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A NOTE ON WITTGENSTEIN, REAL NUMBERS & METAPHILOSOPHY

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For Oswaldo Chateaubriand

One of the most striking and prominent aspects of Wittgenstein's later thought is his conception of the nature of philosophy. However, other aspects of his thought do not seem to be entirely in harmony with these meta-philosophical views. This lack of fit is especially apparent in his philosophy of mathematics. I draw attention to Wittgenstein's views on real numbers as an especially clear case in point. I suggest a way in which we may be able to understand this tension, if not eliminate it, by reminding ourselves of the very early origin of Wittgenstein's views on the nature of philosophy.

One of the most striking, and perhaps notorious, aspects of Wittgenstein's later thought is a preoccupation with the nature of the philosophical activity itself. However, as well as being a very prominent aspect of his thought, to which he devoted some of his most memorable aphorisms, Wittgenstein's views on the nature of philosophy are among his more problematic, in part because of their implausibility, but above all because of the lack of fit between Wittgenstein's avowed metaphilosophical views and his actual practice. This is by no means a new observation. Virtually as soon as his later writings began to become publicly available commentators began to draw attention to this question. Michael Dummett, for example, wrote:

> Wittgenstein's work is full of very general remarks about what philosophy is, such that philosophy should propound no theses, or at least none that could be questioned. This is probably the weakest part of his work, and doubtless affected his manner of presentation; but there is nothing in what he says on any other topic the arguments for which presuppose acceptance of these views, and indeed it seems to me that his actual practice belies them - it is, e.g., quite easy to formulate philosophical theses which Wittgenstein advanced. (Dummett (1960), p.434)

Among the clearest examples of this conflict are to be found in Wittgenstein's philosophy of mathematics. It should be recalled that not only was this the area of philosophy on which Wittgenstein wrote more than almost anything else (at least to judge by the surviving *Nachlaß*), but also that he is reported to have said that his most important work was in this area. (So Rush Rhees reported. Cf. Monk (1990), p.466). The aspect of Wittgenstein's metaphilosophical views which it seems hardest to reconcile with his actual practice is his view that: "Philosophy may in no way interfere with the actual use of language; it can in the end only describe it. For it cannot give it any foundation either. It leaves everything as it is. *It also leaves mathematics as it is ....*" ((1967), p. 124; my emphasis). What is problematic is that Wittgenstein very clearly seems to put forward views about mathematics which are at odds with this general metaphilosophical claim that "philosophy ... leaves mathematics as it is." In particular, in his discussions of set theory and the infinite Wittgenstein very clearly and unambiguously rejects the concept of the completed infinite as incoherent, and accepts the potential infinite as the only coherent concept of the infinite. He writes, for example: "The infinite number series is only the infinite possibility of finite series of numbers. It is senseless to speak of the *whole* infinite number series, as if it, too were an extension."((1975), p.164), and the whole of "Ch. XII" of *Philosophical Remarks* contains many more remarks equally firmly rejecting the con-cept of infinite totalities. (Cf. also the chapter of *The Big Typescript* entitled "Infinity in Mathematics", published as part II of the *Philosophical Grammar* ((1974) pp.451-485)). The problem this gives rise to is that if one rejects the coherence of the completed infinite, as Wittgenstein clearly does, then the whole of set theory is also dismissed as incoherent, since it is only possible to define the higher infinities with which it deals if we accept the completed infinite. The concepts with which set theory supposedly deals are meaningless. Yet in that case Wittgenstein is presumably obliged to dismiss this part of mathematics as incorrect. But, then, how can he pretend to be "leaving mathematics as it is"? Both his critical attitude to the completed infinite and how Wittgenstein appears to think the apparent tension can be defused emerge from the following striking passages, firstly from *The Big Typescript* (1933):

> When set theory appeals to the human impossibility of a direct symbolisation of the infinite it brings in the crudest imaginable misinterpretation of its own calculus. It is of course this very misinterpretation that is responsible for the invention of the calculus. But of course that doesn't show the calculus in itself to be something incorrect (it would at worst be uninteresting) and it is odd to believe that this part of mathematics is imperilled by any kind of philosophical (or mathematical) investigation. ((1974), pp.469-70)

And also, a few years later in his 1939 lectures, commenting on Hilbert's celebrated remark about Cantor's paradise:

> [Hilbert wrote] "No one is going to turn us out of the paradise which Cantor has created."
> I would say, "I wouldn't dream of trying to drive anyone out of this paradise." I would try to so something quite different: I would try to show you that it is not a paradise - so that you'll

leave of your own accord. I would say, "You're welcome to this; just look around you." ((1976), p.103)

Wittgenstein is suggesting that set theory *qua* calculus will not be, and cannot be, affected by his dissolution of the confused interpretation, which essentially involves the incoherent concept of the completed infinite, it is usually given. But it may be possible to find some other less objectionable interpretation of the 'calculus' of set theory. (Cf. his brief reported remarks on such an alternative interpretation (1976), pp.170-1).

However, a much clearer example of a real conflict between the two dimensions of Wittgenstein's thought is his view of real numbers. Here he does *not* suggest that we can reinterpret non-constructive definitions of real numbers which do not conform to his views in some other more acceptable way. They are simply wrong. Wittgenstein enunciates in quite unambiguous terms the principle that a real number can only be defined by a *law*. "A real number is: an arithmetical law which endlessly yields the places of a decimal fraction."((1975) p.228). (Cf. the long discussion in "Chs XVI, XVII & XVIII in *Philosophical Remarks* ((1975), pp. 216-244).[1] Here, then, there seems to be no way in which we can interpret Wittgenstein as leaving mathematics as it is.[2]

If it is indeed impossible to avoid the conclusion that here Wittgenstein really is putting forward views which imply that a substantial portion of classical mathematics, that is mathematics as it is now, is incorrect, is there any way we can understand how he came to embrace such incompatible views simultaneously? I sug-

[1] Da Silva (1993) remains one of the most penetrating discussions of Wittgenstein's theory of real numbers. Cf. also Marion (1998).

[2] And is it completely beside the point to reply, as Mathieu Marion did in discussion, that a large portion of classical mathematics can be reconstructed in constructivist terms. (Cf. Marion (1998) for a useful and up to date discussion of this question.) For Wittgenstein's philosophical "non-revisionism" applies to mathematics *as it is*.

gest that what we need to do is to recall that some aspects at least of his general conception of philosophy were ideas which Wittgenstein arrived at extremely early in philosophical development. Furthermore he did so, and this is the important point, before he had even arrived at any specific first-order philosophical views. As early as 1913, in the *Notes on Logic,* Wittgenstein is enunciating a series of very general, fundamental principles about what philosophy can and cannot do ((1979), p.106), and at least some of them were principles he never abandoned. Since they were such a fundamental part of his thought, I suggest that he was so deeply committed to them that he simply incapable of modifying them, regardless of where his other ideas led him. As Wittgenstein's later ideas matured he moved further and further away from systematic and general views on any philosophical question. However, his views on the infinite and real numbers were never, I suggest, abandoned, and so still provide a test case for those who think there is no problem in harmonising Wittgenstein's theory and practice.

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