BOOK REVIEW

Claire Ortiz Hill & Guillermo E. Rosado Haddock, *Husserl or Frege? Meaning, Objectivity and Mathematics.* (Open Court, 2000), xiv + 315pp.
ISBN 081-269 4171

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Let us suppose that you are a member of the congregation of worshippers of Frege, which you are very likely to be if you are an analytic philosopher. Let us also suppose that you have already heard the name of Edmund Husserl, and are not offended by its very mention (a not uncommon reaction among Fregeans.) Let us suppose further that you know that both philosophers worked around the same time and on closely related issues, that they read some of each other's works, and even exchanged a few letters. Then, I conjecture, you are likely to believe the following myth, that whereas Frege was a clear, precise and original philosopher who changed forever our conception of philosophy, introducing new standards of rigor, Husserl was a thinker whose prolixity was only equalled by his obscurity, and who persisted in conceptual confusions that Frege had so painstakingly exposed, those of psychologism being only the most evident. You may also believe that whereas Frege approached philosophy scientifically, in particular by his use of precise and well defined terminology, Husserl was still the victim of an imprecise philosophical jargon unsuitable for scientific philosophy.

On a more specific issue, you probably know that in 1891 Husserl published a book on the philosophy of arithmetic (*Philosophie der Arithmetik* – *PA*) in which, you may believe, he dared to embrace psychologism many years after Frege had already utterly destroyed it. As if this were not bad enough, Husserl had the nerve to criticize Frege's *Grundlagen*. Small wonder then that (let us suppose you know this too) when Frege published a "devastating" (as Michael Dummett puts it) review of *PA*, in 1894, Husserl had no other alternative than to strike his breast in a remorseful *mea culpa* and finally abandon the errors of psychologism, which he did in his subsequent work, *Logical Investigations* (1900-01), although not in a manner suitable for a gentleman, you may believe, for he nowhere in this work acknowledges the decisive influence Frege and his review had on his finally seeing the light. Bitterness, resentment and ingratitude may be words that jump to your mind with respect to Husserl's attitude in these events.

If you have gone so far as to actually read some of Husserl's books and the secondary literature on them produced by analytic philosophers such as Dagfinn Føllesdal, R. McIntyre, D. W. Smith or Michael Dummett, you may believe that in the works published after *PA* Husserl followed in Frege's steps, trying, for instance, to generalize Frege's notion of linguistic meaning to areas other than language, as if Husserl could never overcome the (never admitted, remember) impact Frege had on him. In short, you may believe that Husserl was hardly anything more than a somewhat erratic satellite orbiting, even if unwillingly, around Frege, whose gravitational field he was unable to escape.

If these suppositions are true about you, this book, which collects a series of important papers by Claire Ortiz Hill and Guillermo E. Rosado Haddock, is just what you need to read. The authors are two outstanding Husserl scholars who have been principally responsible for showing that every single detail of this myth is incorrect. You will be surprised to find, on reading it, that most, if not all, of what you thought you knew about Husserl, the Husserl-Frege affair in particular, was misconceived. Frege's influence on Husserl was in fact much less important than the folklore in analytic circles and most of the studies on Husserl produced in these quarters would have us believe. Both authors are experts on Husserl and Frege and have much to tell us about their relationship and respective contributions to philosophical domains that were of common interest to both Husserl and Frege.

Ortiz Hill and Rosado Haddock give us more than enough detailed historical and textual analysis to support the alternative viewpoint that Husserl was, to say the least, already harboring profound misgivings about psychologism years before the supposedly "devastating" criticism Frege directed against him, and that the semi-official doctrine that Husserl's theory of meaning is based on Frege's views on sense and reference is wrong on many grounds, historical inaccuracy being only one of them.

But this book has more in store for you. Things you may not like if you are, as I am supposing you are, someone who tends automatically to sympathize with Frege and look down on Husserl whenever both their names are mentioned in the same paragraph. One of the things which will be hard to swallow for Frege's admirers is Ortiz Hill's analysis of Husserl's criticism of Frege in *Philosophie der Arithmetik*. She argues that if Frege had paid more attention to this criticism he would probably had shown more caution about taking the road to disaster that began under the apparently clean and well designed gates of extensionalism only to end in the ruins of contradiction.

But, of course, Fregeans are not alone in misinterpreting Husserl. Husserlians themselves have also been very successful in this endeavor. Husserl's philosophical work before the *Logical Investigations* and the philosophical development that led to this masterpiece and the creation of phenomenology have often been disregarded by Husserl experts as uninteresting. Few philosophers in the so-called continental tradition seem to be aware of the fact that throughout his life Husserl had a close interest in issues usually identified with analytic philosophy. Reading this book will also be enlightening for a great number of Husserlians who think that Husserl's work before the *Logical Investigations* is hardly worthy of their attention. They will learn that Husserl developed a theory of formal systems, an elaborate semantics and a surprisingly modern and appropriate philosophy of mathematics, developed with an eye on the most recent trends of the mathematics of his time, in close dialogue with mathematical creators of the rank of Cantor and Hilbert.

Most of the essays in this collection have been published previously over the course of a number of years in various journals. But here they are not merely put together, they are arranged to form an illuminating dialogue. This gives the book a satisfyingly well unified outlook, from which both Fregeans and Husserlians, as well as any philosopher of logic and mathematics can certainly learn much.

Let us give now a brief account of each essay in the collection, before concluding with some more general remarks:

1. *Husserl and Frege on Substitutivity* (C. O. H.)[1]: According to the author "the chief objective of this paper [is] to show Husserl's ability to evaluate Frege's work and pinpoint genuine problems in his reasoning" (p.17). Ortiz Hill emphasizes in particular Husserl's criticism, in *PA*, of Frege's use of Leibniz's law of identity of indiscernibles to define equality. According to Husserl, Leibniz's law defines identity, not equality and, moreover, maybe not even this, for it is conceivable, he thinks, that two different objects may have all their properties in common. Ortiz Hill concludes from Husserl's criticism, and Frege's answer to it, that Frege systematically confuses equality with identity, in this

[1] The initials refer to the paper's author.

way opening the way to a web of problems in many areas of philosophical interest.

In fact, it is not easy to see what Leibniz's principle of identity of indiscernibles purports to give us. Is it a definition or simply a criterion? Is it even true? Leibniz's principle can be stated as follows: $x = y \leftrightarrow (P)\ (P(x) \leftrightarrow P(y))$, in which '*x*' and '*y*' are object-names and '*P*' a variable over the domain of first-order properties. In other words, Leibniz's Law says that two objects are identical if and only if they have all their properties in common. But this surely sounds strange. How can *two* objects be identical? The strangeness of Leibniz's law is in fact inherited from the strangeness of the identity relation. As Rosado Haddock tells us (essay 3), the identity $x = y$ can only make sense if read as a congruence[2] relation between the *senses* of '*x*' and '*y*' determined by sameness of reference.

Now, suppose that '$x = y$' is true. Then although equivalent, the senses expressed by '*x*' and '*y*' can be *different*. Let *P* be a property of objects such that the truth-value of $P(a)$ depends on the *sense* expressed by '*a*' (i.e. *P* is an intensional property). Then $P(x)$ and $P(y)$ may have different truth-values. For instance, although the morning star = the evening star, 'the morning star announces the dawn' is true, but 'the evening star announces the dawn' is false.

This tells us that Leibniz's principle would be false if *P* were not restricted to extensional properties. So, let us suppose that '*P*' refers exclusively to extensional properties, and moreover that Leibniz's principle is a criterion of identity. But if we try to use it in order to recognize an instance of identity we will get ourselves entangled in all sorts of problems. If '*x*' and '*y*' are names with different senses and we de-

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[2] An equivalence relation is a congruence when equivalent objects, according to this relation, can be substituted for one another *salva veritate*. In other words, to say that identity is a congruence relation already implies the acceptance of the truth of Leibniz's principle.

cide to use Leibniz's law in order to verify whether they refer to the same object, we must verify both $P(x)$ and $P(y)$ for *every* extensional property P. Since the truth values of $P(x)$ and $P(y)$ cannot be decided in terms of the senses of 'x' and 'y' alone, we must have access to their referents, but then we will know whether they are the same or not independently of Leibniz's law. Also, if we count the property "is equal to y" among the properties we must test, this will generate an infinite regress, as Husserl noticed in his criticism of Leibniz's principle. If we do not take this property into consideration, why not? Not to mention the fact, also brought up by Husserl in his criticism, that we have an infinite number of properties to test. The primitiveness of the identity relation seems to render Leibniz's principle useless as a criterion of identity.

Matters get worse if Leibniz's principle is taken as a *definition* of identity. As Husserl had also noticed, the primitiveness of the identity relation originates an infinite regress whenever we try to define it. Of course, Leibniz's principle does not fare any better if used, as Frege seemed to want it, as a definition of equality instead. In this case it is just plainly false. As Ortiz Hill, and Husserl, remind us, equality means coincidence with respect to only *some* aspects, and objects that are equal with respect to certain characteristics may differ when our attention is directed to other aspects.

One particular example is illustrative of Ortiz Hill's reasoning. Considering Frege's transformation of the *equality* of two straight lines *with respect to direction* into an *identity* between their *directions*, she says: "Frege believed that by rewriting the sentences of ordinary language, these differences between equality and identity could be made to vanish" (p. 6) and "[i]n these examples he has transformed statements about objects which are equal under a certain description into statements expressing complete identity" (p. 6).

What Frege does in this example is in fact to use a standard method of introducing new mathematical entities (Hermann Weyl called it a definition by abstraction or creative definition): the transformation of a common *aspect* of otherwise different objects into a *new* ideal object, i.e. an equality between objects (the straight lines) is transformed into an identity between *other* objects (their directions.) So, apparently there is no confusion between equality and identity between the *same* objects. All is fine in mathematical domains, Ortiz Hill tells us, but problems may appear in philosophical or practical contexts in which our interest is focused on a particular group of properties and we tend to see them as all the properties that matter. To jump from equality with respect to these properties to full identity will be then always a very present danger.

Frege's notorious Basic Law V, which promotes the ill-fated identification between the mutual subordination of predicates and the identity of their corresponding courses of values, although not an instance of Leibniz's principle, also creates an identity between objects in terms of a lesser form of equivalence between properties. Ortiz Hill believes that this should be enough to put Frege's axiom under suspicion. Although I believe that it is too strong to say that Husserl "pinpointed" *this* particular problem in Frege's system, we can at least, I believe, say that Husserl was sensitive to certain potential weak spots in it. With respect to the inconsistency in Frege's logic, Husserl saw smoke, but did not cry fire. But, as the saying goes, there is no smoke without fire.

2. *Remarks on Sense and Reference in Frege and Husserl* (G. R. H.): the bulk of this paper is devoted to a comparative study of Frege's and Husserl's theories of meaning, which present, as Rosado Haddock notices, marked similarities, but also important differences. Two of the author's conclusions stand out: Husserl's distinction between sense and reference definitively does not come from Frege, and Husserl's turning away from psychologism is definitely not due to Frege's review of *PA*. Rosado Haddock says: "Thus, we may conclude about Føllesdal and

other's statement that it was Frege's influence on Husserl that both turned Husserl away from psychologism and taught him to distinguish between sense and reference, that it is completely unfounded." (p. 33).

In this paper Rosado Haddock introduces Husserl's distinction between states-of-affairs and situation-of-affairs, and shows, by means of this distinction, that Frege's argument that only truth-values could be the reference of sentences (for they are the only things that remain invariant under substitutions in a sentence of expressions with the same reference, but with different senses) is flawed.

A conclusion we can derive from this paper is that Husserl's semantics is not only different from and independent of Frege's, but also, in some aspects, is better.

3. *Identity Statements in the Semantics of Sense and Reference* (G. R. H.): In this paper Rosado Haddock intends to show that "in a semantic theory of sense and reference there is only one sound interpretation of identity statements" (p. 42), namely, identity statements "express the congruence relation, determined by sameness of reference, between the senses of the expressions at each side of the identity sign." (p.43). Moreover, he claims, this "is the only interpretation that does justice to [...] Frege's discussion of identity in *Über Sinn und Bedeutung*" (p.47). The paper closes with a brief discussion of Kripke's account of identity statements.

4. *On Frege's Two Notions of Sense* (G. R. H.): As one might expect from the title, Rosado Haddock concludes "that Frege had two different notions of sense, namely: (i) the notion considered in '*Über Sinn und Bedeutung* ' [...] and (ii) a somewhat unclear notion that appears in '*Der Gedanke* ' and elsewhere, and which has its origins in Frege's old notion of conceptual content" (p.58). According to Rosado Haddock, "without Frege's second notion of sense the famous Principle V, or Basic Law V, of *Grundgesetze der Arithmetik* is completely unintelligible" (p.58).

The author shows that his rendering of Frege's somewhat unclear notion of conceptual content "comes close to Husserl's notion, not completely developed, of a situation of affairs (*Sachlage*)." (p.59).

In the rest of paper Rosado Haddock argues "on behalf of the introduction of Husserlian distinctions in the semantic analysis of mathematics" (p.60). He tries, he claims, "to make precise a semantic notion which is a sort of 'explicans' of Frege's notion of conceptual content and of Husserl's notion of situation of affairs" (p.60), which he calls 'abstract situation of affairs' or 'objective content'. This notion, Rosado Haddock claims (correctly as far as I can judge), is necessary, but maybe not sufficient, for the semantic treatment of mathematics. The notion of objective content, he says, gives us the semantic tools to understand, for instance, the common mathematical phenomenon of equivalent, but seemingly unrelated, mathematical statements, such as the Axiom of Choice and Tychonoff's Compactness Theorem.

5. *The Varied Sorrows of Logical Abstraction* (C. O. H.): In this chapter Ortiz Hill addresses some issues that had already appeared in chapter 1, logical abstraction, identity and Frege's Basic Law V in particular.

In Husserlian terminology we can say that logical abstraction amounts to the reification of non-independent moments, such as color, direction or cardinal number. This is how it works. Suppose R is an equivalence relation defined in a domain D of objects. An equivalence relation is a binary relation that is reflexive, transitive and symmetric, identity being the strongest such relation, in the sense that if $x = y$, then xRy for any equivalence relation R. An equivalence relation R partitions the domain D in mutually exclusive classes, called the equivalence classes determined by R (R-equivalence classes.) If $x \in D$, then the class determined by x, in symbols $[x]$, is defined by: $[x] = \{y \in D: xRy\}$. We can now see R-equivalence classes as new objects such that $[x] = [y]$ iff xRy.

We have that $x = y \rightarrow [x] = [y]$, but the converse is not in general true. Suppose now that we are working in a language in which relations in general cannot differentiate between elements in the interior of *R*-equivalence classes, i.e. let $P(x_1, ..., x_n)$ be an arbitrary *n*-ary relation such that $P(x_1, ..., x_n) \wedge x_1 R y_1 \wedge ... \wedge x_n R y_n \rightarrow P(y_1, ..., y_n)$ – i.e. *R* is what mathematicians call a congruence relation. In this case, as can be easily shown, elements belonging to *R*-equivalence classes can be substituted for each other *salva veritate*, i.e. the identification of *R*-equivalent elements does not conflict with the principle of substitutivity of identicals.

In other words, if an equivalence relation *R* is, with respect to a certain language, a congruence relation, then *R* can be seen, *exclusively in the context of this language*, as identity itself.

A standard application of logical abstraction is the reification of properties. Suppose that *C* is a property of objects in a domain *D* (color, for instance). Suppose that *S* is a criterion of sameness with respect to *C*, i.e. *x* and *y are the same with respect to C* $\leftrightarrow S(x,y)$. Define a binary relation *R* in *D* as follows: $x R y \leftrightarrow S(x,y)$. Obviously, *R* is an equivalence relation. The *R*-equivalence class determined by an object *x* in *D*, $[x]$, is given by $\{y \in D$: *x* and *y* are the same with respect to $C\}$, i.e. the class of all objects that are the same with respect to *C*. This can count as a definition of "the *C* of *x*" (for instance, the color of *x*.) A property of objects is now an object itself, an *abstract object* (for instance, *redness* as the collection of all red objects.)

Let us give an example. Suppose that cardinal numbers are (formal) moments (or aspects) of determinate collections (arguably this is how Husserl sees cardinal numbers in *PA*). We need a criterion in order to determine when two collections have the same number. Equinumerosity is, of course, this criterion. So, the number of a collection is nothing but the collection of all collections equinumerous with it (this is, leaving aside details, Frege's strategy for defining cardinal numbers.)

Logical abstraction is a standard method in mathematics of transforming an equivalence relation – sameness with respect to *C* – into an identity between equivalence classes determined by this relation or, in some cases, those in which we are dealing with a congruence relation, into identity itself. The problem is when we identify sameness with respect to a property *C* with identity in contexts in which this is *not* allowed, i.e. contexts in which there are properties, or relations in general, that can tell apart objects that are the same with respect to *C*. In mathematical contexts, logical abstraction is a way of obtaining new domains from old ones, and is in general unproblematic. Troubles lurk, however, in natural language contexts. Ortiz Hill gives us some good examples of the absurdities that the transformation of properties into objects suitable for extensional treatment can produce. As she says: "No matter how convenient and attractive abstraction may seem to be as a technique for translating expressions into the popular notation of extensional logic, the properties making the difference between equality and identity do not docilely submit to logical measures designed to wipe them out." (p. 88-89). This paper is, in few words, the chronicle of Frege's and Russell's failed attempt to build the foundations of mathematics on extensional logic. Ortiz Hill seems to be asking: if they did not succeeded even in the more amenable domains of the foundations of mathematics, why should we follow their steps in the far more intractable areas of general philosophy?

6. *Frege's Attack on Husserl and Cantor* (C. O. H.): Frege's caustic and very unfair review of *PA*, published in 1894, is the object of this paper. Ortiz Hill intends to show that, in fact, Frege, in this bitter piece of criticism, was not really criticizing Husserl, or not only Husserl, but mainly Cantor. "I will endeavor to show the extent to which Frege used his review of the *Philosophy of Arithmetic* as a forum for attacking Georg Cantor's theory of numbers. By so doing I hope to help put Frege's objections "in the proper light," and so undo some of the damage done to Husserl's book." (p. 95)

7. *Abstraction and Idealization in Edmund Husserl and Georg Cantor prior to 1895* (C. O. H.): For 15 years, beginning in 1886, Husserl worked in Halle, before moving to Göttingen and entering the circle of Hilbert. At that time, in Halle, one of the greatest mathematicians of all times, Cantor, was developing what many believe to be the most original mathematical theory ever conceived, set theory. Considering that the foundation of mathematics was Husserl's concern at this time (his *Habilitationsschrift* of 1887 is entitled *Über den Begriff der Zahl*), it would be truly remarkable if Husserl had not been influenced at all by Cantor in his work, from the time Husserl arrived in Halle to 1895, when he had already developed the ideas that appeared in the *Logical Investigations*. In this paper, Ortiz Hill tries "to shed light on that dark period in Husserl's development by studying the evolution his ideas underwent as this relates to Cantor's philosophizing about abstraction, Platonic ideas, and the concept of number" and focuses "on the important changes which took place in Husserl's ideas during the first ten years in Halle." (p. 109)

With respect to Husserl's philosophical development, which took him from the psychologism of the thesis of 1887 and *PA*, to the Platonic idealism and anti-psychologism of the *Logical Investigations*, which some have attributed to Frege's criticism of *PA*, Ortiz Hill says: "Although his experience of Cantor's work may have acted to pry Husserl away from psychologism and to steer him in the direction of idealism, Husserl said it was Hermann Lotze's work which was responsible for the fully conscious and radical turn from psychologism and the Platonism that came with it." (p. 129)

8. *Did Georg Cantor Influence Edmund Husserl?* (C. O. H.): This paper, which is a natural sequel of the previous one, intends to show how this influence was felt. "[I]n the following pages I [...] show how Husserl's and Cantor's ideas overlapped and crisscrossed during those

years [i.e. Husserl's period in Halle] in the areas of philosophy and mathematics, arithmetization, abstraction, consciousness and pure logic, psychologism, metaphysical idealism, new numbers, and sets and manifolds. In so doing I hope to shed some needed light on the evolution of Husserl's thought during that crucial time in Halle." (p. 137)

Ortiz Hill argues that initially Cantor's and Husserl's ideas about mathematics and philosophy, sets, abstraction and the arithmetization of analysis fit together well. Then a period came in which Cantor's ideas "must have been instrumental in unseating Husserl from his earlier convictions by raising hard questions about imaginary numbers, sets, consciousness and pure logic, idealism, etc." (p. 157). In a third stage, in the author's view, Husserl showed a mixed reaction to Cantor's ideas. He was drawn to some of them (metaphysical idealism and the renunciation of psychologism, empiricism, and naturalism), but turned away from others (Cantor's set theory and Cantor's arithmetization of analysis). There is still, Ortiz Hill believes, a fourth stage of Cantor's influence on Husserl, which "would consist of the assimilation of certain of Cantor's ideas into Husserl's phenomenology." (p. 157) But this wider spectrum of influence is not dealt with in this essay.

9. *Husserl's "Mannigfaltigkeitslehre"* (C. O. H.): The topic of this paper is Husserl's logic, whose highest level is occupied by the *Mannigfaltigkeitslehre*, a term he borrowed from mathematicians in order to designate essentially the metatheory of formal systems and their objective correlates, the *Mannigfaltigkeiten* (which should not be confused with Cantorian sets. Husserl's metamathematics, it must be also stressed, is very different from Hilbert's.) In this paper Ortiz Hill analyzes the reasons Husserl had for constructing the edifice of logic in the stratified way he did, in layers of ever more abstract and formal disciplines, culminating with the *Mannigfaltigkeitslehre*.

Ortiz Hill shows that there were five problems that led to the development of this theory, and, more generally, to Husserl's conception of formal logic: the crucial problem concerning the gap between pure logic and consciousness (which would eventually lead to the putting forward of transcendental logic as a complement to formal logic); the foundation of mathematical knowledge; the problem concerning the psychological analysis of sets; the problem concerning analyticity; and the particularly difficult and important problem concerning imaginary elements in mathematics. Husserl's logic, his *Mannigfaltigkeitslehre* in particular, provided, according to Ortiz Hill, the background against which Husserl could provide answers to all of these problems.

10. *Husserl and Hilbert on Completeness* (C. O. H.): Both Hilbert, in his axiomatisation of geometry and the theory of real numbers, and Husserl, in his attempt to solve the problem of imaginary numbers, introduced notions of completeness into the theory of axiomatic systems. Ortiz Hill's aim here is "to inquire further into the origins of Husserl's ideas on completeness, and then look at how Husserl thought he might provide more secure logical foundation for all knowledge by generalizing insights drawn from his investigations into the foundations of mathematics." (p. 180)

One of the conclusions she reaches is that Husserl "eventually concluded that if a system was complete, then calculating with imaginary concepts could never lead to contradictions" (p. 180). Incidentally, Ortiz Hill mentions how much the idea of justifying calculating with imaginary, i.e. non-referring concepts, was at the same time absurd to Frege and essential from the perspective of the mathematics produced by Husserl's friends Cantor and Hilbert, among others.

Although the author repeats Husserl almost verbatim in the passage quoted above, this conclusion cannot be accepted as stated, for it may induce a misrepresentation of Husserl's actual solution of the problem of imaginary entities in mathematics. The fact is that it is not easy to render Husserl's ideas about completeness and imaginary entities in terms of our much more sophisticated modern treatment of these matters. One hundred years of investigations on the theory of axiomatic systems make any literal reading of Husserl with respect to these problems almost surely contestable. We must try to read the intentions behind his words. Let me suggest an alternative reading:

Since an imaginary concept is imaginary just because it has no meaning from the perspective of a certain system, and moreover an imaginary entity is provably non-existent if this system is complete, then to use it as if it were meaningful and existent is already a blatant contradiction. Therefore, the assertion that such a procedure "never lead to contradictions" can only be false. What Husserl had in mind was something like the following: a system can be safely enlarged by another system, written in an enlarged language, in which imaginary entities were definable, and operations were redefined, provided that the narrower system is complete with respect to the assertions of the narrower language exclusively. Husserl in fact presented more than one version of this solution, but they all share the same basic idea.

Nonetheless, we should not be too critical of Ortiz Hill's incomplete treatment of these difficult questions. This paper was one of the first to dare to venture into what was at the time it was written an almost utterly unexplored aspect of Husserl's thought. On a more personal note, it was this paper that led to the present author's own investigations into Husserl's ideas on completeness[3]. Moreover, Ortiz Hill considers these purely technical questions "rather academic". Her interest lies elsewhere, in the relevance of these problems to Husserl's general philosophy.

Although mentioned in the title, Hilbert's axiom of completeness is not much of an issue here. The reason is that Ortiz Hill does not understand why Husserl insisted that the connection between his

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[3] Which appear in "Husserl's Two Notions of Completeness" (forthcoming in *Synthese*), and "The Many Senses of Completeness" (this issue of *Manuscrito*)

and Hilbert's ideas on completeness was self-evident. In the papers mentioned above I investigate this problem and justify Husserl's belief. I conclude that it is precisely Husserl's notion of a *Mannigfaltigkeit*, the topic of Ortiz Hill's previous paper in this collection[4], that holds the key for the solution of the puzzle.

The author's original reasons for dealing with the questions she addresses in this paper are stated in its concluding section: "I would also like to suggest that approaching Husserl's thought in light of his views on completeness, analyticity, meaning, and identity may also help demystify his phenomenology and so shed light and order where confusion and ineffability have seemed to reign." (p. 194)

11. *To Be a Fregean or To Be a Husserlian: That is the Question for Platonists* (G. R. H.): Rosado Haddock presents here a clear, accurate, if somewhat condensed overview of Husserl's conception of formal logic, his philosophy of mathematics (the epistemology of mathematics and the notion of mathematical intuition in particular) and some aspects of Husserl's semantics. And if this were not enough, Rosado Haddock also provides two applications of Husserl's semantics: how Husserl's distinction between states of affairs and situations of affairs can be used to show the flaw in Church's argument that truth-values are necessarily the referents of statements; and how Husserl's notion of a situation of affairs can be used to adequately assess Frege's Basic Law V.

One of the points Rosado Haddock makes is that Platonists may choose to side with Frege, but if they do, they will not have the benefit of the elaborate account of mathematical knowledge with which Husserl complemented his Platonism, and which is something missing in Frege's. Although he does not say so in so many words, Rosado

[4] But written eight years after "Husserl and Hilbert on Completeness".

Haddock obviously believes that Platonists would be better off siding with Husserl (see essay 15).

There is one point however on which I cannot agree with Rosado Haddock. He says, incorrectly as I believe I have shown in my above mentioned papers, that "[a]s many of his contemporaries, before Gödel's and Tarski's revolutionary writings, Husserl did not distinguish clearly between deductive [or syntactic] completeness and semantic completeness." (p. 202)

12. *Husserl's Epistemology of Mathematics and the Foundations of Platonism in Mathematics* (G. R. H.): This paper complements the previous paper. In it Rosado Haddock says that he "will make a reconstruction and a systematization of Husserl's epistemology of mathematics as based on the notion of categorial intuition." (p. 222) The Sixth Logical Investigation and *Erfahrung und Urteil* (Husserl 1939) are his sources.

The result is indeed a coherent and complete presentation of Husserl's notion of categorial intuition, which lies at the foundation of Husserl's epistemology of mathematics. Rosado Haddock also argues that even after 1905, when Husserl's philosophy was reoriented towards transcendental phenomenology, his basic account of categorial intuition did not change substantially: "Husserl's assessment of mathematical (and other categorial) objectualities in *Erfahrung und Urteil* does not lead to any sort of constructivism, but at most to a refinement of his Platonistic conception." (p. 233)

In one of the appendices to this chapter Rosado Haddock shows how the genesis of well-known mathematical paradoxes (Russell's, Cantor's) can be accounted for in terms of Husserl's epistemology of mathematics. An asymmetry between operations of meaning constitution and the constitution of categorial objectualities is, Rosado Haddock claims, responsible for these paradoxes. According to him, they originate when one presupposes as given an objectuality intended by a meaningful expression, which nonetheless cannot possibly be adequately constituted.

13. *Interderivability of Seemingly Unrelated Mathematical Statements and the Philosophy of Mathematics* (G. R. H.): There are many statements in mathematics that, although equivalent, are seemingly completely unrelated, such as, for instance, the many equivalents of the Axiom of Choice. Rosado Haddock's purpose in this paper is to ask whether this phenomenon has any relevance for the philosophy of mathematics. He thinks that it has.

Rosado Haddock believes that the phenomenon of interderivable statements can only be properly explained from the perspective of Platonism, although not the Fregean variety. He believes that Frege's Platonism, besides having not developed an appropriate epistemology, does not have a good semantics of mathematical statements either. The solution, according to him, is to be found in Husserl, whose semantics provides us with the notion of a situation of affairs. Adapting this notion to a mathematical context, and giving it the name of abstract situation of affairs, Rosado Haddock presents the following solution to the problem this paper addresses: interderivable statements, although referring to different states of affairs, have the same abstract situation of affairs as their referential basis.

14. *On Husserl's Distinction Between State of Affairs (Sachverhalt) and Situation of Affairs (Sachlage)* (G. R. H.): In his well-known paper "Mathematical Truth"[5], Paul Benacerraf states two requirements for an acceptable account of mathematical truth, (i) that the semantic treatment of mathematical statements does not differ considerably from

[5] In *Philosophy of Mathematics: Selected Readings*, P. Benacerraf and H. Putnam (eds.), 2nd ed. rev. (Cambridge, Cambridge University Press, 1983), pp. 403-20, originally published in *Journal of Philosophy* 70 (1973), pp. 61-80.

that of non-mathematical statements and, (ii) that the account of mathematical truth harmonize with a "reasonable epistemology". In this paper Rosado Haddock adds a third requirement, (iii) that a semantics plus epistemology of mathematics must give a satisfactory account of the interderivability of apparently unrelated mathematical statements (like, for instance, the Axiom of Choice and Tychonoff's compactness theorem.) Rosado Haddock finds in Husserl's semantics, in particular in the notions of situation of affairs and state of affairs, a basis for a semantics of mathematical statements that, according to him, conforms to both the first and third requirements above (those that are properly semantic.)

15. *On Antiplatonism and its Dogmas* (G. R. H.): In this last paper of the collection, Rosado Haddock states clearly his commitment to Platonism in the philosophy of mathematics. His strategy to make Platonism a palatable viewpoint has two fronts. In one he criticizes the best known adversaries of Platonist; in the other he presents an alternative account of a Platonist perspective along Husserlian lines.

According to him, behind apparently different anti-Platonist arguments, such as Putnam's Skolemization argument, Benacerraf's ontological and epistemological arguments and the Quine-Putnam indispensability argument, there is in fact a common empiricist prejudice against the existence of mathematical entities and the possibility of our access to them. Rosado Haddock claims that all these purported arguments are contaminated from the very beginning by a bias against the views they intended to attack, namely the existence of mathematical entities and of a non-causal link between us and them (which we can call mathematical intuition.)

Rosado Haddock also argues against Field's philosophy of mathematics by showing that if Field is right, then there are logically equivalent assertions with different logical values. Since this cannot be, mathematics is not, as Field claimed, a mere tool constituted of either false or vacuously true propositions.

This paper is a very adequate way of closing this collection, for it highlights what seems to me a predominant theme of the whole book, the relevance of Husserl's philosophy for the problems on the agenda of Anglo-American analytic philosophy.

It has always puzzled me that Husserl's contributions to the philosophy of logic and mathematics have, in general, been overlooked by philosophers in both the continental and analytical traditions. Husserl was after all a trained mathematician who philosophized in close interaction with other mathematicians in Halle and Göttingen, and who always showed a keen interest in philosophical problems related to the formal sciences. It is my opinion, which I believe the authors of this collection share, that this unfortunate situation has deprived modern philosophy of logic and mathematics of many exciting ideas as well as preventing a correct assessment of the development of Husserl's philosophy. This book is a most welcome and successful effort to change this situation.

In the introduction to the book, Ortiz Hill says that "[t]he principal goal of this collection of papers is to work to integrate Husserl's thought into philosophical discussions in which it rightfully belongs by establishing the legitimate ties between his ideas and those of philosophers and mathematicians who have been more readily accepted into the pantheon reserved for those deemed to have made significant contributions to the field". (p.xi) She also mentions some of the obstacles that, she believes, have prevented an accurate assessment of Hussel's contributions to the philosophy of formal sciences: the ravages that two wars can produce on many fronts, that of intellectual endeavor included, incompetent translations of Husserl into English, insufficient logical and mathematical expertise of phenomenologists in general, and

Husserl's own sparse explicit mention and clear evaluation of the connections between his ideas and those of his contemporaries.

The authors' interest in Husserl in connection with the philosophy of logic and mathematics, Husserl's relation to Frege in particular, go back at least to the 70's. In his Ph.D. dissertation entitled *Edmund Husserls Philosophie der Logik und Mathematik im Lichte der gegenwärtigen Logik und Grundlagenforschung*, (Rheinishe Friedrich-Wilhelms-Universität, Bonn, 1973), Rosado Haddock showed that the Føllesdal's thesis, according to which Frege had a strong influence on Husserl's philosophical development were untenable; and in her master's thesis, entitled *La Logique des Expressions Intentionnelles* (Sorbonne, Paris, 1979), Ortiz Hill also presented some arguments against these views, which were at that time more or less the official doctrine. This shows that the authors of this collection have long been sensitive to the issue of the relations between phenomenology and analytic philosophy, even before it became fashionable after J. N. Mohanty's *Husserl and Frege*[6].

A correct assessment of Husserl's ideas on meaning, objectivity, logic and mathematics, and the relation between Husserl's and Frege's, Cantor's and Hilbert's mathematical and philosophical ideas in the years that witnessed the origin of phenomenology and modern mathematics, which are, I believe, the main points of this book, are tasks whose importance for the philosophy of logic and mathematics, and the history of contemporary philosophy cannot be overestimated. For this reason this book deserves the attention of philosophers belonging to either continental or analytic circles.

I cannot end this already overlong review without a remark about the cover of the book. It shows a photographic reproduction of a sculpture by the French artist Jacqueline Wegmann. It is certainly an intriguing piece of art. It shows a web of subtle but solid wires con-

[6] Bloomington, IN, Indiana University Press, 1982.

necting otherwise isolated pieces of matter, which together compose a balanced and harmonious totality. This is no doubt a suitable illustration of the situation of philosophy in this century. The two dominant philosophical traditions in Europe in the Twentieth Century, phenomenology and analytic philosophy, may appear isolated, but certainly there are strong ties that keep them both as parts of a unified totality. This book helps us to understand these connections.

