Book Review: MACBETH, Danielle. *Realizing Reason: A Narrative of Truth and Knowing*. Oxford: Oxford University Press, 2014, 494 pp., $99.00 (hbk), ISBN 9780198704751

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ABSTRACT
We review Danielle Macbeth's book *Realizing Reason*, published by Oxford University Press in 2014. This extensive book is composed by nine chapters in which Macbeth critically presents the development of mathematical practices in the Western world – from its founding in Ancient Greece's diagrammatic practices to the apogee of mathematical logic in the nineteenth and twentieth-centuries – while offering a revaluation of its present stage by means of a reconsideration of Gottlob Frege's philosophical contributions. In this review, we present a summary of each chapter's contents and make general considerations about them.

Danielle Macbeth's Realizing Reason is a *tour de force* about the history of mathematical knowledge from ancient Euclidean geometry to the late 19th century and early 20th century developments on mathematical logic. It is an ambitious work dealing with a vast array of subjects and philosophical themes while still being able to consistently display a high standard of erudition and originality in areas as diverse as the Philosophy of Mathematics, Language, Science and Mind.

The narrative of the book is complex and multifaceted, but its main thread is two-fold. On one hand, Macbeth aims to develop a novel account of "our being in the world" which gives room for the existence of normative facts in a world which is fully explained by mechanistic causal laws – a profound philosophical dilemma that stands at the center of many authors' works such as Kant, and, more recently, Macbeth's former Pittsburgh colleague, John McDowell (1994). On the other hand, Macbeth argues for a reparation on the perspective with which philosophers should see the practice of mathematics and the mode through which it attains knowledge. The author's objective is primarily to show how the practice of mathematics, in each of its historical stages from the Greeks to the present, by means of its characteristic linguistic notations, enabled thinkers to literally amplify their knowledge, as opposed to merely making explicit what was already implicit in the information one had begun with. Furthermore, Macbeth aims to prove that this much is true even of mathematics as it is currently conceived, i.e. "the practice of reasoning deductively from concepts" (p. 5). One of the author's main challenges is to show how there can be such a thing as an "ampliative deduction", and in order to achieve this feat, Macbeth must break through Kant's conceptual distinctions to open the way for the idea that the knowledge attained by some deductions, which is, *per* definition, analytic, can, at the same time, be synthetic.

Both issues dealt with in the book – the apparent incompatibility of reasons in a world of causes and the notion of ampliative mathematical knowledge – are foundational questions in Philosophy and each can be traced to the early beginnings of philosophical practice itself. It is noteworthy that Macbeth sets out to tackle both at the same time while also showing how the resolution of one question is tied to the resolution of the other (and vice-versa).

The book is divided into three main sections, each composed by three chapters, which chronologically tell the story of reason's development and unfolding from the Ancient Greek's mathematical practice to the present. The first section is entitled Perception, alluding to Macbeth's claim that in the early stages of our intellectual development we have our "primary mode of intentional directedness in perception" (p. 17). This corresponds to a time before the Cartesian turn in the sixteenth century, where "pure intellection", as opposed to the perception of an object, "becomes the paradigmatic mode of intentional directedness and the model even for perception" (p. 18). This intellectual revolution, which led us from bare perception to pure intellection, is the main theme weaving together the three first chapters.

In Chapter 1, Macbeth presents a story detailing how perceptually aware beings, like ourselves, have managed to progress from our ancestors' rudimentary capacities of imitation and of synthesizing procedural knowledge to sophisticated self-consciousness and rationality. Crucial to Macbeth's story is a profound anti-Cartesian stance, according to which we should not make a division between the merely physically describable stuff that is "outside" and the normatively significant, meaningful experiences that are "inside" (p. 20). In explicit opposition to Robert Brandom (1994), Macbeth suggests jettisoning altogether the idea that a world described by means of causes stands in contrast to a world described by means of reasons, as if these concepts were not applicable to things of the same nature. Just as nature acquires biological significance as animals evolve in their environments, e.g. a bunch of leaves becomes *food*, so does nature become socially and culturally significant as intelligent beings begin cooperating, sharing goals and engaging in practices and games among themselves. The last step in that progression is the transformation of social beings into "properly rational beings capable of distinguishing in principle between how things seem and how things are" (p. 56); that is, the acquisition of the capacity to step back from our natural inclinations and to realize that "*anything* we think can be called into question, and improved upon" (p. 49). This final stage of intellectual development, Macbeth claims, depends fundamentally on the coming into being of a natural language, which is, albeit contingent and historical, not an obstacle to objectivity, but constitutive of our access to it.

Notwithstanding their importance, natural languages are intrinsically grounded on our perceptual means of access to the world, and, for that reason, do not reach far enough so as to provide us with knowledge of all there is to be known about in the world. In chapter 2, Macbeth delves deeply into Ancient Greek mathematics - exemplified by Euclidean diagrammatical practice, a methodology that would be the unchallenged orthodoxy in Western mathematical thought for centuries until the Renaissance – in order to make clear how the unfolding of reason takes us ever more far from our immediate empirical reality. Macbeth's central claim in this chapter is that, in Euclidean diagrammatical practice, we do not reason *on* diagrams, but *in* them; in other words, Macbeth claims that an Euclidean diagram does not merely describe a certain course of mathematical reasoning (as, for example, we could describe a mathematical demonstration *on* natural language), it "formulates the *contents of concepts*" in a mathematically tractable way and, for that reason, constitute – as opposed to merely picturing or describing – the reasoning itself. As Macbeth fleshes out that important distinction, it becomes ever clearer how demonstrations in Euclidean geometry managed to amplify our knowledge, often giving rise to discoveries that were not even implicit in what the demonstration had begun with. Differently from an Euler or Venn diagram (or any other types of "picture proofs"), in an Euclidean diagram "what is displayed are the contents of concepts the parts of which can be recombined with parts of other concepts". So, for example, a certain mark in a diagram may be *seen* as either the side of a triangle or the radius of a circle, depending on the perspective that the reasoner impinges on the drawing. The possibility of this "gestalt-shift" (absent in, e.g. Euler and Venn diagrams) is what explains how figures often pop-up in an Euclidean proof - such as when an equilateral triangle appears as if from nowhere in the proof I.1 of the *Elements* - and thus, how "something new can emerge that was not there even implicitly".

Chapter 3 leads us to the radical departure from Ancient thought that happens during the Renaissance with the rise of Modern philosophy, physics and mathematics. Macbeth is particularly concerned with Descartes' influence in the emergence of a new mathematical practice by means of the introduction of the language of elementary algebra. The algebraic method adds a new degree of abstraction to the activity of reasoning, Macbeth argues, since its intentionality is not object-oriented, but directed to the merely potential relations which arbitrary objects may instantiate (p. 132). For example, one begins to interpret geometrical objects in a computationally tractable way, as the arithmetical relationship of some lengths (e.g. a square is some quantity multiplied by itself). By abstracting away from objects, and, thus, from any subject matter in particular, Descartes' language allows "pure intellection to become (at least in intention) an actuality" (p. 149). Similarly to the language of Euclidean geometry, Descartes' algebraic method is not to be conceived as merely a tool through which a course of reasoning can be described or pictured; instead, these symbolic languages present content in a mathematically tractable way, and, because of that, are the matter by means of which reasoning itself is constituted, or, to use Macbeth's terminology, reasoning comes into existence *in* those symbolic languages, as opposed to being merely described *on* them.

The next triad of chapters is entitled "Understanding", referencing the fact that Kant's legacy to Philosophy entails that "pure reason is not and cannot be a power of knowing as Descartes had thought. Not reason but only understanding is a power of judgement, of knowing" (p. 151). This is precisely what chapter 4 is concerned about, more particularly, Kant's Copernican revolution, by means of which our epistemic access to reality is turned upside-down, requiring "the philosopher […] to focus not unthinkingly on the object of knowing but self-consciously on the power of knowing, on what reason requires of objects as objects of knowledge" (p. 199). Macbeth's argues that, as groundbreaking as Kant was, his thought was still pretty much restrained by the scientific, and, most importantly, the mathematical practice of his day, which, absent the revolution that would come in the nineteenth-century, could not ground a proper account of mathematical truth and knowledge – that is, an account of mathematical truth and knowledge answerable to things as they are in themselves, as opposed to things as they merely appear to us.

Chapters 5 and 6 present the new form that mathematics has come to be clothed in by means of the collective effort of intellectuals throughout the nineteenth-century. By means of the work of mathematicians such as Bolzano, Galois and Riemann, Macbeth tells us the story of how mathematics, after twenty-five centuries of development, finally becomes a self-standing discipline, "the work of pure reason wholly unfettered by the contingencies of our form of sensibility" (p. 244). However, not all is well with that sudden reshaping of mathematical practice, since, if mathematics answers to nothing outside of its own activity, as it came to be seen, it starts to look as if mathematics is nothing more than a linguistic game, completely disconnected of any struggle for objectivity.

Indeed, for much of the twentieth-century, Macbeth will go on to argue, a cluster of theses based on (i) the distinction of logical form and semantical content, (ii) a truth-theoretical account of meaning and (iii) a primacy of mathematical logic as the ruler of all formal disciplines will go on to become the orthodoxy in the understanding of mathematics and of its practice. This is, according to Macbeth, a very unfortunate event, since it seems force on us a picture of logic and mathematics as being merely formal disciplines, and, for that reason, completely deprived of intentional properties. Even worse, and this is one of the central points of the book, this is the picture that intellectuals born during the twentieth-century (even the best of them), have accepted without subjecting it to scrutiny, i.e. a picture of reasoning as being purely mechanistic, "nothing more than the rule-governed manipulation of signs with no regard for meaning" (p. 293).

In the last group of three chapters, aptly entitled "Reason", Macbeth purports to analyze the philosophical problems that are engendered by the last great revolution in mathematics, i.e. when it came to be seen as "a practice of deductive reasoning on the basis of defined concepts" in nineteenth-century Germany. Most pressing to the author's concerns is showing that this new conception of the mathematical practice is not purely formal in the sense that it came to be seen by philosophers, but, on the contrary, that it is intrinsically meaningful and often enables us to attain knowledge in the strongest sense of that concept, that is, objective knowledge about things in themselves.

In chapter 7, Macbeth takes the reader to a confrontation, for the first time, with Gottlob Frege's *Begriffsschrift*, a mathematical notation that "was explicitly designed as a notation within which to reason deductively from concepts in mathematics". This long chapter goes at great lengths to explain Frege's concept-script because, as Macbeth defends, one must understand the notation in order to be able *to see* the mode of reasoning embodied within it. The pinnacle of the chapter, however, is Frege's proof of theorem 133 in Part III of the *Begriffsschrift*, which Macbeth presents as being a real example of a deduction that establishes a real extension of one's knowledge. The particularity of that proof is the book's central concern until its very end, namely, the fact that it joins content from two definitions, as opposed to merely joining content from two axioms. That operation of bridging the content from two previously unconnected definitions is precisely what enables that mathematical practice to amplify one's knowledge. Just as figures often pop up in a Euclidean diagram, "as if from nowhere", some deductive proofs link concepts that were independently introduced and which, absent that proof, would display no immediate connection among themselves.

That much gets clearer throughout chapter 8, where Macbeth argues that definitions, although they are, by nature, stipulative, are not epistemically vacuous, since they serve to articulate the inferential content of particular concepts, and that is something one might – objectively – succeed in doing correctly or not. Definitions, however, do not amplify one's knowledge by themselves; it is only in the context of a proof that they are able to forge new links within one's conceptual repertoire:

> proofs without definitions are empty, merely the aimless manipulation of signs according to rules; and definitions without proofs are, if not blind, then dumb. Only a proof can actualize the potential of definitions to speak to one another, to pool their resources so as to realize something new. (p. 387)

The conception of reasoning that we reach by the end of the book is, contrary to the Early Modern simulacrum that we have unreflectively inherited, is neither reductive nor mechanistic. It does not purport to reduce the content of concepts to primitive notions, instead, those contents are displayed in a mathematically tractable way. It is also not mechanistic, Macbeth claims, since the knowledge attained by a deductive proof may be, at the same time, both analytic and synthetic – a fact that makes Kant's dichotomies stand in need of a radical revision.

The book's narrative comes full-circle by the end of chapter 8 and throughout chapter 9, where Macbeth studies the case of physics, about which she draws a parallel between the nineteenth-century revolutions in mathematics and the twentieth-century revolutions in theoretical physics. The underlying theme is that mathematics and physics have both recently undergone profound revolutions, while philosophy "has, until now, remained merely Kantian" (p. 453). The final blow on the Cartesian view that we have inherited from the early moderns involves disentangling the

Sinn/Bedeutung distinction from that of concept and object (a disentanglement that was out of reach for Kant). Only by clarifying those distinctions, we can understand "how a radically mind-independent reality and an unconditioned spontaneity are not only compatible but in the end made for each other" (p. 451).

Realizing Reason suffers from a flaw that is an almost inevitable consequence of its virtues. Macbeth's overambition, i.e. her attempt to leave no stone unturned, leads to her book having a certain *bric-à-brac* quality, since the thread that unites her narrative throughout highly distinct subject matters is usually, but not always, evident. Regardless of that, this book presents innovative theses in a multitude of areas, of special interest being its analysis of Frege's work, which sees his accomplishments from a whole new perspective and as giving rise to a heterodox conception of ampliative deductive knowledge. All in all, *Realizing Reason* is a recommended read for anyone with interests in the broad set of areas encompassing the philosophy of mathematics, mathematical practice, history of mathematics and logic, and who is interested in seeing how the issues on those areas communicate with issues in the philosophy of mind, language and the history of philosophy.

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