

**FREGE'S PUZZLE IS HERE TO STAY:
TRIVIALITY AND INFORMATIVITY IN
NATURAL LANGUAGES**

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Abstract: Frege's puzzling remarks on the beginning of On Sense and Reference challenge us to explain how true identity sentences of the form $a = a$ can differ in cognitive value from sentences of the form $a = b$ when they are made true by the same object's self-identity. Some philosophers (e.g. Almog, Glezakos and Paganini) suggest that the puzzle cannot be set up in the context of natural

languages since natural sentences, unlike those of regimented formal ones, do not wear their logical properties on their sleeves. In this paper we argue that, on the contrary, there exists a notion of coordination between names which is apt to track the relevant logical properties of natural sentences and therefore to set up the puzzle in natural languages. Frege's puzzle is here to stay.

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1. INTRODUCTION

Frege (1892) introduced what would become known as Frege's Puzzle by claiming that identity sentences of the form $a = a$ often have a distinct cognitive value from sentences of the form $a = b$:

$a = a$ and $a = b$ are obviously statements of differing cognitive value; $a = a$ holds *a priori* and, according to Kant, is to be labelled analytic, while statements of the form $a = b$ often contain very valuable extensions of our knowledge and cannot always be established *a priori*.

As the quotation above makes clear, Frege conceived the puzzling predicament as that of explaining how true identity sentences of these two forms can differ in cognitive value (one, trivial, the other, potentially informative) when they are identical in reference and, because of that, made true by the same object's self-identity. Even before advancing his own solution to the puzzle, which involves the postulation of senses, Frege thus makes a substantial assumption: whatever

makes some identity sentences trivial and others potentially informative has to do with their *form*. That assumption severely constrains the types of explanation that can be proposed of an identity's¹ cognitive value, namely, that it must, at some level to be specified, have something to do with its form.

Frege, like most of the “analytic philosophers” of his generation, was primarily concerned with regimented languages (such as the language of first-order logic) in contrast to natural ones. The problem is that, while regimented sentences wear their form on their sleeves, the same cannot be said about natural ones. The former are rigorously disambiguated, so that the logical form of any sentence can be inspected by merely looking at the shapes of the symbols which it is composed of (its superficial syntax). Natural languages, on the other hand, are pervasively ambiguous and misleading homonyms are everywhere. Indeed, one of the most oft-repeated lessons from the early pioneers of analytic philosophy was that one simply cannot infer the form of a natural language sentence from its superficial appearance.

Should we then conclude that Frege's puzzling predicament, as expressed in his famous quotation, does not apply in the context of natural languages? A negative answer to that question requires finding a notion of form applicable to natural language identities which adequately correlate with their cognitive profile.

There are, however, authors who think that this challenge cannot be met. Almog (2008), Glezakos (2009), and Paganini (2016) suggest that we abandon Frege's ship still on its maiden voyage: its assumption that we can learn anything about an identity's potential informativity (or triviality) by

¹ Except in cases where it could be misleading, we'll abbreviate ‘natural language identity sentence’ by ‘identity’.

assessing its form. To be sure, they obviously agree that some identities are informative while others are not, but argue that, given how logically untidy natural language is, there is no notion of an identity's form which could illuminate the issue.

The main objective of this paper is showing that these three authors are wrong, that is, that we can formulate Frege's puzzle in the context of a natural language by appealing to a notion of form. Our objective may, at first, seem merely exegetical - as if our primary focus is understanding Frege. But this is misleading. While we are taking Frege's quotation as the starting point of our investigation, our objective is more general: we want to show which philosophical issue the puzzle is really about and to situate distinct (and often incompatible) solutions to it within a clear framework. We intend, to put it in a few words, to show what Frege's puzzle is a puzzle about and which types of solutions to it are coherent.

Here's a brief overview of the paper. In section 2, we distinguish three notions of form and investigate which of those could possibly be at issue in Frege's original quotation. After arguing that Frege must at least have a *logical* notion of form in mind, we reconstruct his argument in favor of the postulation of senses within the context of natural languages (section 3). According to our reconstruction, Frege's argument - not only his solution but also the mere cogency of its premises - depends on an elusive conception of the logical form of a natural language identity. As we will see, given the messiness of natural languages, it is particularly hard to substantiate that notion. In sections 4 and 5, we experiment with several candidates, and end up agreeing with both Glezakos (2009) and Paganini (2016) that most of these won't do (including the notion of typographical identity, sameness of generic names, sameness of common-currency names, and sameness of private names).

Fortunately, we are lucky enough to find a good contender: the notion of name coordination (section 6). This notion is based on a subject's beliefs about whether two name occurrences *might* be of distinct names or not. Our suggestion then is that we can formulate Frege's puzzling remarks about informative identities within the context of a natural language by employing the notion of name coordination. Against our foes, we thus conclude that Frege's puzzle is also a puzzle about natural language identities, and that Frege's own solution is not circular. It is important to be clear that we don't defend Frege's solution *per se*; we merely show that, while it might not be the best solution, it is at least neither circular nor question-begging.

In the last two sections of the paper (7 and 8), we defend our view from some objections. First, we note that name coordination seems to imperfectly track cognitive profile. There are, after all, uninformative identities whose name occurrences are not coordinated. In order to clear things out, we argue that we should not confuse *trivial* identities (those expressed using coordinated terms) with *merely uninformative* ones - those that are uninformative only because the background information a subject has resolves the truth of the judgement. Trivial identities have coordinated occurrences and thus, unlike merely uninformative ones, can be known to be true just in virtue of their (syntactic) form.

Finally, we defend our view from a potentially fatal objection. Some authors (e.g. Almog 2008, Salmon 2012) have questioned the very existence of natural cases of intrinsic triviality: "I don't know of any natural language example that is of the "a = a" form [...]" (Almog 2008: 572). Identity sentences are uninformative, according to this objection, not because of some intrinsic feature of them (syntactic or semantic) but simply when (and only when) the background information we have resolves their truth. In section 8 we argue that, on the contrary, speakers (and

thinkers) of natural languages *must* be able to detect when two names are intended to be logically coordinated, and that name coordination is apt to track these logical relations, and hence triviality, at the syntactic level. Triviality is, we argue, also a natural language phenomenon.

2. SEMANTIC, SYNTACTIC AND LOGICAL FORM

The question of how we should best characterize the notion of *form* of sentences, especially of natural language sentences, is notoriously tricky (cf. Iacona, 2018). Fortunately, our present purposes allow us to go by with just a small handful of distinctions. The properties of sentences vary on a number of dimensions. The *semantic* properties of sentences are sensitive to their truth-conditions and to the meanings of their components, whatever these are. The *syntactic* properties are sensitive to how many expressions (both tokens and types) it is composed of, and how they are related. The *logical* properties, finally, tell us about the inferential relations of sentences to other sentences in the language. These three dimensions in which properties of sentences can vary give rise to three different notions of form which we may call: semantic, syntactic and logical form, defined as classes of equivalence of the following identity relations.

We shall say that two name occurrences in an identity sentence, n^1 and n^2 , are *semantically* identical iff they share all their semantic properties (whatever these are). They are *syntactically* identical (relative to a criterion of type-identifying linguistic expressions) iff they belong to the same types relative to this criterion. Finally, they are *logically* identical iff they are represented by the same individual constant in a logically *adequate* formalization of the sentences in which they occur (cf. Iacona 2018, p. 70).

While these distinctions possibly collapse onto each other in the case of regimented languages, they need not correlate with each other in the context of natural ones. Identity sentences of the same (syntactic) typographic type, for example, may well belong to different semantic and logical types, as is clear from examples of accidental homonymities. *Vice versa*, sentences of different typographic types can share their semantic or logical properties, as it happens in some cases of anaphora or perhaps with abbreviations ('Severe acute respiratory syndrome coronavirus 2 = SARS-CoV-2'). Finally, as will be clear later, some sentences may share their semantic and logical properties, even if they do not share their syntactic properties.

We defined these three notions of form in such a liberal way that each one is susceptible of being made precise by filling in details about, respectively, semantic, syntactic and logical properties. Referentialists, for example, think that the semantic properties of singular terms are exhausted by their reference. Therefore, the only notion of semantic form that they can conceive of, is one according to which two identity sentences are of the same (semantic) type iff they refer to the same object. Analogously, the notion of syntactic form is liable to various precisifications, depending on one's chosen criterion of linguistic type-identity. As we shall see, there are numerous ways to type-identify linguistic occurrences syntactically, which are more or less suitable depending on the purpose of the categorization. One can, for example, type-identify words typographically (by their spelling) or phonographically (by their pronunciation). To each of these, there corresponds a particular notion of syntactic form. It is of course a further issue whether Frege was, or whether we should, at this juncture, be philosophically concerned with how words are spelled or pronounced (the answer is 'not').

Finally, the notion of *logical* identity (and distinctness) is defined in terms of the *adequacy* of a formalization for logical

purposes. This idea of logical form is meant to capture the capacity of rational agents to detect *transparently* the validity (invalidity) of certain patterns of inference. Consider for example the following unifying argument:

1. Descartes is French
2. Descartes is a philosopher
3. Therefore, there exists at least one French philosopher.

We are sometimes capable of detecting the validity of this and similar kinds of arguments without needing to independently assess whether the two name occurrences of 'Descartes' co-refer. That is, we are sometimes able to “trade on the identity”² of distinct name occurrences. A formalization is *adequate* only if it makes these inferential intuitions syntactically transparent, thus employing a single individual constant for every term occurrence whose co-reference we may take for granted. This happens when we “see” that different word occurrences are meant to be logically identical. As Russell (1998, p. 58) put it: “in a logically perfect language there will be one word and no more for every simple object”.³

Although natural languages are not perfect in this sense, we can still make sense of this notion of logical identity by making reference to the ideal language that *would* capture these natural logical intuitions. It is worth noting that, as

² This term was coined by Campbell (1987).

³ Interestingly, as noted by Iacona (2018: 70), Wittgenstein too makes a similar remark in the *Tractatus* (1952, 5.53): “Identity of the object I express by identity of the sign and not by means of a sign of identity. Difference of the objects by difference of the signs.”

Iacona (2018, p. 70) pointed out, the definition of logical identity does not involve “reference to the syntactic properties of n^1 and n^2 . More specifically, it does not require that n^1 and n^2 instantiate the same graphic type. So it leaves room for the possibility that n^1 and n^2 are logically identical even if they are grammatically distinct, that is, even if they are occurrences of distinct syntactic items”.

Summing up, there are (at least) three relevant notions of form: semantic, syntactical and logical. All of these notions might further divide into different variants depending on how one chooses to characterize, respectively, semantic, syntactical and logical identity (and difference). Which of these notions could - if any at all - help us formulate Frege’s puzzle in the context of a *natural language*?

3. THE SEMANTIC SOLUTION

It is natural to think that, minimally, the notion of form that must be appealed in the formulation of the puzzle about informative identities must relate to the logical notion of form. If an identity sentence is to be (potentially) informative to a subject, then surely this subject must not be capable of deducing it on purely logical grounds. Vice-versa, if an identity is trivial, then the imparted information must have already been resolved by the subject’s logical skills. Thus, we can put forward the following as a minimal characterization of Frege’s initial puzzlement:

[Initial Puzzle] Differences in the cognitive profiles of co-referential identities correlate with differences in their logical form (an identity sentence is trivial if its logical form is $a = a$, but often informative if of the logical form $a = b$). No difference in informativity without difference in transparent logical properties.

Frege's own (well-known) solution to the puzzle was the postulation of extra semantic properties, senses, modes of presentation of a referent - often conceived of as associated with definite descriptions - which can vary even between co-referential terms.

It is natural, now, to think of there being connected with a sign (name, combination of words, letter), besides that to which the sign refers, which may be called the reference of the sign, also what I should like to call the *sense* of the sign, wherein the mode of presentation is contained. (Frege, 1892, p. 1)

Frege's solution can be separated in two steps. First, there's the assumption that the puzzle requires a *semantic* solution, i.e. that the logical form of an identity correlates with its semantic properties. Since referential content, the simplest kind of semantic value, cannot distinguish between identities of different cognitive profiles, one must find a more fine-grained semantic notion that is able to do so. This is the second step, the postulation of senses.

[The Puzzle is Semantic] The logical form of identity sentences supervenes on their semantic form: no difference of logical properties without differences of semantic properties.

[Postulation of Senses] Therefore, since the referential semantic values of co-referring terms is the same regardless of their logical identity (or difference), the terms must differ in another semantic respect: their senses. An identity sentence is trivial if its two occurrences have the same sense, but often informative if they have distinct senses.

Frege's argument can then be characterized as an inference departing from *Initial Puzzle*, passing via the

assumption *The Puzzle is Semantic* and concluding with *Postulation of Senses*. Some brief explanation is in order.

Initial Puzzle encapsulates what we take to be the minimal way of setting up Frege's puzzle about informative identities, i.e. the observation that the cognitive value of an identity sentence correlates with its logical form - in other words, that trivial identities can be represented in an *adequate* regimented language by a single variable flanking an identity sign, while potentially informative ones cannot.

The intermediary premise of the inference, *The Puzzle is Semantic*, is the one that has been targeted more often by critics. Since we wish to remain neutral about the efficacy of this strategy of attack, we shall not delve into its pros and cons. Suffices to say that the intuition that underpins this assumption is that when one "sees" that two occurrences are logically identical, one does so by grasping their meanings. One can see that 'Hesperus is Hesperus' is true simply by grasping that its two name occurrences have the same meaning in some sense to be specified. So any pattern of differentiation as to the logical properties of occurrences must be reflected by an analogous pattern of semantic differentiation between the occurrences themselves.

The conclusion, *The Postulation of Senses*, is a central corollary of the Fregean view: the logical form of an identity, and thus its cognitive profile, is determined by the identity or distinctness of its associated senses.⁴

In order to be puzzled by Initial Puzzle, it is necessary that the notion of logical form to which it appeals be cleared up. As we noted, in formulating the puzzle, Frege was thinking about the case of a regimented ideal language, where

⁴ This reconstruction of Frege's reasoning strives to follow his own text, but it is also compatible with Zalta's (2019) interpretation. We do, however, attempt to flesh out more carefully what Frege's appeal to forms seems to commit him to.

the logical form of any sentence can be assessed just by means of the superficial shape of the symbols which compose it. But we cannot do the same in the context of natural languages. Natural language *typographic* types do not correlate with either logical or semantic types. How, then, can one be puzzled by Initial Puzzle in the context of a natural language if it is not even clear how to assess the logical form of its sentences?

Here's one way of answering this question which obviously does not work: claiming, from the outset, that an identity has the logical form $a = a$ when its occurrences have the same sense. Postulation of Senses is supposed to be the conclusion of an argument which has Initial Puzzle as one of its premises; it would, of course, be circular to a conclusion in order to understand one of its premises.

Frege's *semantic* solution to the puzzle (Postulation of Senses) has been criticized on many fronts. This means that many authors find Frege's way of framing the puzzle (Initial Puzzle) correct but reject either The Puzzle is Semantic or Postulation of Senses. One could reject the former by claiming that logical form is not a matter of semantics (but of e.g. syntax or pragmatics). This is the path chosen, for example, by many referentialists. One could also accept Initial Puzzle and The Puzzle is Semantic but reject Postulation of Senses. This is a fairly common strategy; the notion of sense is left significantly underexplained by Frege himself and its most usual understanding throughout the twentieth century, that of a purely qualitative description, has always been the subject of incessant criticism. Thus, many accept The Puzzle is Semantic, but think that senses are ill-fitted for that task.⁵

⁵ Some contemporary critics of Frege, the relationists, suggest, for example, that his sense-based solution is more committal than it needs to. Relationists argue that identities formalizable by one

These are not the only two ways of blocking Frege's argument. As we mentioned in the introduction, a series of recent papers have consolidated a distinct attack on the Fregean argument. These authors suggest that Initial Puzzle itself sits on contentious grounds. Their pressing question is: why should we assume that the cognitive profiles of identities correlate with any aspect of their form at all?

4. LOGICAL FORM IS NEITHER BASED ON GENERIC NOR COMMON-CURRENCY NAMES: GLEZAKOS (2009)

As we have seen, in order for Initial Puzzle to be puzzling in the context of a natural language, one needs to suggest a way of assessing the logical form of natural language identities and, additionally, show that this way illuminates their cognitive profile. What are the most natural suggestions one could try?

For starters, we could try to assess the syntactical form of natural language identities. Given that an identity of the logical form $a = a$ is formalizable by a single variable, one could suggest that this is so because it is flanked by a single name repeated twice over, as opposed to potentially informative identities, which are flanked by two independent names. Under this interpretation, the puzzling predicament exposed by Initial Puzzle becomes that of explaining "what

variable repeated twice over can be explained as those where the referent is being represented as the same; this irreducibly relational fact can hold between two name occurrences even if they do not possess any distinct intrinsic feature (like being associated to distinct senses). Thus, these authors claim that Frege's solution to his own puzzle is a bit of an overkill: we can do the same with less. See Fine 2007 for an influential defense of this view, and Gray 2017 for a recent survey of the area.

is the source of the epistemic difference between true identity sentences that contain a single name twice, and those that contain two names” (Glezakos, 2009, p. 205).

Quick reflection shows that ‘name’ is not a univocal notion and that there might be distinct ways to classify name-occurrences as belonging to the same category or not. To each of these ways there corresponds a different notion of syntactic form. Is there one such notion that foots Glezakos’ explanatory bill? The first step towards answering that question is disambiguating a few conceptions of names.

Kaplan (1990) proposes an influential distinction between generic and common-currency names. The distinction might sound technical, but it is easy to grasp. Generic names are the names which namesakes share. Thus, Diogenes the Cynic (the notorious philosopher who slept in a large ceramic jar) and Diogenes Laërtius (the famous biographer of Greek philosophers) share the generic name ‘Diogenes’. However, there is also an intuitive sense according to which Diogenes the Cynic’s name, ‘Diogenes’, is not the same as Diogenes Laërtius’. This is because they have distinct common-currency names. A common-currency name can be defined in terms of generic names and a certain history of use that starts at their original context of introduction (usually under the form of a baptism). Thus, these two Diogenes have distinct common-currency names because, even though they are the same generic name, they have, to summarize, distinct origins (one was introduced in the former Diogenes’ baptism on the 5th century BC, the other, on the second’s, around the 3rd century AD).

The definition of these two types of names could be complicated in several ways but this need not concern us at the moment.⁶ This rough-and-ready understanding is

⁶ For just one example, it is not clear whether ‘JFK’ and ‘John F. Kennedy’ are distinct common-currency names or not (the

enough for our purposes. In the remainder of this section, we show that the conception of logical identity appealed to in Initial Puzzle cannot be understood as being based on sameness of generic or common-currency names.

In order to see that, consider the thought experiment described by Kripke (1979) about a subject, Peter, who, upon hearing several people talk about a man called ‘Paderewski’ who, not only is a proficient pianist but also a prominent politician, infers that these people must have been talking about two distinct men who are namesakes.⁷ We could then imagine Peter learning about his confusion after being addressed by a subject who told him

(1) Paderewski is Paderewski.

Given that Peter might learn something valuable from that utterance, the identity uttered is informative. However, the identity sentence uttered is composed by two occurrences of what seems to be the same name, ‘Paderewski’ – the two occurrences are not only co-referential but also have the same spelling.⁸ Furthermore, these two occurrences end up being classified as of the same name no matter which of the two conceptions of names

example comes from Unnsteinsson 2019). The former was originally just an abbreviation of the latter but, as time passed by, it gained a life of its own, so to say. Other tricky cases are those where a name gradually changes its referent, like in Evans’ (1982) well-known ‘Madagascar’ case. These tricky cases need not concern us.

⁷ The assumption usually made to secure the rationality of Peter’s inference is that he believes no politicians have musical talent.

⁸ As Glezakos (2009, p. 205) notes, this reference + phonographical criterion seems to be tacitly assumed in Frege’s (1892) footnote B.

defined above one adopts. Surely the two occurrences of Paderewski's name in (1) have to be categorized as occurrences of the same generic name and also of the same common-currency name.

To be sure, things could have been different. We could, for instance, tell a different story where the occurrences in (1) are co-referential but not (instances of) the same common-currency name. One could imagine, for instance, that baby Paderewski was baptized as 'Paderewski' by his biological parents, but then abandoned in the front door of the local county's orphanage with no letter nor information. Paderewski is swiftly adopted by a couple of wealthy benefactors. Never having been able to find out whether the baby had already been given a name before, the benefactors decide to just come up with their own name for their newest son. As chance would have it, they decide to call Paderewski 'Paderewski'. Baby Paderewski is then christened 'Paderewski' twice over. If (1) was uttered by a social service worker in an attempt to explain that the boy abandoned in the orphanage is the one later adopted by the wealthy benefactors, then the two name-occurrences of 'Paderewski' in (1) would be of distinct but co-referential common-currency names.⁹

There's nothing paradoxical with the idea that someone could bear two common-currency names that are instances of the same generic name, but this won't be of any help to our initial problem. In the original story of Paderewski, (1) allowed Peter to acquire important information even though its two name-occurrences were not only of the same generic

⁹ Kaplan (1990, p. 114-115) discusses a variation of the same story under the rubric of the Mischievous Babylonian. His protagonist decides to call both the morning star and the evening star 'Phosphorus', and thus ends up giving Venus two common-currency names which are instances of the same generic one.

name, but also of the same common-currency name. How then can we explain the informativity of an identity sentence in terms of the names it contains?

We believe Glezakos' pessimism is unwarranted.¹⁰ The main reason is that she fails to consider private or idiolect-specific conceptions of names corresponding to how a subject takes his language to be. A notion in that vicinity might very well give rise to a conception of syntactic form which makes sense of Initial Puzzle. In a recent paper, Paganini (2016) attempts to come to Glezakos' rescue. She argues that no notion of name sameness, private or public, seems to be able to do the job that is required. If successful, she will have even more definitively shown that Initial Puzzle is not puzzling at all.

5. FREGE'S PUZZLE AND SAMENESS OF PRIVATE NAMES: AGAINST PAGANINI (2016)

In the previous section, we saw there can be informative identities that contain both the same generic and the same common-currency names. However, as Paganini (p. 524) notes, both the generic and the common-currency ways of categorizing names are insensitive to how speakers themselves take their own languages to be. They do not take into account the fact that subjects like Peter would describe themselves as possessing two distinct 'Paderewski' names, that is, that they *believe* they are distinct common-currency names:

It may be objected that the notions of name I have considered are public; they have to do with the social

¹⁰ Although somewhat influential, her paper is notoriously very short, totaling just 6 pages.

character of names. Since an identity sentence may be informative for one speaker and not for another, it may be the case that what is relevant in order to evaluate the information drawn from an identity sentence does not have to do with social characteristics of names, but with private ones, i.e. with what any individual speaker believes to be names. (Paganini, 2016, p. 524)

Paganini's remark is in order, and the hypothesis to be tested now is that informative identities are flanked by occurrences of distinct *private* names. How do we individuate private names? As a first pass, Paganini suggests that a speaker possesses as many private names as she believes. If a speaker takes two name occurrences to be of distinct common-currency names (if e.g. she thinks they were introduced independently from each other), then she has two private names which she associates with each name occurrence. Thus, since Peter believes that the pianist called 'Paderewski' is distinct from the politician called 'Paderewski', then he also believes "their" names to be distinct common-currency names. It thus follows that Peter has two private names 'Paderewski' regardless of the fact that, objectively speaking, he is taking one name for two.

Private names thus afford us a way of characterizing the syntactic form of an identity, such as (1), that nicely correlates with its cognitive profile. (1) is of the form $a = b$ and thus informative, the hypothesis goes, because - for Peter, at least - its two name occurrences are of distinct private names.

Unfortunately, this strategy's success is short-lived. Paganini presents a handful of clever counterexamples to the thesis that identities are only informative when a subject believes they contain distinct common-currency names (and thus has two private names). Take, for instance, the case of 'Mao Zedong' and 'Mao Tse-Tung' (p. 525). Imagine that

Sally, a native speaker of English is, for the first time, reading about the contemporary history of China. We can suppose that she finds sentences like ‘Mao Zedong commanded the Long March’ in some of the books, and sentences like ‘Mao Tse-tung was the founding father of the People’s Republic of China’ in others. Assuming that none of the texts she reads explicitly claim that Mao Zedong and Mao Tse-tung are the same person, we can very well conceive of the possibility that, after all of that reading, Sally not only doesn’t know that Zedong and Tse-tung are the same person, but also doesn’t know whether ‘Mao Zedong’ and ‘Mao Tse-tung’ are the same common-currency name or not. She may, for example, wonder: “Are the two name occurrences transliterations of the same Chinese characters or are they transliterations of different Chinese characters?” (p. 525). Thus, at the end of the day, Sally has a hunch that ‘Mao Zedong’ and ‘Mao Tse-tung’ might be the same common-currency name, but still falls short of believing it.

If Sally, continuing her research, were to finally encounter a passage of a text explicitly stating ‘Mao Zedong is Mao Tse-tung’, she would obviously have her knowledge expanded. She would no longer be in doubt about whether these name occurrences refer to distinct people. However, we cannot explain the informativity that sentence would have for Sally by means of her possession of two private names associated with each name occurrence. By assumption, Sally suspects but does not believe that ‘Mao Zedong’ and ‘Mao Tse-tung’ are distinct common-currency names. This, of course, means that she does not have two private names associated with each of the occurrences in an identity sentence which was informative to her – this is enough to falsify our previous hypothesis that an identity’s informativity walks hand in hand with distinctness of private names. This shows that private names, as Paganini has defined them, do not track the informativity of name-involving identity sentences.

This case shows that private names are not the notion we need, but it does suggest a different way. As Paganini (p. 527) herself notes, even if the subject in the story is uncertain about whether 'Mao Zedong' and 'Mao Tse-tung' are the same common-currency name, it is still compatible with her beliefs that they are. That is, she at least believes they *might* be distinct.¹¹ This suggests the following hypothesis: an identity is informative just when a subject believes its name occurrences might be of distinct common-currency names. Identities of the form $a = a$ would then be those about which a thinker believes the two occurrences are of the same name; while those of the form $a = b$ would often be those about which a thinker is at least uncertain about whether they're of the same or not.¹²

This interpretative hypothesis, which we judge correct, is considered by Paganini (*ibid.*) but briefly dismissed. She considers a similar case about a non-philosopher subject, Sally again, that is trying to establish whether Descartes is Cartesius. Just as in the previous case, Sally not only doesn't know whether Descartes and Cartesius are the same, but also doesn't know whether these two name occurrences are of the same common-currency name or not. However, Paganini tries to give this story a new twist by supposing that Sally believes that the names are connected somehow, instead of merely having no opinion on the issue:

¹¹ Just as Paganini (p. 526) suggests, we take the epistemic modal sentences such as "S believes that x might be F" to mean either "it is compatible with S's beliefs that x is P" or "S believes that it is compatible with her own beliefs that x is P" (Yalcin 2007, p. 996-997).

¹² A thinker is at least uncertain about p if that thinker either believes not-p or believes not-p might be the case.

When someone does not have beliefs concerning whether “Cartesius” and “Descartes” are somehow related, it is compatible with her beliefs both that they are name occurrences of the same name and that they are name occurrences of different names. The case is different when someone is uncertain. This is the case with Sally: she believes that “Cartesius” and “Descartes” are name occurrences connected in some way, but she is uncertain how they are connected; for this reason there is no fact of the matter about whether it is compatible with Sally’s beliefs that “Cartesius” and “Descartes” are instances of different names. (Paganini, 2016, p. 527)

Armed with the assumption that it is indeterminate whether Sally’s beliefs are compatible with ‘Descartes’ and ‘Cartesius’ being distinct (common-currency) names, Paganini concludes that we must reject the view under consideration. But should we grant Paganini that assumption? What does it mean for Sally to believe that the name occurrences are *somehow* connected? All of the ways of answering that question we are able to come up either entail that, according to Sally, the name occurrences might still be of distinct names, or that they might not.

For example, Sally might believe that ‘Cartesius’ is but a Latinization of ‘Descartes’. But if this is what she believes (it would be a true belief, by the way), then it is clearly incompatible with her beliefs that these name occurrences are of distinct common-currency names, i.e. if one occurrence is just a different version of the other, then they are the same name *tout court*. Maybe what Sally believes is that the name ‘Cartesius’ was given by a philosopher to her son in homage to the notorious philosopher Descartes. This is one way the occurrences could be related without them being the same. However, it would then follow that Sally

believes them to be distinct common-currency names with independent contexts of introduction.

Perhaps Paganini wants us to think of a scenario where Sally has as much evidence for believing that the name occurrences are the same as she does for believing that they're not. Here's one way that could happen: two experts in early modern philosophy in which Sally deposits equal trust give her incompatible testimonies: one tells her that 'Descartes' and 'Cartesius' are connected but distinct names, while the other says that they're just versions of the same name. Since Sally trusts them equally, she is at a loss on how to adjust her beliefs. However, it is easy to see that this scenario doesn't give us what Paganini wants. Even if Sally is conflicted, it isn't indeterminate whether she thinks the two names might be distinct or not. If she decides to believe the first expert, then she believes the names are distinct; if she believes the second, then she believes the names are the same; if she decides to suspend her judgement on their testimonies, then she just believes whatever she believed prior to talking to them. All of these possibilities are compatible with the view we are considering.

One may summarize these observations concisely by appealing to the plausible principle that if it is neither determinately true nor false that a subject *S* believes a proposition *p*, then *S* ought to believe that both *p* and not *p* are compatible with her beliefs. If Sally's information is insufficient to settle the identity (or difference) of the names 'Cartesius' and 'Descartes' it is thereby also insufficient to exclude that they are different names. And this, contrary to what Paganini claims, is enough to guarantee that her beliefs are compatible with the names actually being different. Thus, Paganini's objection has no teeth against our interpretative hypothesis.

6. LOGICAL FORM AS NAME COORDINATION

We have just defended a hypothesis which links Frege's original remarks about the logical form of identity sentences, Initial Puzzle, to a subject's beliefs about whether its name occurrences might be of distinct common-currency names or not. The question we faced was: how do we assess the logical form of a natural language identity? The answer we are now ready to give is: an identity has the form $a = a$ when the subject believes its name occurrences might not be distinct, and $a = b$ otherwise.

This then helps us account for one of our primary objectives, that is, showing that there is a notion of syntactic form which correlates with Frege's puzzle of informative identity. The notion of syntactic form that emerges from our discussion is "idiolect-specific", i.e. it is related to how a subject takes his own language to be. It is a manner of type-individuating the expressions of an idiolect with respect to whether the subject believes they might be instances of distinct public names or not. For ease of exposition, we might say two name occurrences are *coordinated* when the subject believes they might not be of distinct names; and *not coordinated* otherwise. Name coordination is then the syntactic notion by means of which we type-individuate names and make sense of Initial Puzzle.

The syntactic notion of name coordination we are invoking is not wholly original. Indeed, it can be seen as a restriction to proper names of what has been variously called 'strict coreference' (Fine 2007)¹³, 'grammatically determined

¹³ Fine (2007) eventually opts for the term 'semantic coordination' which is where we find inspiration for our own choice of terminology. It should be emphasized that we are not assuming, as Fine does, that name coordination is a semantic (as opposed to syntactic, for example) relation.

coreference' (Fiengo and May 2006), 'explicit coreference' (Taylor 2015), and 'de jure codesignation' (Pryor 2017), and 'de jure coreference' (Pinillos 2011, Recanati 2012, 2016). All of these authors had a common goal in mind: characterizing that relation which holds between two representations when their co-reference is given for free, so to say. Name coordination is just that but restricted to names.

Taylor (2015, p. 240) makes claims that are not far from our own. He employs the notion of explicit co-reference - for all purposes very similar to our own notion of name coordination - with the objective of individuating names and showing what Frege's puzzle is about:

A name (type) is, in effect, a set of (actual and possible) name tokens such that all tokens in the set are guaranteed, in virtue of the rules of the language, to co-refer one with another. Call such a set a chain of explicit co-reference. I take it to be a linguistically universal fact about the linguistic category NAME that numerically distinct tokens of the same name share membership in a chain of explicit co-reference and numerically distinct tokens of two type distinct names will be members of disjoint chains of explicit co-reference — even if the two tokens are coincidentally co-referential.

There is, then, widespread agreement that Frege's puzzling remarks in the beginning of *On Sense and Reference* have to do with the distinction between identities whose co-reference can be taken for granted, and those others whose co-reference we might have to discover.¹⁴ This distinction,

¹⁴ See also Heck (2012) for a similar assessment. Regardless of accepting that Frege's puzzle is about name coordination (although, of course, he does not use that term), Heck goes on to advance a formal/syntactic solution to it. Whether Frege's puzzle must be solved semantically or syntactically (in other words, whether name coordination is a semantic or a syntactic relation) is

we argue, can be analyzed in terms of our notion of name coordination. Thus, here's a way to reconstruct Frege's argument in a natural setting while avoiding any charge of circularity, and thus escaping from both Glezakos and Paganini's charges:

1. [Initial Puzzle] Differences in the cognitive profiles of co-referential identities correlate with differences in their logical form (an identity sentence is trivial if its logical form is $a = a$, but often informative if of the logical form $a = b$). No difference in informativity without difference in transparent logical properties.

1.5 [Logical Form is Name Coordination] A natural language identity has the logical form $a = a$ iff its name occurrences are coordinated, and $a = b$ if they aren't. No difference in logical form without difference in name coordination.

2. [The Puzzle is Semantic] The logical form of identity sentences supervenes on their semantic form: no difference of logical properties without differences of semantic properties.

2.5 [Name Coordination is Semantic] Whether two name occurrences are coordinated supervenes on their semantic form: no difference in coordination without differences of semantic properties. *[follows from the three previous propositions]*

3. [Postulation of Senses] Therefore, since the referential semantic values of co-referring terms is the same regardless of their logical identity (or difference), the terms must differ in another semantic respect: their senses. An identity sentence is trivial if its two occurrences have the same sense, but often informative if they have distinct senses.

an issue which we remain neutral about. See Gray (2017) for an in-depth exploration of this question.

Logical Form is Name Coordination links Frege's initial remark (Initial Puzzle) with the recently introduced notion of name coordination. The suggestion is that coordination (or lack thereby) plays the (syntactic) role of an in-the-head ideal language in the definition of logical identity (and difference). *Name Coordination is Semantic* follows from Frege's assumption that the solution to the puzzle must be semantic. Finally, Postulation of Senses is now conceived as an attempt to *explain* name coordination. In summary, our reconstruction conceives of Frege's Puzzle (in its natural language manifestation) as that of explaining the relation of name coordination, and shows how Frege's own solution, the postulation of senses, is a coherent candidate.

It should be noted that we are not endorsing the sense-based explanation of name coordination (Postulation of Sense), but merely showing its internal cogency. We are also neutral about whether the solution to the puzzle must be semantic or not - we merely aim to show that Frege's argumentation can be set out in a natural language context in a non-circular.

There is, however, a dangerous threat to this reconstruction of the argument which must be addressed. As Frege's own quotation makes clear, identities that have the form $a = b$ are only *often* informative, not always. But, then, an identity might be constituted by name occurrences that are not coordinated and still be uninformative. Hasn't something gone wrong?

7. UNINFORMATIVE BUT NOT TRIVIAL

Frege, we have seen, said that identity sentences of the form $a = b$ are only *often* informative, not always. Why? Well, the answer is obvious: any informative sentence can only be informative to a subject *once*, namely, before that subject

learns the information the sentence imparts. Even the most informative sentence becomes uninformative after one learns the information it conveys.¹⁵ While this remark seems innocent enough, it could give rise to a particularly pertinent criticism of our reconstruction of Frege's puzzle.

Our view, remember, is that the cognitive difference between identity statements correlates with their name occurrences being coordinated or not. More precisely, we have argued that the syntactic notion of coordination (between the name occurrences of an identity sentence) is sufficient for triviality. However, as we've just noticed, coordination is not *necessary* for triviality. If one knows all about Samuel Clemens' pen names, then one will not learn anything new from the sentence 'Mark Twain is Samuel Clemens', even if the two occurrences in that identity are clearly not coordinated (they subject knows, after all, that they are instances of independently introduced common-currency names with distinct histories of use). These considerations apply just as well to Frege's own celebrated example: surely, no one well informed finds the sentence 'Hesperus is Phosphorus' informative any more.

But this, our objector may tell us, shows that what makes an identity uninformative cannot be the subject's beliefs about its name occurrences, e.g. the fact that these occurrences are not coordinated. Now, everybody agrees that, for any identity sentence a subject might encounter, if she knows that its name occurrences co-refer, then that identity will be uninformative to that subject regardless of there being name coordination between its occurrences or

¹⁵ Just to be sure, in this paper we take no stand on how to characterize the information conveyed by an informative identity sentence. A few examples of authors who do are: Perry's (2001) reflexive propositions, García-Carpintero's (2006) presupposed contents, and Chalmers' (2004) primary intensions.

not. From that, one could claim that the real explanatory factor here seems to be knowledge of co-reference, a type of knowledge that can come about quite independently of any *syntactic* property of the relevant sentence.

Is there a principled way to amend the connection between cognitive profile and name coordination? The distinction needed, we suggest, is that between *trivial* identities and *merely uninformative* ones. The former are those whose truth can be known just on the basis of its syntactic form. The latter, on the contrary, are those whose truth is known by a subject, but where her knowledge is based on facts which are extrinsic to the syntax of the identity itself. Intuitively, the difference that we are highlighting is that between an explicit self-identity 'Mark Twain is identical to *himself*' and an identity - already known to be true by the relevant subject - which contains two name occurrences which are not coordinated, such as 'Mark Twain is Samuel Clemens' as heard by a literary scholar.

The distinction between trivial and merely uninformative identities is intuitive enough. But is it principled? Are there really intrinsically trivial identities? Some think not (Almog 2008, p. 567):

[...] informativeness lies rather in a relation between (1) the background information we have and (2) the target judgment. If the information we have resolves the truth of the judgment, it is uninformative; if not, it is informative. The informativeness is thus not intrinsic to the identity judgment. The informativeness rests in whether the in-the-head information I do have can settle the truth value of the undecided judgment.

This view of informativity (and lack thereby) stems from the contention that, outside regimented formal languages, there is no garden variety of logical form that is primitively transparent in the relevant sense, i.e. that Initial Puzzle itself

must be rejected. As Almog puts it, “‘a = a’ and ‘a = b’ are not sentences encountered outside a logic (or algebra or group theory etc.) textbook; no one on the Santa Monica Beach or on CNN asserts the schematic sentence ‘a = b’” (Almog, p. 553). If Almog is right, then our whole project of trying to associate the cognitive profile of identity sentences with coordination among name occurrences would be bound to fail.

Can the distinction between trivial identities and merely uninformative ones be motivated without begging the question? We think it can. Drawing from an argument by Campbell (1987), in the next section we make a few considerations which point in this direction. We think that the distinction between trivial identities (constituted by coordinated name occurrences) and merely uninformative ones (constituted by name occurrences which are not coordinated but are known to co-refer) should be acknowledged by authors of any persuasion. To see why this is so, consider the following argument.

8. TRADING ON IDENTITY

Suppose that thinkers were always uncertain about whether two occurrences are instances of the same common-currency name or not. In other words, suppose no name occurrences were ever coordinated in our sense. This would give rise to a peculiar skeptical scenario where thinkers could always reasonably wonder whether distinct name occurrences co-refer with each other or not. If this were the case any inference of the following structure - where superscripts serve to differentiate between the (uncoordinated) occurrences - would come out as being invalid:

- I. Descartes¹ is French
- II. Descartes² is a philosopher
- III. Thus, there exists at least one French philosopher

This inference would be invalid because the two occurrences of 'Descartes' are not coordinated, and thus they could be - for all that the subject knows or believes - instances of independent common-currency names. Since they could be instances of independent names, they could very well fail to co-refer. Thus, the subject is not rationally allowed to draw a unifying inference (III) without first certifying that the two occurrences of 'Descartes' co-refer. This fact, however, cannot be expressed by appending a disambiguating premise 'Descartes¹ = Descartes²'. Superscripts, we said, track name *occurrences*, not types. Since the superscripts 1 and 2 have already been "used up", the inference should be represented as follows:

- I. Descartes¹ is French
- II. Descartes² is a philosopher
- III. Descartes³ = Descartes⁴
- IV. Thus, there exists at least French philosopher

But, wait: it is obvious that this solution would give rise to the same problem as before. Premise III would only make the inference valid if the subject was somehow sure that its name occurrences are instances of the same common-currency names as the two previous premises. However, we are assuming that no name occurrences are ever coordinated - so, for all that the subject knows, 'Descartes³' and 'Descartes⁴' might be instances of common-currency names that have nothing to do with 'Descartes¹' and 'Descartes²'.

One could, of course, add even more identity premises producing even more occurrences, but it should be obvious that one would quickly find oneself in the same predicament:

absent a relation of name coordination between occurrences, the inference will always fall short of validity. The sheer possibility of reasoning in a way that is transparently valid would, consequently, be led into a regress.

The moral that Campbell (1987, p. 276) influentially extracted from these observations was that “we need an account of when an inference may simply trade upon the co-reference of two singular tokens” if the possibility of singular reasoning is to be established at all. In other words, thinkers must, at some point, be able to treat distinct occurrences as being *primitively* “of the same name” without need of additional information. The emphasis on ‘primitively’ is important. If a thinker treats distinct occurrences (in an inference) as being co-referential only on the basis of a further belief to the effect that these tokens refer to the same thing, one would then still need to explain how come the occurrences in that further belief are connected to the occurrences in the premises of the inference.

Thinkers are able to treat two distinct name occurrences as primitively co-referential when these occurrences are coordinated. Coordination then supervenes on the basic competence that subjects have of trading upon the identity of certain occurrences; a competence which is, furthermore, conceptually required by the mere idea that singular reasoning can sometimes be *transparently* valid. The link between the transparency of thoughts and the rationality of agents is aptly expressed by MacFarlane (2004, p. 21): “we require logical validity to be formal because we require it to be transparent, and we require it to be transparent because of the reasons and responsibilities to which it gives rise.”

These observations show that there must be a garden variety of *transparent* logical identity that is not conceptually dependent on the logic textbook formal notion of validity. On the contrary, it is such natural language, pre-theoretic notion of transparent validity which constitutes the

conceptual ground of the abstract formal notion. As Shapiro (2005, p. 669) put it: “neither proof-theoretic consequence nor model-theoretic consequence is primary. Instead, they illuminate the various informal, pre-theoretic notions of logical consequence”.¹⁶

It will be remembered that our proposal is to distinguish *trivial* identities (those expressed using coordinated terms) from *merely uninformative* ones - those that are uninformative only because “the background information we have [...] resolves the truth of the judgement” (Almog, *ibid.*). Only the former type of identities, we argue, is of the form $a = a$. Almog, we have seen, claimed that there are no natural language examples of the ‘ $a = a$ ’ form. If he is right, our account of triviality is doomed. The above detour through the notion of transparently valid inferences, we hope, will suffice to convince the reader that this is not true. Even in a day-to-day conversation in the street one must be able to detect arguments that are *transparently* valid. If the analysis that we have offered in this paper is correct, this means that speakers of natural languages must be able to detect when two names are (intended to be) logically identical. And we have argued that what tracks logical identity at the syntactic level is our notion of name coordination.

¹⁶ Moreover, the same primitive cognitive capacity of trading upon the identity of certain tokens grounds both the syntactic notion of coordination *and* the notion of logical identity, which therefore coincide: the notion of logical identity as we have defined it is necessarily co-extensive with the syntactic notion of coordination, since they are both grounded on the same capacity to “trade on identity”.

9. CONCLUSION

Our starting point was Frege's original suggestion that the informativity of an identity sentence correlates with its *logical* form. This idea proved difficult to apply in a natural language context. As we have seen, the sheer idea that natural language sentences have a logical form (in the sense that is necessary to pose Frege's puzzle) is not without issue. We made several attempts to come up with a criterion for assessing a natural language identity's form - typographical form, sameness of generic name, sameness of common-currency name, sameness of private name - but none worked. Finally, we found a suitable candidate: a natural language identity is of the form $a = a$ when its name occurrences are coordinated, and $a = b$ otherwise.

Name coordination then allowed us to pose Frege's puzzle in the context of natural languages. The puzzling predicament is precisely explaining how it can be the case that some identities have coordinated occurrences while others do not. Given that understanding of the puzzle, Frege's semantic solution can be seen to avoid circularity. It might not be the best solution to the puzzle, of course, but it does not, we showed, beg any questions.

In the final sections of the paper, we noted that name coordination seemed to only imperfectly track cognitive profile. There are, after all, uninformative identities whose name occurrences are not coordinated. In order to clear things out, we argued that we should not confuse trivial identities with merely uninformative ones. Trivial identities are precisely those that Frege was concerned with, i.e. the ones he would ascribe a logical form $a = a$. These have coordinated occurrences and thus can be known to be true just in virtue of their syntax. Merely uninformative ones, on the other hand, are those that contain non coordinated occurrences which are known to co-refer (on extrinsic

grounds). While these identities would not teach anything new to a subject who knows about their occurrences' co-reference, their unformativity does not follow from their syntax - it had to be learned from other means.

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