it is almost entirely tied up with the *Begriffsschrift* (Frege (1979), p. 184) and includes what Dummett has called "the apologetic remark, 'the extension of a concept or the class is not for me the first thing' " (Dummett (1981), p. 22).

Dummett concludes that Frege must have been moved to ask himself what his work had achieved at that particular time "because he had finally come to recognize that his ambition to set beyond doubt the derivation of arithmetic from logic had irrevocably failed. Instead of completing the project, he had to acknowledge that it could not be accomplished; and he wanted to take stock of what survived from the disaster, what truths he had nevertheless established" (Dummett (1981), p. 22). "His task now", Dummett writes, "was to salvage from the wreck whichever of his ideas remained undamaged, those, namely, not dependent on the notion of a class or extension of a concept…. We may thus set the date of his discovery that his solution of Russell's contradiction would not work…."[5] (Dummett (1991), p. 6).

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[5] Hans Sluga also has concluded that by 1906 Frege "was beginning to think that the theory of sets was undermined by the contradiction. He concluded that there was no use for sets or classes anymore" (Sluga (1980), 169-70).

In addition to the above mentioned writings dated 1906, an "Introduction to Logic" (Frege (1979), pp. 185-96) of August 1906 and the kindred "A Brief Survey of my Logical Doctrines" (Frege (1979), pp. 197-202) have survived.[6] However, with the exception of the incomplete draft of the article about Schoenflies and another incomplete draft of a 1924/5 article (Frege (1979), pp. 267-74), all Frege's unpublished writings after 1906 in which he wrote of Schoenflies, Korselt, extensions, identity and Russell's paradox are lost (Veraart (1976), pp. 98-101). Missing are Frege's offprint of Schoenflies' 1906 article with critical remarks that Frege made, notes on Russell's paradox, set theory, identity, extensions (Veraart (1976), p. 98); a 35 column supplement to *Basic Laws* called "Basic Law V replaced by (Basic Law) V'" (p. 97); two versions ("58 columns, 18 columns and 70 pages") of a piece from about 1906 about constructing concepts and extensions of concepts called "On Two Odd Concepts" (p. 98); and directly related to that solution to Russell's paradox that Frege proposed in the appendix to *Basic Laws II*; and an undated piece about the impossibility of predicating a predicate of itself; (Veraart (1976), p. 98; Scholz and Bachmann (1936b), p. 28).

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[6] There is confusion about the date of "17 Key Sentences on Logic" (Frege (1979), 174-75). The editor of Frege's *Schriften zur Logik und Sprachphilosophie aus dem Nachlass* dates it before 1892 (Frege (1978), p. 174 n. 1). His *Nachgelassene Schriften* (Frege (1969)) and English edition give "1906 or earlier" because according "to a note of Heinrich Scholz's the manuscript should be dated around 1906" (Frege (1979), p. 174 and note), but remarks that it could have been written earlier. However, according Scholz's note as reproduced by Veraart (Veraart (1976), p. 89), the piece dates from about the same time as "My Basic Logical Insights", which, in accordance with a note by Scholz, the English edition dates 1915 (Frege (1979), p. 251 and note). Veraart reproduces Scholz's note for that piece as reading "from the war years" (Veraart (1976), p. 89). Be that as it may, the piece shows no kinship with Frege's other writings of 1906, which cling to certain specific themes. 1906 might have been a typographical error for 1916, a war year.

Of particular interest to the present investigation is the loss of materials dealing with the subjects discussed in Frege's 1906 letters to Husserl. Missing are remarks by Frege on a lecture given by Hugh McColl on "The Calculus of Equivalent Statements and Integration Limits" published in 1901 (p. 94); a lecture on the *Begriffsschrift*; a draft for a brief overview of his logical teachings on thoughts, equipollent propositions, coloring or illumination, disassociating assertoric force from the predicate, negation and the hypothetical mode of sentence composition (p. 89), apparently covering much the same material as that dealt with in the surviving, posthumously published texts from 1906 on the subject (Frege (1979), pp. 185-96, 197-202); and, of course, Husserl's letters themselves.

3. HUSSERL'S 1906-1907 CORRESPONDENCE WITH FREGE

Husserl initiated this exchange of letters by sending Frege a copy of "A Report on German Writings in Logic from the Years 1895-1899, Fifth Article" (Husserl (1904)), a review by Husserl of a long 1895 article about propositions without subjects and the relationship of grammar to logic and psychology by Anton Marty (Marty (1895)), a prominent member of Franz Brentano's school. There is no indication of an accompanying letter.

Frege replied in a letter dated October 30 to November 1, 1906 (Frege (1980b) pp. 66-70) that he did not have the time to go into Husserl's review thoroughly, but would make some observations that had occurred to him in reading it. Echoing remarks made in the above mentioned "Introduction to Logic" and "A Brief Survey of my Logical Doctrines", Frege begins his observations by complaining that logicians needlessly complicate matters by not heeding his writings and then he goes on to discourse about the need to divorce logic from psychology and from language and grammar, leitmotifs that recur as he writes on.

The last three quarters of his letter is devoted to a discussion of equipollent propositions. Logicians, he stresses, must engage in proper logical analyses for which equipollent propositions differ only with regard to form. Equipollent propositions, he maintains, have something in common in their content, what he calls the thought that they express. It alone is of concern to logic. The rest is "the coloring" and "the illumination" of the thought. In proper logical analysis a single standard proposition for each system of equipollent propositions is all that is needed and any thought could be communicated by such a standard proposition. Given a standard proposition everyone would then have the whole system of equipollent propositions and could make the transition to any one of them.

He then turns to the question of whether 'If A then B' is really equipollent to 'It is not the case that A without B', something that Husserl had disputed in his review of Marty. The answer, Frege claims, is to be found in his *Begriffsschrift* without further ado. It must be understood, he explains, that hypothetical constructions are generally composed of improper propositions each proposition is only an "indicative part and each indicates the other", so that neither the antecedent alone, nor the consequent alone expresses a thought, but the whole propositional complex. Taking the letters 'A' and 'B' to stand for proper propositions, we have four combinations: A is true and B is true; A is true and B is false; A is false and B is true; A is false and B is false, of which the first, third and fourth, but not the second, are compatible with the proposition 'If A then B'. By negation, Frege concludes, we obtain: A is true and B is false, or: A holds without B holding, just as on the right-hand side.

Frege then goes on to say that replacing 'A' and 'B' by the proposition '$\Phi(a)$' and '$\Psi(b)$', 'If $\Phi(a)$ then $\Psi(a)$' acquires generality of content, and its negation cancels this generality and says that there is an object (say Δ) such that and $\Psi(\Delta)$ is false. The proposition '$\Phi(a)$ does not hold without $\Psi(a)$ holding' is now understood to read: 'In general,

whatever *a* may be, '$\Phi(a)$' does not hold without '$\Psi(a)$'. By negation we obtain: 'It is not in general so that, whatever *a* may be, '$\Phi(a)$' does not hold without '$\Psi(a)$'. In other words: 'There is at least one object (say Δ) such that $\Phi(\Delta)$ is true while $\Psi(\Delta)$ is false'. We get the same as on the left-hand side. So in each case we therefore have an equipollence, Frege concludes.

Scholz's notes tell us that in the lost November 10th response to Frege's letter, Husserl referred to Bolzano and expressed his views on equipollent propositions and coloring, as well as on logic. A continuation of that letter followed on November 16th. It was in this lost letter, according to Scholz's notes, that Husserl first expressed his views on 'the paradox', by which Scholz thought that he might have meant Russell's paradox.[7]

In a letter dated December 9th (Frege (1980b), pp. 70-71), in which there is no mention of Bolzano or 'the paradox' and half of which is about equipollency, Frege writes that Husserl's second letter has prompted him to reply that it seemed to him that for logical analysis to be possible an objective criterion was necessary for recognizing a thought again as the same. Frege said that he considered the only possible means of deciding whether proposition *A* expressed the same thought as proposition *B* to be that if by using only purely logical laws it could be established, without knowing whether the content of *A* or *B* was true or false, that *both* the assumption that the content of *A* was false *and* that of *B* true and the assumption that the content of *A* was true and that of *B* false led to a logical contradiction, then nothing could belong to the content of *A* as far as it was capable of being judged true or false, which did not also belong to the content of *B*. For, there would be no reason at all for any such surplus in the content of *B* and such a surplus would not be logically self-evident either. In the

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[7] This mention of Bolzano is found in German editions of Frege's correspondence but not the abridged English edition (Frege (1980a) p. 71 note).

same way, nothing could belong to the content of *B*, as far as it was capable of being judged true or false, except what also belonged to the content of *A*. Thus, Frege concluded, what may be judged true or false in the contents of *A* and *B* was identical, and this alone, the thought expressed by both *A* and *B*, was of concern to logic. This, Frege maintained, was the only means of judging when two propositions express the same thought, or what part of the content of a proposition is subject to logic that he thought existed.[8]

In reply, Husserl wrote another two part letter, dated December 21, 1906 - January 13, 1907. According to Scholz, the first part was a continuation of his previous letter in which he expressed his views about hypothetical constructions and brought up the "paradox" once again; the second part was a reply to Frege's letter. There is no indication that Frege ever replied.

Now, it is quite telling that in these letters Frege has only referred to theories that he developed in his *Begriffsschrift* of 1879. Indeed, it is important to cogitate on this, for it lends support to Dummett's interpretation of Frege's writings of that year. Not only are neither *The Foundations of Arithmetic* nor *The Basic Laws of Arithmetic* ever mentioned in these letters, but there is no mention of such characteristically post-*Begriffsschrift* theories as the famous semantical distinction between sense and reference that had figured so prominently in his 1891 correspondence with Husserl, or Basic Law V and the extensions that in 1894 he had castigated Husserl so severely for eschewing and that

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[8] In the last long paragraph of his letter Frege comments on Husserl's statement in his letter that "the form containing 'all' is normally so understand that the existence of objects falling under the subject and predicate concepts is part of what is meant and is presupposed as having been admitted". Now the word 'all', the presupposition of existence and Russell's paradox of the set of all sets that is not a member of itself is one of the themes of Husserl's unpublished writings on sets and Russell's paradox. Unfortunately, though, entering into this here would take us much too far afield.

Frege came to blame for undermining the foundations of his logical edifice (Frege (1980c), p. 214). Frege certainly does appear to be in full retreat.

4. HUSSERL AND A PARADOX OF MATERIAL IMPLICATION

In the letter to Husserl, Frege affirmed the equivalence of 'If A then B' and material implication because of objections that Husserl had voiced regarding to Marty's and Brentano's conviction that 'If *A* then *B*' equaled 'No *A* (or *A* is not true) unless *B*'. "There is not even equivalence (or equipollence) here," Husserl had complained in his review of Marty, "and the test of negation, which surely must yield propositions that once again are of the same truth value, rejects it. Negation of the left hand yields: *A can* be true without *B* being true; and of the right: *A is* true without *B* being true". It is only a predisposition to think that belief signifies acknowledgment or denial of existence and is appropriately expressed by the existential "is" or "is not", Husserl argues, that makes it seem that every hypothetical judgment is also purely and simply an existential judgment and can therefore be expressed as an existential proposition without undergoing a change in meaning. There are only a few cases, in which the transition to equipollent existential propositions so approaches natural thinking that the temptation to confuse *logical analysis* (analysis of what is logically contained in the sense of the proposition and is to be logically deduced from it), with *genuine signification analysis* (analysis of what is actually contained in the sense), proves favorable for the existential interpretations, he maintains (Husserl (1904), pp. 299-300).

In a note, Husserl wrote of an "interesting and significant paradox" that he had chanced upon about 10 years earlier and had led him to undertake a thorough study of hypothetical judgments. He formulates this paradox in the following way: "From the proposition, If *A* then *B*, there obviously follows, No *A* (or *A* is not true) unless *B*. It is

also evident that from the second proposition the first follows. Thus it is totally certain that the propositions are equivalent. But the test of negation on both sides… proves, again with evidence, that the two propositions are *not* equivalent: a complete contradiction." He then adds: "Now *A* and *B* can signify propositions, but also concepts. Thus we perhaps more precisely read: If the proposition *A* is true, so is the proposition *B*; or, If something is *A*, then it is *B*; or If *A* exists, then *B* exists…." (Husserl (1904), p. 299 n.).[9]

In another note to the expression "*genuine signification analysis*" that he has used, Husserl informs readers that by this he means "the purely grammatical" in the sense of his concept of "pure grammar" of the Fourth Logical Investigation (Husserl (1900/01), pp. 493-529). In that investigation about the distinction between independent and non-independent meanings and the idea of pure grammar, Husserl particularly stresses the importance of drawing the line between *meaning analysis*, which distinguishes what is meaningful and intelligible from what is nonsensical, and *logical analysis*, which it is a matter of purely formal, objective compatibility. The latter distinguishes what is formally consistent from what is formally inconsistent, or logically contradictory or paradoxical. For Husserl, *logical* laws were inviolable, analytic laws that showed what held for objects in general, what could be said of the objective validity of meanings purely on the basis of their form alone.

To illustrate his point, Husserl gives examples of meaningless combination of words like: "a round or". In comparison, "round

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square" has meaning, but as a logical contradiction fails the logic test (Husserl (1900/01), p. 517). His main point concerning Marty and material implication would then be that although a hypothetical construction could be *meaningfully* translated into a material implication that that would still run counter to the laws of *logic*, and so court paradox.

This difference between what was meaningful and what was logically admissible was no minor issue for Husserl. The gap between the two had been impressed upon him as he tried to complete *The Philosophy of Arithmetic* at a time when he maintained close personal and professional ties with the creator of set theory, Georg Cantor, who in those days was busy exploring the paradoxical world of transfinite numbers (Hill and Rosado Haddock (2000), pp. 109-36; 137-60; 161-178). Although in the review of Marty it was a matter of meaningful combinations of symbols leading to logical paradox, during the 1890s Husserl had struggled particularly hard with the opposite question of how in mathematical contexts formalization yielded combinations of symbols, like $\sqrt{2}$ or $\sqrt{-1}$, that arithmetically speaking were nonsensical, but could nevertheless be used in calculations without engendering logical contradiction. By 1891, Husserl was already rejecting Frege's answer that such signs or combinations of signs were unfit for scientific use and should be rejected outright (Hill and Rosado Haddock (2000), pp. 182-85).

5. FREGE ON GRASPING LOGICAL OBJECTS AND RECOGNIZING THEM AS THE SAME AGAIN IN A PURELY LOGICAL MANNER

Now "Husserl's" paradox of material implication may well be 'the paradox' of the missing letters and it is perfectly possible that Husserl was only writing to Frege about it and not at all about anything as dramatic as Russell's paradox. However, for Frege, 'the' paradox was Russell's paradox and, though unmentioned in these letters, it is present just below the surface for it and equipollency were very much on Frege's mind that year. So it is

worthwhile to inquire further into links between Russell's paradox and what Frege and Husserl had to say to one another about what is ostensibly the main topic of the letters, equipollency.

The bridge between Frege's 1906 ideas about equipollency and Russell's paradox is to be found by examining the evolution of Frege's ideas about Basic Law V. And study of Frege's December 9th letter actually turns up an important clue as to how to his ideas about equipollency and Basic Law V are connected. There Frege wrote: "an objective criterion is necessary for recognizing a thought as the same, for without it logical analysis is impossible" (p. 70). And he concluded that there was not "any other means of judging what part of the content of a proposition is subject to logic, or when two propositions express the same thought" than through the application of his equipollency test (p. 71).

Frege had placed the need to apprehend logical objects and to recognize them as the same again at the very heart of his plan to derive arithmetic from logic in *Foundations*. And it was this need that had inspired the leap of faith in extensions that had led him to mandate the paradox producing Basic Law V. Let us take a closer look at his reasoning. In *Foundations*, he defined the problem as follows:

> Since it is only in the context of a proposition that words have any meaning, our problem becomes this: To define the sense of a proposition in which a number word occurs.... we have already settled that number words are to be understood as standing for self-subsistent objects. And that is enough to give us a class of propositions which must have a sense, namely those which express our recognition of a number as the same again. If we are to use the symbol *a* to signify an object, we must have a criterion for deciding in all cases whether *b* is the same as *a*.... In our present case, we have to define the sense of the proposition "the number which belongs to the concept *F* is the same as that which belongs to the concept *G*" (Frege (1884), §62).

Reasoning on, Frege adopted a modified version of Leibniz's principle of substitutivity of identicals as his definition of identity (§65).

He recognized, however, that this definition would only afford a means of recognizing an object again in case it should crop up in some other guise, but did not provide for all cases (§66). Adopting this way, he acknowledged, required presupposing that an object can only be given in one single way. But were that so, he realized, all identities would merely amount to a statement that whatever is given to us in the same way is to be reckoned as the same, which is a principle so obvious and so sterile as not to be worth stating. No conclusion could in fact be drawn from it which was not the same as one of the premises. However, if we are able to make use of identities in such diverse fields, he realized, it is surely because we can recognize something as the same again even though it is given in a different way (§67).

So, lucidly recognizing the inadequacy of theories that he had tested up to that point, Frege felt pushed to introduce the extensions[10] that became an indispensable part of the logic of *Basic Laws* (Frege (1893), pp. ix-x). It was Basic Law V that was to guarantee the needed passage from concepts to their extensions. Part Va of that law asserts that if two functions always have the same value for the same argument, then they have the same graph; whatever falls under either one of the two concepts will fall under both. So they are equal in extension. According to Vb, two functions having the same graph, always have the same value for the same arguments; if concepts are equal in extension then whatever falls under one falls under the other (Frege (1980c), pp. 159-60, 218 n. F; Frege (1893), §§9, 21).

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[10] It is unfortunately impossible to review here all the steps in Frege's reasoning leading up his introduction of extensions, but I have done this elsewhere, notably in *Rethinking Identity and Metaphysics, Foundations of Analytic Philosophy* (Hill (1997)) and in "Husserl and Frege on Substitutivity" and "The Varied Sorrows of Logical Abstraction", both anthologized in (Hill and Rosado Haddock (2000)).

It was this law that Frege ultimately blamed for Russell's paradox (Frege (1980b), pp. 130-31; Frege (1980c), pp. 214-24). In his 1903 appendix about it, Frege defines the question raised by Russell's finding as being: "Can we always infer from one concept's coinciding in extension with another concept that any object that falls under one falls under the other likewise?" (Frege (1980c), p. 214).[11] And he answers himself writing:

> If in general, for any first-level concept, we may speak of its extension, then the case arises of concepts having the same extension, although not all objects that fall under one fall under the other as well.

This, however, really abolishes the extension of the concept, in the sense we have given the word. We may not say that in general the expression 'the extension of one concept coincides with that of another' means the same thing as the expression 'all objects that fall under the one concept fall under the other as well, and conversely.' (Frege (1980c), p. 221). The only place where the mistake could lie, Frege had found, was in his law *Vb*, which must therefore be false. Along with *Vb*, V itself collapses, but not *Va* (Frege (1980c), p. 219).

In the 1906 article on Schoenflies and the logical paradoxes of set theory, Frege explained how in sentences of the form 'If something is a Φ, then it is a Ψ and if something is a Ψ then it is a Φ' we designate mutual subordination which has strong affinities with the first level relation of identity-equality. And this, he further contended, almost ineluctably compels us to transform a sentence in which this mu-

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[11] Regarding this in 1904 David Hilbert wrote of Frege and the paradoxes: "he accepts among other things the fundamental principle that a concept (a set) is defined and immediately usable if only it is determined for every object whether the object is subsumed under the concept or not, and here he imposes no restriction on the notion 'every'; he thus exposes himself to precisely the set-theoretic paradoxes that are contained, for example, in the notion of the set of all sets…." (Hilbert (1904), p. 130).

tual subordination is asserted of concepts into a sentence expressing an equality. However, he was aware that it would be necessary to assume an unprovable law authorizing the desired transformation. And he realized that such a law was not as self-evident as is desirable for a law of logic. "And if it was possible for there to be doubts previously," he then confessed, "these doubts have been reinforced by the shock the law has sustained from Russell's paradox" (Frege (1979), pp. 180-81).

No longer defending the transformation and his law, in notes published in a 1912 article by Phillip Jourdain, Frege acknowledged that:

> when classes are introduced, a difficulty (Russell's contradiction) arises. In my fashion of regarding concepts as functions, we can treat the principal parts of Logic without speaking of classes, as I have done in my *Begriffsschrift*…. Only with difficulty did I resolve to introduce classes (or extents of concepts) because the matter did not appear to me quite secure – and rightly so, as it turned out. The laws of numbers are to be developed in a purely logical manner. But numbers are objects, and in logic we have only two objects, in the first place: the two truth-values. Our first aim, then, was to obtain objects out of concepts, namely, extents of concepts or classes. By this I was constrained to overcome my resistance and to admit the passage from concepts to their extents. I fell into the error of letting go too easily my initial doubts…. (Jourdain (1912), p. 191 n. 29).

"The prime problem of arithmetic", Frege had concluded in his study of Russell's paradox in the 1903 appendix to *Basic Laws II* (Frege (1980c), pp. 214-24), "may be taken to be the problem: How do we apprehend logical objects, in particular numbers? What justifies us in recognizing numbers as objects?" (p. 224) "And even now", he had agonized in the beginning of the appendix, "I do not see how arithmetic can be scientifically established; how numbers can be apprehended as logical objects, and brought under review; unless we are permitted – at least conditionally – to pass from a concept to its extension" (p. 214).

In his 1906 letter to Husserl, Frege is still wrestling with the problem of how to grasp logical objects and to recognize them as the same again in a *purely logical manner*. This is evident in his theories about equipollent propositions, material implication, in the importance that is still placing on using logic alone to recognize a thought as the same again. Bitten by paradox he has abandoned the course of reasoning adopted in *Foundations* and *Basic Laws* and retreated to the seemingly safer ground of the *Begriffsschrift*.

6. HUSSERL ON PURE LOGIC AND ACTUAL CONSCIOUSNESS

Grasping logical objects and recognizing them as the same again was also one of Husserl's lifelong concerns. But his most basic philosophical convictions had never allowed him to be seduced by the paradox producing line of reasoning that led Frege to embrace extensions and to try to legitimate his involvement with them through Basic Law V. Of the direction that Frege's thought had taken in *Foundations*, Husserl bluntly stated in *The Philosophy of Arithmetic* that he could not see how it might signify an advance in logic and that it was such as to make one wonder how anyone would could hold them as true but in passing (Husserl (1891a), pp. 121-22).

Husserl's conception of pure logic was substantially different from Frege's. What Frege deemed purely logical analyses, Husserl considered a blind manipulation of symbols. He was firm in the conviction that "logic must not be a mere formal (mathematical) theory… but requires phenomenological and epistemological elucidations in virtue of which we not merely are completely certain of the validity of its concepts and theories, but also truly understand them" (Husserl (1903), p. 215). His phenomenology would "afford access to the 'sources' from which the fundamental concepts and ideal laws of pure logic proceed and back to which they must be traced in order to procure the necessary clarity and distinctness for a critical epistemological understanding

of logic"[12]. So Husserl would not have found the ideas about pure logical analysis, equipollency and recognizing when two propositions express the same thought that Frege outlined in his letter any more convincing than his ideas about material implication and equipollency.

Late September 1906 actually found Husserl calling for "a new and large work" to analyze, in connection with the phenomenological theory of judgment, "the essences of the various forms of propositions, which belong, from another viewpoint to the domain of pure grammar". Listed are investigations "concerning pure logic (and pure grammar), the logical calculus… the essence of the categorical propositions well as the existential" and a "comprehensive work on hypothetical judgments and hypothetical inferences" (Husserl (1906), p. 495), all themes of the correspondence with Frege initiated a month later.

Late September 1906 also found Husserl reminiscing about how while working on the logic of mathematical thought and mathematical calculation in the 1890s he had been "tormented by those incredibly strange realms: the world of the purely logical and the world of actual consciousness – or… the phenomenological and also the psychological". He had had no idea of how to unite them, he recalled, and yet believed that they had to interrelate and form an intrinsic unity. So he had wracked his "brains concerning, on the one hand the essence of representation and judgment, the theory of relations and so on, and on the other hand, concerning the elucidation of the interrelationships between the formalism of mathematics and logic" (Husserl (1906), pp. 490-91).

The extension of his efforts to the whole domain of the purely logical had been occasioned by his work on the logical calculus during the winter of 1890 (Husserl (1906), pp. 490-91). In an article of that year, "On the Logic of Signs (Semiotic)" (Husserl (1890), pp. 20-51), we find him inquiring into the logical justification of symbolic reason-

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[12] (Husserl (1900/01) Introduction to Volume II, §1, my translation.

ing. It begins with the question: "How is it that one can speak of 'concepts' which one, nevertheless, does not authentically (*eigentlich*) possess, and how is it not absurd that the most certain of all the sciences, arithmetic, is to be based upon such concepts?" (p. 20). He wanted to know "by what *right* do we operate in our practice of judging…, using symbols instead of the true concepts?" (p. 37). He complained that one might "search logical works in vain for light on what really makes such mechanical operations, with mere written characters or word signs, capable of vastly expanding our actual knowledge concerning the number concepts…" (p. 50). His painful struggle to understand and come to terms with paradoxical or seemingly paradoxical results of blindly reasoning with symbols was really the crucible in which many of the most important ideas of the *Logical Investigations* became purified.

Hints of Husserl's answer that logic must not be a mere formal theory, but requires phenomenological and epistemological elucidations in virtue of which we truly understand its concepts and theories (Husserl (1903), p. 215) were already evident in his interesting reaction to Ernst Schröder's attempt to show in *Vorlesungen über die Algebra der Logik* that bringing all possible objects of thought into a class gives rise to contradictions. In his 1891 review of that work, argued that it was the *blind* reasoning with the symbols that left one vulnerable to contradiction. He lucidly stressed that:

> in the cases where we simultaneously have, besides certain classes, also classes *of* those classes, the calculus may not be blindly applied. In the sense of the calculus of sets as such, any set ceases to have the status of a set as soon as it is considered as an element of another set; and this latter in turn has the status of a set only in relation to its primary and authentic elements, but not in relation to whatever elements *of* those elements there may be. If one does not keep this in mind, then actual errors in inference can arise. (Husserl (1891b), pp. 84-85).

For Husserl, logicians could only submit to a logic which they had thought through and thought through with insight. An epistemology of

pure logic had to be developed. On this note, let us turn to our last area of divergence between Frege and Husserl on the matter of equipollency.

7. FREGE ON EQUIPOLLENCY AND BASIC LAW V

Guillermo E. Rosado Haddock has made some thought provoking observations about Husserl's and Frege's semantics, equipollency, and Basic Law V that illustrate some of the main theses of this paper. Remember that in his letters to Husserl, Frege maintained that if it could be established, without knowing whether the content of *A* or *B* was true or false, that *both* the assumption that the content of *A* was false *and* that of *B* true, and the assumption that the content of *A* was true and that of *B* false, led to a logical contradiction, then nothing could belong to the content of *A* as far as it could be judged true or false, which did not also belong to the content of *B*, and nothing could belong to the content of *B*, as far as it was could be judged true or false, except what also belonged to the content of *A*. That meant, Frege believed, that what could be judged true or false in the contents of *A* and *B* was identical.

Now Rosado Haddock has used the fact that Frege's distressing Basic Law V, or Principle V, is an identity statement between statements, and thus a dramatic illustration of Frege's 1906 ideas about equipollency, to show how Frege's semantics of sense and reference proves inadequate with respect to that law. Husserl's semantics of sense and reference, Rosado Haddock maintains, is more fruitful, more detailed than Frege's and can serve to explain some confusions incurred by him (Hill and Rosado Haddock (2000), pp. 58, 202, 213-15). Let us first apply Frege's ideas on equipollency to Basic Law V before turning to Rosado Haddock's ideas about Husserlian and Fregean semantics.

Since readers are familiar with the content of Basic Law V as discussed above, and since for Frege equipollency should be decided independently of the content of the statements anyway, for the purposes of this discussion we can abbreviate the law to read $Va = Vb$. Now for Frege the reference of a statement was a truth value, and according to the equipollency test in his letter to Husserl, Va and Vb would only prove equipollent were both true or both false. For Basic Law V to be the true law that it would have had to have been for Frege's system to work, both the reference of Va and the reference of Vb would obviously had to be T. If that could have been so, then Basic Law V would also pass Frege's equipollency test.

However, Frege ultimately concluded that although the truth value of Va was T, the truth value of Vb was F. This being so, then the assumption that the content of Va is false and that of Vb true leads to a logical contradiction, but the assumption that the content of Va is true and that of Vb false does not. So according to Frege's theory something belongs to the content of Va as far as it is capable of being judged true or false that does not also belong to the content of Vb, and something may belong to the content of Vb, as far as it is capable of being judged true or false that does not belong to the content of Va.

Frege would thus be obliged to conclude that what may be judged true or false in the contents of Va and Vb was not identical and his law failed his equipollency test. Since in his letter to Husserl, Frege says that he thinks that his equipollency test is the only possible means of judging what part of the content of a proposition is subject to logic, or when two propositions express the same thought, and that without such an objective criterion logical analysis is impossible, he is once more tacitly confessing that Basic Law V could not fulfil the role that it was designed to fulfil.

8. HUSSERL ON STATES OF AFFAIRS AND SITUATIONS OF AFFAIRS

Now, in a number of articles,[13] Rosado Haddock has argued that the official Fregean semantics of sense and reference according to which the sense of a statement is the thought that it expresses and its referent is a truth value cannot adequately deal with certain important semantical issues. To make his point, he has drawn attention to Husserl's semantical distinction between state of affairs (*Sachverhalt*) and situation of affairs (*Sachlage*).[14] He attributes certain ambiguities present in Frege's theory of sense to "the fact that – contrary to Husserl – he lacked the notion of a state of affairs, which… lies between the thought and the situation of affairs and prevents them from collapsing into each other" (p. 215).

In the *Logical Investigations*, Rosado Haddock explains, Husserl, like Frege, distinguished between the sense and the reference of statements. But for Husserl the referent of a statement was "not a truth value, but a state of affairs (*Sachverhalt*). Although it is true that the predicates 'is true' and 'is false' apply to thoughts, this does not make them, according to Husserl, the reference of assertive sentences. Thus, the sentence 'The earth is round' refers to the state of affairs that the earth is round. Different sentences can refer to the same state of affairs by means of different propositions (thoughts)" (p. 34).

Rosado Haddock takes Frege's well known example of 'The morning star is a planet' and 'The evening star is a planet' to illustrate Husserl's point. For Frege these statements express different thoughts and when a proper name is substituted for another in a statement, only the truth value remains invariant. For Husserl, however, the referent of both statements is the state of affairs that Venus is a planet, which also

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[13] Rosado Haddock, 1986, 1991, 1992, 1996a, 1996b, 1999. All but 1996a are anthologized in Hill and Rosado Haddock 2000, whose pages I cite in this section.

[14] Husserl began alluding to this distinction in the Fourth Logical Investigation § 11. Other references are: (Husserl 1908), §§ 7, 30b); (Husserl 1939), §59).

remains invariant when going from one of the statements to the other (pp. 34, 209).

In addition, though, Rosado Haddock explains, Husserl came to teach "that there are important invariance relations between sentences that are not adequately described either as relations of invariance of states of affairs, or as relations of invariance of truth value, nor as relations of invariance of thought" (p. 61). Using an example from arithmetic, $5 + 3 > 6 + 1$, Rosado Haddock points out that the state of affairs referred to in this case is that number 8 is greater than the number 7 and that it remains invariant if we substitute the expression '9 - 1' for the expression '5 + 3' or the expression '3 + 4' for the expression '6 + 1'. He then looks at $6 + 1 < 5 + 3$, which is not obtained from $5 + 3 > 6 + 1$ by mere substitution of an expression for another expression differing in sense but having the same referent. Here '< ' and '>' do not refer to the same state of affairs. But apart from the truth value, which they share with any other true statement whatsoever, Husserl has pointed out, these statements have in common the *abstract* situation of affairs, a certain proto-relation of which they are categorizations that also remains invariant under transformations of statements in which expressions are substituted for expressions with different sense but the same reference (p. 209).

Thus Husserl, Rosado Haddock emphasizes, has proposed an invariance principle stronger than Frege's (p. 37). For the state of affairs and the situation of affairs remain invariant under substitutions that affect only the senses but not the reference (p. 35). It is unprovable, Rosado Haddock points out in support of Husserl's new distinction, "that by transforming statements into statements by way of substituting expressions for expressions with different senses but the same reference only the truth value remains invariant" (p. 211). And it is easy, "to imagine sorts of transformations of sentences that affect the state of affairs and the situation of affairs without affecting the truth value of the sentence" (p. 36)

Moreover, the situation of affairs is a sort of referential basis of the state of affairs that even remains invariant under transformations of statements that could change the state of affairs referred to by the statement (p. 209). When we say that we have the same physical law expressed in two different but equivalent ways, what we mean according to Husserl (Husserl (1908), § 30b), Rosado Haddock points out, is that the situation of affairs is the same, even though the states of affairs that the two expressions refer to may vary (p. 210). It seems unavoidable, Rosado Haddock deems, "for an adequate semantic analysis of mathematics to distinguish the abstract situation of affairs of mathematical statements both from the state of affairs referred to by them and from their truth value" (p. 63).

Looking at Frege's highly problematic Basic Law V (pp. 58-60; 208-15), Rosado Haddock observes that, in spite of what some scholars have conjectured, the "statements at either side of the identity sign of Principle V have different senses. Moreover, none is obtained from the substitution of an expression for another expression having different sense but the same referent. Thus they do not even refer to the same state of affairs. But if Principle V were true, its two sides would have in common not only the truth value – which they would have in common with denumerably many statements expressible in conceptual notations – but also the situation of affairs" (p. 214).

Rosado Haddock's conclusion? Frege's lack of the notion of a state of affairs lying between the thought and the situation of affairs and preventing them from collapsing one into the other contributed to the semantical confusion that contaminated Basic Law V (p. 215). Comparing Husserl's and Frege's choices, Rosado Haddock finds that "Frege's candidate has the advantage of simplicity. Actually, it is the simplest possible choice. But it has the disadvantage of not being very informative, since it obviates important semantic relations between sentences not reducible to sameness or difference of truth value (in the actual world). Both of Husserl's candidates are informative and non-

trivial" (pp. 37-38). "Thus, after all," Rosado Haddock finds, "also as a semanticist Frege fell short of being a Husserlian" (p. 215).

9. CONCLUSION

The paradox of Husserl's lost letters and Scholz's notes is probably only the paradox of material implication that Husserl discusses in his article on Marty. Husserl's use of the definite article and the singular form of the noun (in spite of what the English translation says) indicates that he was referring to a specific paradox and that Frege knew which one this was. The fact remains, though, that for philosophers and mathematicians in 1906 (including Husserl, as his unpublished writings on Russell's paradox show) *the* paradox was Russell's paradox and the topics that Frege chose to address in the extant letters are plainly connected with his ruminations about the paradox derived within his own system.

Here I have discussed three deep-rooted and longstanding differences between Husserl's and Frege's basic approaches to pure logic that are present beneath the surface of what is said in the letters that we have. These differences concern Husserl's ideas about avoiding paradoxical consequences by shunning three potentially paradox producing practices that Frege espoused. Specifically, Husserl saw the need for: (1) correctly drawing the line between meaning analyses and logical analyses; (2) an epistemology of pure logic; (3) a subtler understanding of the semantics of statements than Frege ever proposed.

My study is part of a larger, ongoing project to lend insight into the questions that Russell's paradox raises for logic and epistemology once signaled by Kurt Gödel (Gödel (1990), p. 258), a secret admirer of Husserl's work (Wang (1986), (1987), (1996)). Like Husserl, Gödel believed that the certainty of mathematics was to be secured not by the manipulation of physical symbols, but by acquiring a deeper understanding of the abstract concepts that lead to the setting up of these

systems and by seeking insight into the solvability and actual methods of solving all meaningful mathematical problems. He saw phenomenology as "a procedure or technique that should produce in us a new state of consciousness in which we describe in detail the basic concepts we use in our thought, or grasp other basic concepts hitherto unknown to us" (Gödel (1995), p. 383). He believed that Husserl's theories could "safeguard for mathematics the certainty of its knowledge" and "uphold the belief that for clear questions posed by reason, reason can also find clear answers" (Gödel (1995), p. 381).

The judgment that David Hilbert passed on Frege's efforts can help set some of Husserl's ideas into perspective. Explicitly contrasting his own views with those of Frege, on more than one occasion, Hilbert stressed that Frege's efforts were bound to fail because:

> No more than any other science can mathematics be founded by logic alone; rather, as a condition for the use of logical inferences and the performance of logical operations, something must already be given to us in our faculty of representation [*in der Vorstellung*], certain extralogical concrete objects that are intuitively [*anschaulich*] present as immediate experience prior to all thought. If logical inference is to be reliable, it must be possible to survey these objects completely in all their parts, and the fact that they occur, that they differ from one another, and that they follow each other, or are concatenated, is immediately given intuitively, together with the objects, as something that neither can be reduced to anything else nor requires reduction.

This to be the basic philosophical conviction that Hilbert considered requisite for mathematics and for all scientific thinking, understanding, and communication in general (Hilbert (1927), pp. 464-65; Hilbert (1925), pp. 376, 392).

Frege's entire line of reasoning regarding pure logic always ran counter to Husserl's deepest convictions that philosophical logicians could not be satisfied with developing pure logic as a mere formal, mathematical theory, as an expanding system of propositions with naive objective validity, but that they had to go further and strive for

philosophical clarity with respect to their propositions and objective forms of validity. They needed insight into the essence of the ways of knowing involved in the use and application of these propositions and into the way logicians interacted with both the objective structures of formal logic and mathematics and those of extralogical reality. From a Husserlian perspective, failure to investigate these matters would and did leave Frege vulnerable to paradoxical consequences that he could not have overcome and still hold fast to his views about pure logic. For Husserl, Frege's ideas about pure logic were shortsighted, if not downright blind.

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