CDD: 142.7

HUSSERL'S CONCEPTION OF LOGIC

JAIRO JOSÉ DA SILVA

Department of Mathematics,
State University Paulista (UNESP),
Campus de Rio Claro,
Av. 24-A, 1515, Jd. B. Vista,
13506-900 RIO CLARO, SP
BRAZIL

jairomat@linkway.com.br

This paper presents and discusses Husserl conception of logic, formal logic in particular. A special emphasis is giving to Husserl's idea of a theory of manifolds as the closure of the thematic field of formal logic. Husserl's own version of logicism in the philosophy of mathematics is also presented and some aspects of his conception of formal logic are highlighted and contrasted with Frege's.

This paper deals with the concept of logic, formal logic in particular, in the philosophy of Edmund Husserl. Although a contemporary of Frege, with whom he exchanged a few letters, shared a common agenda of logical and philosophical problems and maintained a brief polemic over the philosophy of arithmetic and related issues of a logical nature, such as the definition of identity and the theory of abstraction (see Hill (1991)), Husserl's conception of logic is by far less known and still less influential than Frege's. Despite the fact that Husserl's attack on psychologism in logic, which constitutes most of the *Prolegomena* to his *Logical Investigations* (henceforth *LI*), is far more elaborated and detailed than Frege's.

But Frege was not the only major logician of the time with whom Husserl engaged in dialogue. There is hardly a logician of any importance of last century or the first decades of this century whom Husserl does not mention and often discuss extensively. Schröder, Jevons, Wundt, Boole, Peirce, Voigt, Marty, Bergmann, are only some of the logicians whose work Husserl knew and discussed in print. Moreover, after moving to Göttingen in 1901, Husserl entered the circle of Hilbert and there was certainly some mutual influence between them, whose full extent is still to be measured. If for nothing else the place and time in which Husserl elaborated his conception of logic, and the intellectual weight of the logicians he engaged in dialogue with, are already reasons enough to recommend a study of this somehow neglected chapter of modern logic.

What I intend to do here is little more than introduce the reader to this chapter, and a natural starting point for it is Husserl's theory of denotation for statements. Whereas for both Frege and Husserl the sense of a statement is the thought (or the proposition as Husserl preferred to call it) it expresses, the denotation of statements is, for them, a matter of disagreement. Unlike Frege, for whom statements denote truth values, Husserl prefers to embrace the certainly more intuitive point of view that they denote states of affairs (*Sachverhalten*), which is how Husserl calls complex objective situations constituted by objects standing in determinate relations to each other. States of affairs are examples of what Husserl calls categorial objectualities[1], which are entities that, albeit fully objective, require the action of consciousness for

---

[1] The neologism "objectuality" translates Husserl's technical term *Gegenstand*, which has a specific phenomenological meaning, namely the objective correlate of an intentional experience. The term *Objekt*, translated by "object", means simply the object in the natural, pre-phenomenological attitude, i.e. that which is fundamentally different from the subject who knows it.

their constitution[2]. In the pursuit of knowledge consciousness provides a given, pre-categorial material with a (logical) form, i.e. a particular formal "structure" which makes it into a state of affairs. Husserl calls this pre-categorial basic material that provides the basic "stuff" for a state of affairs, but not only one state of affairs, a situation of affairs (*Sachlage*). We can think of states of affairs as situations of affairs plus logical forms, these forms being the imprint of consciousness on situations of affairs. Situations of affairs are considered as pre-categorial objectualities not because consciousness does not have a role to play in their constitution, but because they are passively, rather than actively constituted by consciousness[3].

For example, "3<5" and "5>3" are two different expressions that denote two different, although obviously related, states of affairs. According to Husserl both of them originate from the same situation of affairs, which we nonetheless cannot accurately describe. Situations of affairs have the curious property of being somehow ineffable. They cannot be perfectly expressed by propositions, for propositions express only states of affairs, hence they cannot be expressed at all, for there are no other means to express them but propositions. We can think of states of affairs as situations of affairs "seen" from a certain perspective. Since situations of affairs can only be seen from a certain particular perspective,

[2] The notion of constitution is a central and difficult notion of Husserl's philosophy, in broad terms it can be understood as "the dispensing of meaning in pure consciousness". Nonetheless, the notion of constitution cannot be interpreted in the context of subjective idealism (see, in particular, *Logical Investigations*, §55.) The fact that an objectuality requires for its constitution the action of consciousness does not imply that its reality is confined to subjectivity. In fact, objective reality itself demands complex series of constitutive acts.

[3] Active and passive constitution of objectualities, such as, for instance, sets, is a topic extensively dealt with in Husserl's *Experience and Judgement* (*Erfahrung und Urteil*, 1939.)

there is no way we can see a situation of affairs previously to its embodiment as a state of affairs. A situation of affairs is some sort of common nucleus of equivalent states of affairs, although we must resist the temptation of making it into an *abstractum* obtained from equivalent states of affairs. States of affairs presuppose situations of affairs, not the opposite. Husserl considered this distinction between states and situations of affairs relevant for certain questions in the philosophy of science, such as for instance the problem raised by different although equivalent formulations of the same physical laws[4].

But I am not concerned here with Husserl's semantics, or even with his theory of meaning, but rather with Husserl's logic. For him logic is, in its most general sense, the theory of science[5], which is essentially an architectural arrangement of propositions (to use Husserl's metaphor). By this he means that the form *par excellence* of science is that of a theory, in which propositions follow deductively one from some others, and ultimately from first principles. Therefore, Husserl believes, logic cannot be indifferent to the fact that propositions denote states of affairs rather than truth values.

Logic, as the theory of science, Husserl thinks, must be concerned not only with propositions, how they can be formed so as to fulfill their task of denoting a possible state of affairs, i.e. so as to be meaningful, how they can be combined into a single meaningful enunciation, how they can be combined so as to preserve

---

[4] Rosado Haddock also saw the possibility of applying this distinction to the philosophy of mathematics. See, for instance, his "Husserl's Relevance for the Philosophy and Foundations of Mathematics" (Rosado Haddock (1997)).

[5] Logic is "a universal a priori doctrine of science. It [must] be concerned with the formal – in the widest sense – a priori character of all sciences as such, that is it [must] be concerned with what unifies them in a priori generality, with that to which they are necessarily connected to the extent that they are truly sciences" (*FTL* §55.)

their truth, or how they can be validly obtained from given assumptions, but also, and as emphatically, with objects. But only insofar as these objects are completely indeterminate with respect to their nature and are thought exclusively as the basic stuff on which logical forms are imprinted. We may say that, for Husserl, objects interest logic only insofar as they are the basic constitutive elements of states of affairs, which interest logic only insofar as they are bearers of logical forms. Or better still, logic is interested only in the *concept* of object and all that can be said about it *a priori*.

One aspect of the science of logic is, for Husserl, formal logic, whose task is the study of the formal laws concerning, on the one hand, propositions and theories and, on the other, their objective correlates, i.e. the objects, states-of-affairs and manifolds (the objective correlates of theories) they refer to. Accordingly, Husserl distinguishes two major sub-domains of formal logic, that concerned with propositions and theories, which I will call the logic of propositions, and that concerned with objects conceived in utmost generality, objects though only as instances of the concept of "something whatsoever" (*Etwas-überhaupt*) and manifolds, which Husserl calls formal ontology.

For Husserl formal logic is an *a priori* science, and as such it can only be a conceptual, as opposed to a factual, science. Therefore, both the logic of propositions and formal ontology have concepts as objects of study. Husserl calls the concepts each of them is concerned with, respectively, meaning categories (for the logic of propositions) and object categories (for formal ontology). Some examples of meaning categories are the concepts of name, concept, proposition, the concept of meaning itself, etc.; examples of object categories are the concepts of object, property, relation, order, collection, combination, state of affairs, whole and part, number, etc.

Husserl distinguishes three levels of increasing generality and abstractness within each domain of formal logic. The following schema gives a brief characterization of each of them:

The logic of propositions, whose ruling concept is the concept of meaning, has to do with the meaning categories, and can be divided in the following three levels:

The first level, the level of so-called morphology, has the task of identifying the basic (syntactic) categories in which the basic elements of meaningful propositions fall, the so-called categories of meaning, and establishing the formal laws that regulate the composition of elements of these categories to form complex propositions with (grammatical) sense. Husserl calls this theory pure grammar, its goal is to establish the *laws of formation* of meaningful assertions. However, these laws are not designed to prevent formal counter-sense such as "Socrates is a man and Socrates is not a man", they are designed only to prevent senselessness such as "is a man not".

The second level, which Husserl sometimes calls apophansis, has the task of investigating the basic formal laws and theories referring to the fundamental categories of meaning. It includes the investigation of the laws that prevent counter-sense (i.e. formal inconsistency) of complex compound propositions and the laws that guarantee the preservation of a unitary meaning (i.e. consistency) for theories. In other words, the task of apophansis is to investigate the formal laws that guarantee "objective validity" (*LI, Prolegomena* §68) for propositions and theories. It includes the establishment of "logical laws" such as *tertium non datur* or the law of non-contradiction and also a theory of formal deduction, i.e. a theory whose task is to establish *laws of transformation* or laws of formal derivation. Traditional syllogistic belongs to this level of formal logic[6].

[6] Husserl says, in *FTL* §14, that this level of formal logic also includes what he calls the 'analysis' of formal mathematics. Husserl is here draw-

In *Formal and Transcendental Logic* (*FTL*) Husserl distinguishes two levels inside apophansis: the logic of non-contradiction, or consequence logic, and the logic of truth. The latter provides us with the formal *a priori* laws pertaining to the *concept* of truth, which includes the laws that guarantee the preservation of truth in complex propositions and formal derivations. The former gives us the laws regulating exclusively the constitution and preservation of meaning. Logical laws such as *tertium non datur* or the law of non-contradiction and logical principles, such as *modus ponens* and *modus tollens*, have different formulations, and ultimately different justifications, in the logic of consequence and in truth logic. In the former these laws and principles are formulated so as to guarantee exclusively the preservation of meaning for composite propositions and theories; in the latter they are formulated so as to guarantee the preservation of truth.

The third level includes the theory of deductive systems considered exclusively from the perspective of their form (the theory of possible forms of theory). Its task is to investigate "the essential classes (or forms) of theories and their corresponding laws of relation" (*LI, Prolegomena* §69). Supposedly, as can be inferred from some of Husserl's own investigations, it also includes at least some aspects of metamathematics, such as the study of properties of theories, such as completeness. This level includes the theory of the *concept* of theory. I will come back to this level of the formal logic of propositions soon, now let me give an sketch of the corresponding levels of formal ontology.

---

ing our attention to a subtle distinction. We can, by a change of focus, consider a formal mathematical theory, such as formal mathematical analysis, from two complementary perspectives, either as a theory teleologically concerned with objects, or as a theory concerned exclusively with propositions considered as pure senses. From the former perspective a formal theory belongs to formal ontology, but from the latter it belongs to apophansis.

 XXII(2), pp. 367-397, October.

Formal ontology, whose main concept is the concept of object in its utmost generality (*Etwas-überhaupt*), has do to with the object categories, and can also be stratified in three levels:

The first level corresponds to a correlate of morphology of the logic of propositions. Its task is to identify the basic categories in which the fundamental "building blocks" of possible objective states of affairs fall, and establish the formal laws according to which these "blocks" can be combined in order to form complex objective state of affairs. These laws are the objective correlate of the formation rules of pure grammar and can be obtained simply by a change of focus, i.e. a shift of attention from propositions to their correlate states of affairs.

The second level, the correlate of apophansis, has the task of establishing the fundamental laws that pertain to the basic object categories (discerned in the previous level) and develop their theories. This level includes most of formal mathematics, such as set theory and arithmetic. At this level the objective categories are themselves objects of study.

The third level of formal ontology comprehends the theory of manifolds. Its task is to investigate manifolds, the objective correlates of formal non-interpreted axiomatic systems. From the standpoint of present day knowledge, I believe, we can include here not only all the particular formal theories of manifolds, the so-called abstract algebras, but universal algebra and some aspects of metamathematics, such as model theory, as well.

Husserl's conception of logic has, as can be seen from merely glancing at the above schema, remarkably distinctive features. Let's point out some of them. The most conspicuous seems to be Husserl's inclusion of ontology, even if only as formal ontology, within the scope of formal logic. This inclusion of formal ontology into logic can go as far as to be a complete submission of the latter to the former. Due to the strict correlation between categories of meaning and categories of object, which induces a similar correspondence between the logic of propositions and formal ontology, we can, as Husserl noticed (*Hua XXIV*- pp.51-4), conceive the whole of pure formal logic as formal ontology.

Formal ontology is, in few words, the totality of things that can be said about objects considering *exclusively* that they are objects. For instance, objects are able to stand in relation to each other and objects can be operated upon so as to produce other objects. These relations and operations being thought of as indeterminate relations and operations satisfying only certain specific formal properties, such as commutativity or associativity. Therefore the study of the so-called formal manifolds, i.e. domains of completely non-specified objects subject to equally non-specified relations and operations, which are supposed only to satisfy certain formal properties, is part of formal ontology.

Objects can be collected into sets, hence set theory, the study of the formal properties and conceptual laws concerning sets, is also part of formal ontology, and *a fortiori* formal logic. Notice that the inclusion of set theory within the realm of formal logic has different rationales in Husserl and Frege. The fact that sets are logical *objects*, for they are obtained by logical abstraction from concepts, justifies for Frege the inclusion of set theory into logic; for Husserl, set theory, which is the theory of the *concept* of set, belongs to logic simply because the concept of set has universal applicability, i.e. anything whatsoever can be collected into a set.

Since the whole of formal mathematics falls within the range of formal ontology, Husserl manages in one sweeping move to bring the totality of formal mathematics into the domain of formal logic. Not only arithmetic, the formal theory of the *concept* of number, or set theory, but also the theory of any particular formal manifold, conceived exclusively as a non-specified domain of objects subjected to non-specified operations and relations satisfying only certain particular properties, such as group theory or the theory of Riemann continuous manifolds, is part of logic. Any manifold is in fact a particular concept which does not discrimi-

 XXII(2), pp. 367-397, October.

nate the objects that fall under it with respect to their nature. Any collection of objects can, for instance, constitute a group, this is what qualifies the concept of group as a logical concept. So, the study of a manifold is the study of a concept. Consider the following quotation:

> The definition of a manifold determines a concept of manifold, a species of manifold, and represents a manifold in general as an object of this concept; or better: a concept of classes for manifolds or a species (*Hua XII* p. 493).

It is precisely the universal applicability of this concept that qualifies it as a logical concept.

The criterion of universal applicability is also responsible for the elimination of certain mathematical disciplines from the realm of formal logic. Geometry, for instance, as the mathematical theory of physical space, or better the mathematical theory of a mathematical surrogate of physical space is not a logical science. The reason is that the concept of space does not have universal application. But if we take geometry and eliminate its reference to physical space, thus obtaining a formal theory only formally identical with geometry (a theory equiform to geometry, as Husserl sometimes calls it), we are back within logic.

Since Frege's conception of logic does not make room for any type of ontology, being limited to a logic of propositions, the reduction of arithmetic to logic demanded of him painstaking efforts in order to show that arithmetical objects were nothing but logical objects and that arithmetical assertions were reducible to logical assertions. This process deformed arithmetical assertions and objects beyond recognition. I believe that anyone will agree that to think of the number one, for instance, as an infinite class abstracted from a concept is far less intuitive than thinking of it as a concept applicable to arbitrary singletons containing, each of them, an arbitrary object, as Husserl did[7]. So, Husserl's conception of logic seems more appropriate than Frege's for a more "natural" version of the logicist thesis.

Formal mathematics contains more than simply theories of formal manifolds, it also contains their metatheories. We may be interested in how different formal theories, or how different manifolds, the objective correlate of these theories, relate to each other. Husserl provides an entire domain of formal logic to accommodate these inquiries. I will have more to say about this domain shortly.

Another interesting characteristic of Husserl's conception of logic is its stratification. Each level corresponds to a higher degree of generalization and abstraction. On the level of morphology we are only concerned with the establishment of basic categories and their correlation. On the subsequent level we are concerned with the theories of these categories. On the superior level we are concerned with the theories of these theories. In particular this seems to point towards a clear distinction between theory and metatheory in Husserl's conception of logic.

Contrary to what may seem at first sight, the dichotomy between the logic of propositions and formal ontology cannot be construed as a dichotomy between syntax and semantic. Within each domain of formal logic, the logic of propositions or formal ontology, there are syntactical and semantic notions in play. For instance, the study of a formal manifold, a task for formal ontology, is nothing but the study of a particular formal system of axi-

---

[7] The nature of numerical attributions is another important point of disagreement between Frege and Husserl. For the former, numbers are attributes of concepts, for the latter they are attributes of collections, not necessarily conceived as extensions of concepts. Husserl criticized Frege's theory of logical abstraction of *Foundations of Arithmetic* in his *Philosophie der Arithmetik* (*PA*, 1891), Frege, in turn, criticized Husserl's theory of abstraction of *PA* in his review of 1894, as an example of the psychological theories of abstraction he abhorred (Frege (1894)).

oms, which is purely syntactic. After all, there is no other way to approach a formal manifold but through the system of axioms that defines it. Also the study of the notion of truth, a semantic notion, is a task for truth logic, a branch of the logic of propositions.

But, although still not clearly drawn, a distinction between a syntactic and a semantic approach to logic is, at least potentially already detectable in Husserl's conception of logic. For instance, the distinction drawn in *FTL* between consequence logic, which includes a logical theory of deduction, and truth logic, the logical theory of truth, or rather the logical theory of the purely formal notion of truth, although far from presenting a clearcut separation between syntax and semantics, at least points in the right direction.

Another relevant aspect of Husserl's conception of logic is the inclusion of a theory of manifolds. Husserl's original term for this theory is *Mannigfaltigkeitslehre,* defined as the study of manifolds, the objective correlate of formal theories. Depending on the focus, on senses or objects, the theory of manifolds includes both a theory of purely formal deductive systems, as a part of the formal logic of propositions, and a theory of manifolds properly speaking, the objective correlate of formal deductive systems, as a sub-area of formal ontology. This domain of formal logic deserves a special attention, so I devote the next section to it.

1. THE CONCEPT OF MANIFOLD AND THE THEORY OF MANIFOLDS

A formal theory is, for Husserl, only in intention a theory, for they lack what characterizes proper theories, the reference to a completely determinate domain of objects. A formal theory is, for Husserl, nothing but the formal "skeleton" of a possible theory. Nonetheless, Husserl believes, formal theories still "intend" (if consistent) to describe domains of objects, which are, however, left completely indeterminate with respect to the nature of their elements. These domains are determinate only with respect to their forms by precisely the formal theories they are correlated with. A formal manifold (*Mannigfaltigkeit*) is what Husserl calls, borrowing from mathematical terminology, the objective correlate of a formal theory, the domain of indeterminate elements conceived exclusively as a domain regulated by a theory having this form.

> The *objective correlate* of the concept of a possible theory defined only by its form is the concept of a *possible domain of knowledge that must be governed by a theory of such a form.* The mathematician calls ... such a domain a manifold (*LI Prolegomena* § 70).

The first systematic study of mathematical manifolds appears with Riemann in "On the Hypotheses which Lie at the Foundations of Geometry" (1854). In this work the concept of manifold is introduced as a generalization of the concept of space. Since geometry is Riemann's first concern, he considers only continuous manifolds. The concept of manifold generalizes the notion of space in two ways. Firstly, the dimension of a manifold can be any natural number, secondly a manifold "is susceptible of various metric relations [and] space [...] constitutes only a particular case of a triply extended magnitude" (*op. cit.*, p. 413). This notion constitutes a powerful mathematical instrument for it allows the mathematician, among other things, the simultaneous investigation and the systematic organization of various different notions of "space" and their corresponding "geometries".

This aspect of the Riemannian theory of manifolds impressed Husserl particularly. In *LI Prolegomena* §70 he mentions Riemann and the theory of *n*-dimensional Euclidean and non-Euclidean manifolds explicitly and says:

> From the manner in which the different types of manifolds that are similar to space can be transformed one into another by means of a variation of the degree of curvature, the philosopher who knows the first principles of the theory of Riemann-Helmholtz can conceive how the pure forms of theory which belong to types that present marked differences are united by a law.

It is clear from the above quotation that Husserl attributes to his *Mannigfaltigkeitslehre* a task similar to that accomplished by Riemann with respect to formal manifolds obtained by generalization of the concept of space. Moreover, we can safely suppose, I believe, that the investigation of particular formal manifolds is for Husserl also a task for the theory of manifolds. That is, the task of investigating formal manifolds, and ideally all *possible* formal manifolds, as well as the investigation of the a priori laws concerning their mutual relations, belongs to the theory of manifolds, as a province of formal logic.

If we generalize Riemann's concept of manifold in order to take into consideration not only continuous quantities but any collection, no matter of what cardinality, discrete or continuous, of elements whose particular nature is always left out of consideration, in which we define relations and operations only formally by certain "axiomatic forms"[8], we have the present day concept of a (formal) mathematical structure. The concepts of ring, group, module, vector space, topological space, and the like are different notions falling all under the common concept of a mathematical structure. That is, any particular notion of a mathematical structure is itself a concept under which many different, non-isomorphic structured domains of objects fall. For instance, many different collections of objects, in which many different binary operations can be defined can all fall under the concept of group, provided these operations satisfy certain formal properties. Any mathematical structure, conceived not as a particular structured

[8] Husserl reserves the designation of "axiom" for certain "laws of essential being" pertaining to concepts that apply to objects properly speaking, i.e. axioms are true assertions about a given concept. He prefers the expression "forms of axioms" to refer to the non-interpreted basic expressions that characterize a formal theory, i.e. the collection of formal expressions from which all the assertions of the formal theory follow analytically. These forms of axioms can be seen as an implicit definition of the relations and operations of the formal manifold in question.

domain of specified objects, but as a form, or still a concept, is an example of what Husserl calls a formal manifold.

To investigate a given mathematical structure, that is, to develop its theory, means to derive, not necessarily in an explicitly given formal language, the consequences of the (non-interpreted) axiomatic forms that more or less arbitrarily define this structure, in the sense of being the statements that implicitly characterize this particular structure and the only ones that can be assumed without proof. So, to investigate a mathematical structure, that is, to develop its theory, means simply to derive in a systematic way all the purely formal consequences of the axiomatic forms that characterize this structure.

Consequently, we cannot identify Husserl's notion of a formal manifold with the present day notion of a model for a formal theory. A formal theory can have many non-isomorphic models, but it determines only one formal manifold. Any model for a formal theory can be investigated by means other than those provided by this theory, but the formal manifold determined by a formal theory cannot be investigated independently of this theory. Moreover, a model is a domain of specified objects, operations and relations, a formal manifold is a domain of purely formal objects, operations and relations characterized only by certain formal properties.

Despite of this intimate correlation between a formal theory and the manifold it determines, so as to make it impossible to investigate one independently of the other, Husserl insists on distinguishing these two notions carefully. His reason for this is to point out the fact that formal theories and manifolds correspond to different, although intimately correlated notions. A formal theory is, in a sense, a complex formal proposition, and a formal manifold is, in this sense, the complex formal state of affairs this complex proposition denotes. One can develop a formal theory in a mechanical way, obtaining by chains of formal deductions all the necessary analytic consequences of the (forms of) axioms that characterize this theory, but by doing so, Husserl believes, one does not necessarily do science (and we can extract from this observation a criticism of rules-of-the-game interpretation of the Hilbertian variety of formalism). Science is concerned with knowledge and as such its interest is ultimately focused on objects. Therefore the study of formal systems of axioms is teleologically oriented towards objective domains, even though they are kept completely indeterminate as to their nature and conceived only as contentless formal manifolds regulated by the formal theories under study.

The distinction between a formal theory and its manifold serves, in Husserl's thought, an epistemological purpose, it emphasizes the fact that any theory, formal or not, is always directed towards objects. The development of a formal theory in itself, independently of any possible domain of objects described by a theory having the form this formal theory circumscribes, is, for Husserl, not a task for formal logic, which is and must remain orientated towards epistemology.

Considering the strict correspondence between a formal theory and the formal manifold it determines, the theory of manifolds must also be conceived as the theory of all *possible* formal theories (which Husserl sometimes calls forms of theory, in order to emphasize their dual aspect of *forms* and forms *of* theories) and their mutual correlation. A deductive theory, the only form of theory that interests logic, either interpreted or non-interpreted, is characterized as a "architectural" structure of propositions logically connected among them and logically founded on self-evident axioms, or forms of axioms in the case of formal theories. Hence a theory of possible forms of theory must be primarily a theory of deductive systems.

Husserl says:

> [The] distinct forms [of theory] do not lack mutual relation. There must be an order of proceeding according to which we can construct the possible forms [of theory], determine their

> regular connections, and eventually convert one into another by varying certain fundamental determining factors, etc. There must be, if not in general at least for the forms belonging to certain determined classes, general principles that rule within the prescribed limits the regular genealogy, combination and transformation of these forms (*LI Prolegomena* §69)

Husserl is here setting a goal for the theory of manifolds, understood now, by a shift of focus, as the theory of all possible forms of theory. Namely, to generalize for the theory of deductive systems what Riemann accomplished for the theory of continuous manifolds. What Husserl has in mind, I believe, are the "family ties" the formal theories which mathematicians create have among them. For instance, the formal theory of Euclidean *n*-dimensional manifolds is derived by generalization from axiomatic geometry, the mathematical theory of 3-dimensional physical space (or rather its mathematical surrogate). Some theories of the so-called abstract algebra (field theory in particular) are equally derived from the axiomatic theory of real numbers. The theory of topological vectors spaces is created by the combination of two independent theories, the theory of vector spaces and the theory of topological spaces. Another example, particularly important for Husserl for it offered him the key to the solution to the problem of imaginary entities in mathematics, is the creation of the theory of imaginary numbers from a suitable extension of a suitable theory of the real numbers. For Husserl, the mutual relations formal mathematical theories display must obey some general principles which demand theoretical investigation. This is one of the tasks he assigns to the *Mannigfaltigkeitslehre*.

Incidentally, this task bears a close resemblance to the approach to mathematics characteristic of the Bourbaki group. According to Bourbaki, mathematics is nothing but the study of formal theories, or, correlatively, mathematical structures, some of which are fundamental or "mother" structures from which other structures are generated in some uniform way (see Rosado Had-

 XXII(2), pp. 367-397, October.

dock's "Husserl's Relevance for the Philosophy and Foundations of Mathematics", Rosado Haddock (1997)).

The theory of manifolds, considered either as the theory of formal objective domains or, correlatively, as the theory of formal deductive systems, is itself, according to Husserl, a mathematical theory, in fact the highest achievement of mathematics. Consider the following quotations:

> Mathematics is, in the highest and widest sense, the science of theoretical systems in general, abstracting from what is being theorized about in given theories of different sciences (*Hua XII*, p. 430).

> Mathematics is, in its highest ideal, a theory of theories, the most general science of possible systems in general (*Hua XII*, p. 431).

As already noticed, it is not easy to draw a line separating the theory of (formal) deductive systems from the theory of (formal) manifolds properly speaking, as these theories are presented in either *LI* or *FTL*. This is a consequence of the fact that in these texts Husserl defines a formal manifold simply as the objective correlate of a formal deductive theory. But in some earlier drafts Husserl prepared for a series of two talks he gave in Göttingen in 1901 and related texts of the same period, Husserl presents a more restricted notion of a formal manifold. This restricted notion was required by the problem of imaginary numbers in mathematics, a problem that had been in his mind since 1890.

At that time Husserl conceived the formal domain associated with a formal axiomatic system as the domain of all formal (materially indeterminate) objects[9] that can be *singularized* by this

[9] A formal object is an "anything whatsoever (*Etwas-überhaupt*)" determined exclusively in terms of formal operations and relations with other formal objects. It is, so to speak, a mere *locus* determined only by its formal properties vis-à-vis other equally empty *loci*.

system. That is, all the (formal) objects that can be denoted or described by the system in question (see *Hua XII* pp.430-88). So, no element can figure in a formal manifold if not *explicitly* required to be there by the axioms of the system. Let's call this the restricted notion of formal manifold, as opposed to the general notion of formal manifold I have been discussing, a notion that does not require that the elements of a formal manifold must be singularized by the system whose correlate they are, provided that they stand in formal relations that obey the formal properties designed by the formal system in question.

Either notion of formal manifold naturally raises a fundamental question: can we conceive a formal manifold that is completely determinate by its theory, in the sense that this theory needs no further extension in order to determine its manifold, so that nothing "remains open" in it? Husserl's answer is "yes", but depending on which notion of manifold we choose, the general or the restricted, the notion of a complete or definite ("*definit*" is Husserl's own term) formal manifold admits two variants: (1) if we understand by a formal manifold simply the correlate of a formal system of axioms, a definite or complete manifold is simply the correlate of a complete system of axioms, that is, a manifold is completely determined provided that its theory can decide any question that can be raised about the manifold, (2) if we understand the notion of formal manifold according to the earlier characterization, a definite or complete manifold is the correlate of a formal theory that is not only completely described *but also completely determined* by this theory. In other words, a definite manifold is, in this sense, a domain that is maximal with respect to the system of axioms that determines it, i.e. a domain that cannot be enlarged and still be described by the same system, as well as a domain for which any question referring *exclusively* to the objects of the domain can be answered by the system in question[10].

When Husserl abandoned the notion of a manifold as the domain of (formal) objects that are singularized by a system of axioms in favor of the notion of a domain as simply the objective correlate of the system in question, thus admitting that some objects of the domain may not be required to exist by the formal system, provided that they satisfy the formal laws prescribed by the system, the restrict notion of definiteness disappeared from his texts. A formal manifold conceived simply as the objective correlate of a formal system is definite, from *LI* on, when it is completely described by the system in question, i.e. when any question that can be raised about it, provided that this question can be expressed in the language in which this system is written, can also be answered by the system. This is tantamount to saying that a formal manifold is definite simply when its formal theory, that is, the theory whose correlate this manifold is, is complete[11].

---

[10] This distinction is useful in understanding Husserl's claim that the notion of definiteness, as he understands it, is correlated with the notion of completeness involved in Hilbert's axiom of completeness for geometry and the theory of real numbers (cf. *Ideas* §72). However, Husserl does not discuss explicitly how we are supposed to accomplish the "aboutness" that his restrict notion of definiteness requires, i.e. how we can tell apart the statements *referring exclusively to the domain of a theory* from the collection of all statement of the language in which this theory is written. In my paper "Husserl's two notions of completeness" (to appear) I discuss this problem.

[11] A complete formal theory determines *ipso facto* a definite formal manifold. There is in fact no real difference between the "syntactic" notion of completeness for the theory and the "semantic" notion of definiteness for its corresponding manifold. There is, for Husserl, only a difference of perspective between these two notions. More precisely, the notion of syntactic completeness (for formal deductive systems) and semantic definiteness (for formal manifolds) are only two aspects of the same notion. This explains, I believe, the apparent confusion between syntactic and semantic completeness that some commentators (such as S.

Husserl needed the restricted notion of definiteness for he chose to characterize an element that is imaginary from the perspective of a formal deductive system simply as an element which does not belong to the formal manifold determined by this system. Hence, in order to guarantee that imaginary elements would not be admitted (i.e. that they would be really imaginary) in the formal manifold determined by a formal axiomatic system, this manifold was entitled to contain *only* those elements whose "existence" was required by its system.

Although Husserl did not maintain the more restricted notion of a formal manifold in his later works, preferring to abandon the idea that a system of axioms had any business in the definition of the elements of its formal objective domain, in *Ideas* §72 he presents the notion of completeness for a *structure*, which seems to me a new, more palatable, version of the earlier notion of relative definiteness. It is clear from the context that by "domain" or "manifold" he has in mind a structured collection of objects properly speaking, not only formal objects (he mentions explicitly the case of 'spatial formations', the objective domain of geometry). Such a domain is, he defines, "definite" when it is possible, by considering the "essential nature" of the domain in question, to single out a *finite* number of concepts and propositions from which it is possible to derive all the truths referring to this domain. I.e. a mathematical structure, understood as a structured domain of determined objects, is "definite" when all the propositions of the language of this structure that are true in it are consequences of a finite theory, which is tantamount to saying that the complete theory of the structure in question is finitely axiomatizable.

A system of axioms, Husserl also says, is "definite" when it "exhaustively defines" a manifold, i.e. when it determines a manifold that is definite in the above sense. As Husserl defines it here

---

Bachelard, for instance) detected in the way Husserl deals with these notions. See *FTL* §31.

the notion of definiteness applies exclusively to axiomatic systems that are categorical (i.e. have a unique model) and complete. Husserl seems unaware of the fact that in general axiomatic systems do not single out a unique objective domain in which the axioms of the system are all true, i.e. axiomatic systems can have non-isomorphic models. On the other hand we have to keep in mind that Husserl does not share our prejudices in favor of first-order logic, so for him categorical theories would not seem to be such a rare phenomenon. A more sympathetic interpretation is to read the uniqueness implicit in this definition of definiteness for an interpreted theory as simply a way of singling out the structure *intended* by this theory. Hence, according to this interpretation, a (interpreted) theory is definite when it is both the complete theory of its *intended* model and can be finitely axiomatized.

When Husserl tries to extend this definition to non-interpreted axiomatic systems things get more complicated. In the same paragraph Husserl says that "[t]he definition remains as a system even when we leave the material specification of the manifold fully undetermined [...] The system of axioms is thereby transformed into a system of axiomatic forms, the manifold into a form of manifoldness, and the discipline relating to the manifold into a form of discipline." By abstracting from the nature of the elements of a particular mathematical structure, thus obtaining a formal manifold only formally determined by its corresponding formal theory, we break the referential link that existed between an *interpreted* theory and its intended interpretation. A formal theory does not have an intended interpretation. Also, it becomes impossible to differentiate between an assertion in general expressed in the language of an axiomatic system and an assertion referring exclusively to the manifold this system determines.

Hence, Husserl's definition of definiteness for formal manifolds in *Ideas* §72 can only be read as follows: a formal manifold is definite just when any sentence of the language of its corresponding formal system is decided in it, either as a consequence of this system or as a contradiction to it, and moreover this formal theory is finitely axiomatizable. Apparently, this definition introduces a new element with respect to the notion of definiteness presented in earlier texts, which I have been discussing here, namely finite axiomatization. However, I do not believe that Husserl conceived the idea that any theory could have an infinite number of axioms, that is, for him, a theory is always a *finite* theory. Hence, the notion of definiteness presented in *Ideas* is the same notion of *LI* and *FTL*.

The notion of a definite theory seemed to Husserl to embody the very ideal of human rational understanding. The supreme task of man's reason is, it seems, to try to submit the infinite complexity of objective domains to the finite powers of human rationality. When successful these efforts produce *theories* (which are the ideal form of the expression of human reason) in which *all* the infinitely many (hence completeness) true assertions about the domains in question are derived from *finite* sets of axioms (hence finite axiomatization). These two requirements together, completeness and finite axiomatization, express the two main aspects of *human* reason, its actual finitude and its potentially infinite power. The fact that Husserl sets definiteness as an *ideal* for all theories, formal or not, shows that he recognizes as a goal for human theoretical efforts the complete submission of the infinitude of the world to the finitude of human reason[12].

To the very end of his life Husserl considered the notion of definiteness a central notion of his philosophy. To the point of considering in *The Crisis of European Science and Transcendental Phenomenology*, written only few years before his death, as the highest

---

[12] In *Crisis* Husserl tells us that the *locus* of human theoretical reason is not in fact any particular individual mind, but a collective consciousness that cuts across history and binds together a community of co-workers that, despite separation in time and space, produces science through coordinated efforts. To set definiteness as an ideal for theories is, in fact, to set a goal for this community.

ductive systems or as the theory of their objective correlates. Given that the objects of the theory of manifolds are either themselves theories or correlates of theories, the theory of manifolds is on a superior level of abstraction and generality with respect to the formal theories of the first level.

With the inclusion of the theory of manifolds, Husserl believes, the thematic field of formal logic is finally complete. This broad conception of formal logic, encompassing logical syntax, apophansis, formal ontology, formal mathematics in particular, and the theory of manifolds constitutes, Husserl believes, the realization of Leibniz's ideal of a *mathesis universalis.*

CONCLUDING REMARKS

Both Frege and Husserl faced the problem of providing foundations for arithmetic. But whereas for Frege this was primarily a mathematical problem, for Husserl it was basically a philosophical, or more specifically an epistemological question. In *Philosophy of Arithmetic* Husserl approaches this question from a psychologistic perspective, one that was acidly though not fairly criticized by Frege in his review of this work (1894). Incidentally, by the time *PA* was published, Husserl was already harboring profound misgivings about the correctness of such an approach to the philosophy of mathematics or to philosophy as a whole. A few years later, with the publication of the *Logical Investigations,* his rejection of psychologism was complete. Among the problems that showed Husserl the limitations of psychologism one stands out, that of imaginary entities in mathematics. It was precisely this problem, or at least this was one among the problems that reoriented Husserl's entire philosophy of mathematics towards a variant of formalism, which was eventually to grant a distinguished position to formal mathematics and, in particular, the theory of manifolds and deductive systems.

The evolution of Husserl's thought in the philosophy of arithmetic, which started with the epistemological task of tracing system or as a contradiction to it, and moreover this formal theory is finitely axiomatizable. Apparently, this definition introduces a new element with respect to the notion of definiteness presented in earlier texts, which I have been discussing here, namely finite axiomatization. However, I do not believe that Husserl conceived the idea that any theory could have an infinite number of axioms, that is, for him, a theory is always a *finite* theory. Hence, the notion of definiteness presented in *Ideas* is the same notion of *LI* and *FTL*.

The notion of a definite theory seemed to Husserl to embody the very ideal of human rational understanding. The supreme task of man's reason is, it seems, to try to submit the infinite complexity of objective domains to the finite powers of human rationality. When successful these efforts produce *theories* (which are the ideal form of the expression of human reason) in which *all* the infinitely many (hence completeness) true assertions about the domains in question are derived from *finite* sets of axioms (hence finite axiomatization). These two requirements together, completeness and finite axiomatization, express the two main aspects of *human* reason, its actual finitude and its potentially infinite power. The fact that Husserl sets definiteness as an *ideal* for all theories, formal or not, shows that he recognizes as a goal for human theoretical efforts the complete submission of the infinitude of the world to the finitude of human reason[12].

To the very end of his life Husserl considered the notion of definiteness a central notion of his philosophy. To the point of considering in *The Crisis of European Science and Transcendental Phenomenology*, written only few years before his death, as the highest

[12] In *Crisis* Husserl tells us that the *locus* of human theoretical reason is not in fact any particular individual mind, but a collective consciousness that cuts across history and binds together a community of co-workers that, despite separation in time and space, produces science through coordinated efforts. To set definiteness as an ideal for theories is, in fact, to set a goal for this community.

goal of the theory of manifolds the study of *definite* manifolds (*Crisis* §9).

2. MATHEMATICS AND LOGIC: THE *"MATHESIS UNIVERSALIS"*

One of most distinctive features of Husserl's logic is how readily it admits formal mathematics into its realm. But, like Frege, Husserl is not willing to grant access into logic to the whole of mathematics. Many mathematical sciences, such as geometry and mechanics, are not considered part of formal logic simply because they are "contentual" sciences which do not have the universal applicability that characterizes logical disciplines. That is, these sciences deal with concepts, like for instance those of space and force, which have a content of their own, i.e. a given well defined and limited range of application.

Husserl's conception of mathematics in general, and in particular formal mathematics can be stated in a simple formula: mathematics is an *a priori* conceptual science, which is characterized by a method rather than a subject matter. Like pure logic, mathematics is essentially eidetic analysis.

> *The world of pure logic and mathematics is a world of ideal objects*, a world of "concepts", as it is also usual to say. *All truth here is nothing but analysis of essence or concept*; that which is required by the concepts and cannot be dissociated of their content, of their meaning, becomes known and established (*Hua XXIV* p. 50)

But not any "analysis of essence or concept" is mathematics. The use of the right method, in which two aspects stand out, formalization and axiomatization, makes all the difference. Phenomenology, for instance, which is also an eidetic science, cannot be "mathematized" because its concepts do not have the exactness, the sharpness of boundaries that would allow the application of the mathematical method (see *Ideas* §§71-75).

As *a priori* knowledge mathematics is essentially distinct from empirical sciences, based on experience and induction. Mathematical methodology is fundamentally different from the methods of empirical science. Mathematics begins either with some self-evident truths concerning some concept it is interested on, be it the concept of physical space, number or the concept of theory itself, or, in the case of purely formal theories, with an implicit definition of the concept the theory is about, like for instance the concept of a *n*-dimensional non-Euclidean manifold, and proceeds deriving by purely logical means all the mediate truths concerning the concept or all the consequences of its implicit definition by the axiom-like statements the formal theory is based on.

Geometry, for instance, is the mathematical science of the concept of physical space, its goal is to unveil the essential laws pertaining to our concept of space. Likewise, the task of arithmetic is to obtain an ideally closed and logically articulate system of truths concerning the concept of number. Or, in more technical terms, a complete theory containing all the truths relative to the notion of number. What makes arithmetic part of logic and excludes geometry from it is not a purported methodological difference between them, which does not exist (the unity of mathematics is a unity of *method*), but the fact that the concept of number is a logical (in fact formal ontological) concept, whereas the concept of space is not. The reason, according to Husserl, is that the concept of space does not apply to any object whatsoever, whereas the concept of number does.

Mathematics is, for Husserl, twofold. There is formal mathematics, which is part of logic, but there is also "contentual" mathematics, such as geometry, mechanics or mathematical physics, which is not logic, but that is as much an *a priori* conceptual science as formal mathematics. The field of formal mathematics is also split in two levels. On the first level there are the theories regarding formal ontological categories, like set and number, and on the second the theory of manifolds, either as the theory of deductive systems or as the theory of their objective correlates. Given that the objects of the theory of manifolds are either themselves theories or correlates of theories, the theory of manifolds is on a superior level of abstraction and generality with respect to the formal theories of the first level.

With the inclusion of the theory of manifolds, Husserl believes, the thematic field of formal logic is finally complete. This broad conception of formal logic, encompassing logical syntax, apophansis, formal ontology, formal mathematics in particular, and the theory of manifolds constitutes, Husserl believes, the realization of Leibniz's ideal of a *mathesis universalis*.

CONCLUDING REMARKS

Both Frege and Husserl faced the problem of providing foundations for arithmetic. But whereas for Frege this was primarily a mathematical problem, for Husserl it was basically a philosophical, or more specifically an epistemological question. In *Philosophy of Arithmetic* Husserl approaches this question from a psychologistic perspective, one that was acidly though not fairly criticized by Frege in his review of this work (1894). Incidentally, by the time *PA* was published, Husserl was already harboring profound misgivings about the correctness of such an approach to the philosophy of mathematics or to philosophy as a whole. A few years later, with the publication of the *Logical Investigations*, his rejection of psychologism was complete. Among the problems that showed Husserl the limitations of psychologism one stands out, that of imaginary entities in mathematics. It was precisely this problem, or at least this was one among the problems that reoriented Husserl's entire philosophy of mathematics towards a variant of formalism, which was eventually to grant a distinguished position to formal mathematics and, in particular, the theory of manifolds and deductive systems.

The evolution of Husserl's thought in the philosophy of arithmetic, which started with the epistemological task of tracing back arithmetical concepts (I mean, concepts of the arithmetic of the *natural* numbers) to their original sources, a task Husserl believed could be accomplished by psychology[13], ended up in a variant of formalism in which arithmetical theories in general are conceived as nothing but formal theories that define implicitly their concepts. On the other hand, Frege's foundational project for arithmetic never questioned what was for him an obvious fact, that arithmetic is the science of numbers, i.e. a mathematical discipline whose task is to *explain* what number are and derive their properties from the basic truths such an explanation brings to light. That is, Frege never abandoned the view that axioms are explicative in favor of the formalist view that they constitute in fact implicit definitions.

Regardless its failure Frege's project, as a foundational project in mathematics, was from the very beginning already out of touch with the mathematics of the time. Mathematics was by then already moving clearly towards formalism in the sense Husserl understood it. As a formal theory that implicitly defines its concepts arithmetic needed no longer any foundation. This approach, on the other hand, creates new metamathematical problems that Frege completely failed to understand, as his correspondence with Hilbert shows. Husserl not only understood clearly these new problems and the new mathematical theories they required but also reserved for them the highest position in the edifice of formal mathematics, and *a fortiori* logic.

Frege and Husserl have also very different conceptions of logic. For Frege there is no room for an ontology, not even a formal ontology, in logic, whereas for Husserl, as we have seen, formal ontology is not only an important part, but maybe even, under a certain interpretation, the whole of logic. Since arithmetic and

---

[13] Since for Husserl "constitution" is not a synonym of "creation", even in his earlier "psychologistic" period, we cannot infer from the theory presented in *PA* that for Husserl numbers are mental objects.

all the formal theories of mathematics can, so to speak, "cling" to any domain of objects whatsoever, they are not theories about any objective domain in particular. So, for Husserl, they fall squarely into formal ontology, and consequently are nothing but pure formal logic.

Frege has a completely different characterization of logic. For him logic deals exclusively with assertions, and certainly not with objects, no matter how abstractly they are considered[14]. Therefore, to reduce any theory like arithmetic to logic could only mean to derive its assertions from purely logical assertions, and its concepts from purely logical concepts. Whereas for Husserl the unrestricted scope of numerical variables is enough to put arithmetic into the realms of logic, for Frege the carrying out of the logicist program demands much greater efforts. Despite its evident merits, Frege's conception of logic is obviously much less convenient for the logicist thesis than Husserl's more accommodating views.

But Husserl's conception of logic has still other merits. Let's ennumerate some:

1. A distinction between a syntactic and a semantic approach to logic is, at least potentially, already detectable in it.
2. Husserl sees clearly that it is logic's task to present a complete set of formation and transformation rules. Morphology has the task of determining the basic components of meaningful propositions and presenting the laws which regulate their meaningful composition. One of the tasks of consequence logic is to present a theory of deduction.

---

[14] Of course, I am not considering logical objects, the peculiar sort of objects Frege created in order to blur the distinction he himself had carefully drawn between objects and concepts.

3. The distinction between the theories of basic concepts such as set and number and the theory of manifolds, whose objects are precisely these theories, points towards a clearly marked separation between theory and metatheory.
4. The sketches Husserl prepared for the talks given in Göttingen in 1901 and related texts of the same period (*Hua XII*, pp. 340-500) show his efforts in trying to characterize the formal domain of objects determined by a formal axiomatic system from any collection of arbitrarily given formal objects. Although incomplete and far from satisfactory in many points, these efforts reminds us of Henkin's strategy of defining a model for a theory from constants. Of course I am not claiming that Husserl had any clear conception of formal semantics, but he certainly noticed that a formal theory and a realization of it are two different things and also realized that we can construct level by level a domain of objects in which the theory is true (he calls this a constructive or mathematical manifold.)

These is, I believe, enough evidence to support the conclusion that, unlike to Frege's, Husserl's conception of logic antecipates many problems and conceptual distinctions of present day logic. Moreover, Husserl's brand of logicism in the philosophy of mathematics seems much more natural and closer to our intuitions than Frege's. If mathematics, or part of it, is nothing but logic we should be able to see this clearly looking at mathematics as it presents itself to us, not after strenuous efforts to redefine beyond recognition its main concepts.

Finally, the delimitation of the many provinces of formal logic was only the beginning of Husserl's efforts to tackle the many philosophical problems raised by logic. He also set himself the problem of rooting objective logic in the only source of objectivity, transcendental consciousness. Beyond formal logic there is, for him, transcendental logic and here is where truly radical philosophy begins. Needless to point out how completely alien to Frege's perspective (and how absurd this would seem to him) is the project of a transcendental logic. But this is a matter for another paper.

REFERENCES

BACHELARD, S. (1968). *A Study of Husserl's Formal and Transcendental Logic* (Evanston, Northwestern University Press).

DA SILVA, J.J. (1993). Husserl's Philosophy of Mathematics, *Manuscrito* 16, nº 2, 121-148.

FREGE, G. (1894). Review of Husserl's *Philosophie der Arithmetik, Zeitschrift für Philosophie und philosopische Kritik*, 103, pp. 313-332, repr. in Frege (1984).

———. (1971). *Écrits logiques et philosophiques* (Paris, Seuil).

———. (1980). *Philosophical and Mathematical Correspondence* (Oxford, Blackwell).

———. (1984). *Collected Papers on Mathematics, Logic, and Philosophy*, ed. Brian McGuinness (Oxford, Blackwell).

———. (1986; 1884). *Foundations of Arithmetic* (Oxford, Blackwell).

HILL, C.O. (1991). *Word and Object in Husserl, Frege and Russell: The Roots of Twentieth-Century Philosophy* (Athens, Ohio University Press).

HUSSERL, E. (1970). *Husserliana*, Vol. XII (The Hague, Martinus Nijhoff).

———. (1972; 1891). *Philosophie de l'arithmétique* (Paris, PUF).

———. (1970; 1900-01). *Logical Investigations* (New York, Humanities Press).

———. (1972; 1913). *Ideas – General Introduction to Pure Phenomenology*, (New York, Collier-Macmillan).

———. (1969; 1929). *Formal and Transcendental Logic* (The Hague, Martinus Nijhoff).

———. (1976; 1954). *La crise des sciences européenes et la phénoménologie transcendentale* (Paris, Gallimard).

———. (1970; 1954). *Expérience et jugement* (Paris, PUF).

———. (1984). *Husserliana*, Vol. XXIV (The Hague, Martinus Nijhoff).

———. (1998;1984). *Introduction à la logique et à théorie de la connaissance (1906-1907)*, Fr. tr. of *Hua XXIV* (Paris, Vrin).

———. (1995). *Articles sur la logique (1890-1913)*, including *Hua XII*, pp. 340-500 (Paris, PUF).

———. (1987). *Correspondance Frege-Husserl* (Mauvezin, TER).

RIEMANN, B. (1959). "On the Hypotheses which Lie at the Foundations of Geometry", in (Smith, D. E., ed.) *A Source Book in Mathematics* (New York, Dover) pp. 411-425.

ROSADO HADDOCK, G. (1997). "Husserl's Relevance for the Philosophy and Foundations of Mathematics", *Axiomathes* (1-3), 125-132.

———. (1999). "To Be a Fregean or To Be a Husserlian: That is the Question for Platonists", *Contemporary Mathematics*, 235, pp. 295-312.