QUINE'S PROXY-FUNCTION ARGUMENT FOR THE INDETERMINACY OF REFERENCE AND FREGE'S CAESAR PROBLEM

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Abstract: In his logical foundation of arithmetic, Frege faced the problem that the semantic interpretation of his system does not determine the reference of the abstract terms completely. The contextual definition of number, for instance, does not decide whether the number 5 is identical to Julius Caesar. In a late writing, Quine claimed that the indeterminacy of reference established by Frege's Caesar problem is a special case of the indeterminacy established by his proxy-function argument. The present paper aims to show that Frege's Caesar problem does not really support the conclusions that Quine draws from the proxy-function argument. On the contrary, it reveals that Quine's argument is a non sequitur: it does not establish that there are alternative interpretations of our terms that are equally correct, but only that these terms are ambiguous. The latter kind of referential

indeterminacy implies that almost all sentences of our overall theory of the world are either false or neither true nor false, because they contain definite descriptions whose uniqueness presupposition is not fulfilled. The proxy-function argument must therefore be regarded as a reductio ad absurdum of Quine's behaviorist premise that the reference of terms is determined only by our linguistic behavior.

Introduction

Frege's Caesar problem arises, for example, when we introduce the notion of direction by means of the following contextual definition:

(D) The direction of line x = the direction of line y if and only if x and y are parallel.

(D) does not decide in all cases whether an identity statement of the form 'the direction of line *a* = N' is true or false; it leaves open, for instance, whether 'The direction of the Earth's axis = Julius Caesar' is true or false. As a consequence, (D) does not completely determine the reference of proper names of the form 'the direction of line *a*'. (D) is compatible with the interpretation of 'the direction of the Earth's axis' as a proper name of Julius Caesar.[1] For the same reason, (D) does not completely determine the reference of the abstract general term 'x is a direction', either.[2] Put briefly, (D) does not decide whether Julius

[1] See Frege 1884, §§ 55, 66.

[2] See Frege 1884, § 68. In Frege's words, we do not obtain a concept of direction 'with sharp limits of its application'.

Caesar is a direction, and if so, which one. This problem is usually referred to as 'Frege's Caesar problem'.

To establish this kind of indeterminacy in a more rigorous form, Frege used in a parallel case his much-acclaimed permutation argument. The problem in this context is that axiom V of the system of *Grundgesetze*, which can be transcribed as

(V) $\dot{\varepsilon}\ \varphi(\varepsilon) = \dot{\varepsilon}\ \psi(\varepsilon)$ if and only if for all x: $\varphi(x) = \psi(x)$,

does not completely determine the reference of the abstract singular terms of the form '$\dot{\varepsilon}\ \varphi(\varepsilon)$'.[3] Let f be a one-to-one function mapping each object of the domain to another object such that not every value-range is mapped to itself. In Quine's terminology, f is a 'proxy function' assigning to each value-range a unique object that can be regarded as a 'proxy' of that value-range. Since f is one-to-one, the proxies of the value-ranges fulfill the criterion of identity for value-ranges in (V) as well. This argument shows that (V) does not decide whether the value-ranges themselves or their proxies are the referents of the proper names of value-ranges. Frege concluded from this that we are free to identify the two truth values with any two arbitrarily chosen value-ranges without violating (V).

Quine used a similar argument to establish the more general thesis that the reference of all terms is indeterminate. The function *the space-time region of x* is a proxy function mapping physical objects one-to-one to the space-time regions they occupy. It can be used to replace the physical objects to which a theory refers with the space-time regions they occupy without changing that theory in any relevant way. From this he drew the conclusion that we can interpret

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[3] See Frege 1893, §§ 9, 10.

the concrete term 'Fido' both as a name of the dog known as 'Fido' and as a name of the space-time region of this dog: the two interpretations are equally correct, although they are incompatible.

In a late paper, Quine mentioned that, with regard to abstract terms, the referential indeterminacy established by his proxy-function argument is already familiar from Frege's Caesar problem:

> [...] think of a body in the scientific framework of space and time. Insofar as you specify the precise sinuous filament of four-dimensional space-time that the body takes up in his career, you have fixed the object uniquely. We could go farther and *identify* the object, a chipmunk perhaps, with its portion of space-time [...]. Next, we might identify space-time regions in turn with the sets of quadruples of numbers that determine them in some arbitrarily adopted frame of coordinates. We can transfer sensory connotations now to this abstract mathematical object; and still there is no violence of scientific evidence. [...] Thus we can come to terms somewhat with the indeterminacy of reference, as applied to bodies and other sensible substance, by just letting the sensory connotations of the observation sentences carry over from the old objects to their proxies. In the case of abstract objects such as numbers, devoid of sensory connotations, the indeterminacy of reference is already familiar. It is seen in Frege's so-called Caesar problem: the number 5 may be Julius Caesar. We happily use numbers without caring whether they be taken according to the Frege-Russell constructions or Ackermann's or von Neumann's. (Quine 1995, p. 259)

In the literature, Frege's argument for the referential indeterminacy of abstract terms has yet not been critically compared with Quine's more general proxy-function argument. The main goal of the present paper is to fill this gap. It is instructive to make this comparison because Frege's discussion of the Caesar problem reveals that Quine's proxy-function argument is a *non sequitur*: it does not really establish that there are different interpretations of our terms that are equally correct, but only that these terms are ambiguous. The latter kind of referential indeterminacy implies that almost all sentences are either false or neither true nor false, because they contain definite descriptions whose uniqueness presupposition is not fulfilled. Quine's argument must therefore be regarded as a *reductio ad absurdum* of his behaviorist premise that the reference of terms is determined only by our linguistic behavior. In sections 1 and 2, I briefly recapitulate the two arguments, and, in section 3, I criticize Quine's argument on the basis of Frege's discussion of the Caesar problem. My general reading of Quine is based on Gibson's interpretation, according to which Quine's behaviorist conception of language is the key to understanding his thesis of the indeterminacy of reference.[4] With respect to Frege, I mainly stick to the standard reading of him as a Platonist, but I argue that he is committed to a structuralist version of Platonism, which is also hinted at by Quine's remarks.[5] My reading of the Caesar problem basically agrees with the interpretation recently proposed by Salmón.[6] Because of the paper's broad scope, I cannot justify here all aspects of my reading of Frege and Quine, but I have done so extensively in other papers.[7]

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4 See Gibson 1982 and 1988.

5 The standard reading goes back to Dummett 1981, chap. 20. It has been criticized by Weiner 1995 and others.

6 See Salmón 2018.

7 See Greimann 2003, 2014 and 2021.

1. Frege's Caesar Problem

Frege's logicist program of reducing arithmetic to logic was intended to provide arithmetic with a better foundation. Following Quine, we can divide this program into a conceptual and a doctrinal part.[8] The task of the former is to define the concepts of arithmetic in terms of purely logically concepts, and the task of the latter is to derive the laws of arithmetic from purely logical laws. The Caesar problem affects the conceptual part. It arises mainly in two contexts: the definition of the (cardinal) numbers in *Grundlagen* (1884), and the introduction of the value-ranges in the first volume of *Grundgesetze* (1893).

In *Grundlagen*, Frege argues that his first and his second heuristic attempt to define the numbers are defeated by the problem. The first attempt is to define the numbers by the inductive definition

(I) The number 0 belongs to the concept F if and only if for all x: not F(x). The number 1 belongs to the concept F if and only if not for all x: not F(x) and for all y and z: if F(y) and F(z), then y=z. The number n+1 belongs to the concept F if and only if there is an x such that F(x) and the number n belongs to the concept of being a y such that F(y) and y≠x. (Frege 1884, § 55)

This definition can be regarded both as a contextual definition of the singular terms '0', '1', '1+1', etc., and as an explicit definition of the second-order predicates 'the number 0 belongs to the concept of being an x such that F(x)', 'the number 1 belongs to the concept of being an x such that F(x)', 'the number 1+1 belongs to the concept of

8 See Quine 1986b, pp. 69-70.

being an x such that F(x)', etc. In the first case, the Caesar problem does not arise. However, Frege wants to construe the numbers as objects, and not as concepts of second order. For this reason, he considers the singular terms as the *definienda* of (I). In this case, the following problem arises:

[W]e can never – to take a crude example – decide by means of our definitions whether any concept has the number Julius Caesar belonging to it, or whether that same familiar conqueror of Gaul is a number or not. Moreover we cannot by the aid of our suggested definitions prove that, if the number a belongs to the concept F and the number b belongs to the same concept, then necessarily a = b. Thus we should be unable to justify the expression 'the number that belongs to the concept F', and therefore should find it impossible in general to prove a numerical equality, since we should be quite unable to achieve a determinate number. It is only an illusion that we have defined 0 and 1; in reality we have only fixed the sense of the phrases 'the number 0 belongs to', 'the number 1 belongs to'; but we have no authority to pick out the 0 and 1 here as self-subsistent objects that can be recognized as the same again. (Frege 1884, § 56)

The problem is that the singular terms '0', '1', '1+1' are ambiguous when they are defined by (I), because (I) does not sharply delimit the extension of concepts like *x is the number belonging to the concept of being a moon of Jupiter*. This is the Caesar problem with regard to (I).

The second heuristic attempt, sketched in §§ 62 to 67, is analogous to the definition of directions by (D). It consists in defining the numbers contextually, by Hume's principle

(H) The number belonging to the concept F = the number belonging to the concept G if and only if the concepts F and G are equinumerous [*gleichzahlig*], i.e., if there is a one-to-one function f such that for all x: F(x) if and only if G(f(x)).

Frege's main objection to the contextual definition of directions by (D) is that it cannot be used to decide whether, say, England is the same as the direction of the Earth's axis (cf. § 66). For analogous reasons, the contextual definition (H) cannot be used to decide whether, say, England is the number belonging to the concept of being a moon of Jupiter. This is the Caesar problem, again.

In his discussion of the Caesar problem, Frege stresses the context principle, 'Only in the context of a sentence words have any meaning' (see § 60). Although this principle is one of Frege's most influential contributions to the philosophy of language, we still do not know exactly what role it plays in his foundation of arithmetic. On the one hand, the textual evidence strongly suggests that its main task is to justify contextual definitions. On the other hand, Frege rejects such definitions, just because of the Caesar problem.[9]

His solution to the Caesar problem with regard to the cardinal numbers in *Grundlagen* is to define them by the explicit definition

(N) The number belonging to the concept F = the extension of the concept of being equinumerous to F.

Clearly, this solution presupposes that the reference of singular terms of the form 'the extension of the concept F' is completely determined. Otherwise, the problem of

[9] See Dummett 1991, chap. 16 and 17.

referential indeterminacy would reoccur. However, in *Grundgesetze*, Frege identifies concepts with characteristic functions. He accordingly construes the extensions of concepts as extensions of functions. The latter are called 'value-ranges' (*Werthverläufe*) by him.[10] In this framework, the solution of the Caesar problem by (N) presupposes that the reference of proper names of a value-range is completely determined. Surprisingly, Frege assumes that the Caesar problem also affects these proper names. In § 3, he introduces the concept of value-range by means of the following informal explanation: the words 'the function $\varphi(\xi)$ has the same value-range as the function $\psi(\xi)$' are coreferential (*gleichbedeutend*) with the words 'the functions $\varphi(\xi)$ and $\psi(\xi)$ always have the same value for the same arguments'. This amounts to the explanation that the intended notion of value-range fulfills the principle

(V) $\dot{\varepsilon}\, \varphi(\varepsilon) = \dot{\varepsilon}\, \psi(\varepsilon)$ if and only if for all x: $\varphi(x) = \psi(x)$,

which is axiom V of the system in *Grundgesetze*. Note that the introduction of the concept of value-range by (V) is parallel to the introduction of the concept of number by (H) and also to the introduction of the concept of direction by (D). They are all instances of the scheme '$f(x) = f(y)$ if and only if $R_{eq}(x,y)$', where R_{eq} is an equivalence relation such as being parallel, being equinumerous, and having the same values for the same arguments.[11] The Caesar problem with regard to

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[10] See Frege 1893, §§ 2 and 3.

[11] For simplicity's sake, I have ignored here the difference between first level and second level functions. Strictly speaking, axiom V is not an instance of (S), because the function *the value-range of the function* $\varphi(\xi)$ is a second level function.

the introduction of value-ranges is that (V) does not fix the truth values of mixed identity statements like

(T) $\dot{\varepsilon}$ — (ε) = for all x: x = x.

Since the sentence 'For all x: x = x' is a proper name of the True, (T) is true if and only if the value-range of the function — (ξ) is identical to the True.

To show that (V) does not determine the truth value of (T) completely, Frege sketches an argument in § 10 that has come to be known as the 'permutation argument'. Let X be a bijective function on the domain of discourse whose value for a value-range as argument is not always that same value-range. Since X is bijective, it is always true that x = y if and only if X(x) = X(y). Consequently, the values of X also fulfill the criterion for being the value-range of a function φ contained in (V):

(V') X($\dot{\varepsilon}$ $\varphi(\varepsilon)$) = X($\dot{\varepsilon}$ $\psi(\varepsilon)$) if and only if for all x: $\varphi(x)$ = $\psi(x)$.

Hence, (V) does not decide whether the value-ranges themselves or the corresponding values of X are the referents of the value-range terms.[12]

The question arises why the referential indeterminacy established by the Caesar example is a problem for Frege at all. Why must we reject definitions like (H)? There are several possible answers:

1. (H) does not inform us about what kind of entities the numbers really are, whether, for instance, the number

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[12] For a more detailed reconstruction of the permutation argument, see Bentzen 2019, section 2.

belonging to the concept of being a moon of Jupiter is a set, or a person, or something else (metaphysical problem);
2. (H) leads to a violation of the law of excluded middle, because it does not achieve a sharp demarcation of the concept of number (logical problem);
3. (H) does not provide us with a means of recognizing a number as the same object again when it is not given to us by a description of the form 'the number belonging to the concept F' (epistemological problem);
4. (H) does not rule out that arithmetic has unintended models (model-theoretical problem);
5. (H) does not allow us to prove 'England is the number belonging to the concept of being a moon of Jupiter' nor its negation (proof-theoretical problem).

For our purposes, only the metaphysical and the logical background problem are relevant. We do not need to discuss to which extent Frege aimed at solving the other background problems.

In contrast to (H), the explicit definition (N) informs us about which sort of entities the numbers are: numbers are extensions. For this reason, it is widely assumed that the Caesar problem is a metaphysical problem. According to the Platonist view, there are facts determining which objects the numbers are, whether they are sets, and if so, which ones, whether Julius Caesar is a set, and so on. A materially adequate definition of number must correctly specify which objects exactly the numbers are.[13]
However, in § 107 of *Grundlagen,* Frege explicitly says that he does not consider the application of the notion of extension

[13] See Benacerraf 1965, pp. 285 ff. The discussion of the Caesar problem in Schirn 1996, Wright 1983, and in Wright and Hale 2001 is also based on the metaphysical reading of the problem.

in his definition to be of 'decisive importance'.[14] He would be satisfied with any definition that agrees with Hume's principle. This claim corresponds to the structuralist view that co-extensiveness of *definiens* and *definiendum* is not a condition for the material adequacy of definitions of number; it suffices that the extensions of *definiens* and *definiendum* are isomorphic.[15] This weaker criterion of adequacy allows us to identify numbers with different sorts of objects. It is based on the structuralist assumption that, from a mathematical point of view, mathematical objects are exhaustively defined by their structural properties. Any sort of objects can be regarded as a version of the (cardinal) numbers as long as it satisfies (H). Mathematics abstracts away from the metaphysical nature of the objects it describes.

Frege is well aware of the fact that extensions have properties that cannot be applied to numbers: we do not say, for instance, that 'one number is more inclusive than another, in the sense in which the extension of a concept may be more inclusive than that of another' (§ 69). Nevertheless, (N) is materially adequate, he argues, because it agrees with (H). To be sure, Frege's structuralism is weaker than the standard one, which derives from Benacerraf's influential criticism of Frege's Caesar problem.[16] According

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[14] For this reason, Benacerraf revised in a later paper his realist reading of Frege. He recognized that '[d]efinitions that are adequate to *his* purposes need not preserve reference' (1981, pp. 44) and concluded from this that 'a straight-forwardly 'realist' construal of Frege's intentions or accomplishments will fail to do justice to his practice' (1981, pp 63).

[15] See Quine 1964, pp. 209-211.

[16] The *locus classicus* is Benacerraf 1965. We can ignore here other forms of mathematical structuralism because they are not relevant for our discussion.

to Benacerraf, any object can be considered to be the number 3:

> To *be* the number 3 is no more no less than to be preceded by 2, 1 and possibly 0, and to be followed by 4, 5 and so forth. [...] *Any* object can *play the role of* 3; that is, any object be can the third element in some progression. [...] The search for which independently identifiable particular objects the numbers really are (sets? Julius Caesars?) is a misguided one. (Benacerraf 1965, p. 291)

Contrary to Benacerraf, Frege cannot consider Caesar as a number, because he assumes that mathematical objects are abstract entities that exist eternally in a Platonic realm of reality.[17] He sees this as a natural assumption of common sense, which prevents the common man from confusing Julius Caesar with the direction of the Earth's axis.[18] Frege is thus committed to a Platonist version of structuralism. It rejects, on the one hand, the hyper-Platonist assumption that the Platonic realm is like a museum in which numbers are exposed and each number is discriminated from all other abstract objects by a label that specifies which number it is, but it insists, on the other hand, that only abstract objects (and more specifically logical objects) can be considered as numbers.

Quine also reads Frege as a structuralist. He defends Frege's identification of numbers with extensions (classes) arguing that, from an arithmetical point of view, the differences between extensions (classes) and numbers are 'don't-cares'. Frege's definition of number must not be understood as a 'conceptual analysis' claiming that the

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[17] See, for instance, Frege 1884, § 61.
[18] See Frege 1884, § 66.

concept of extension is contained in the hidden deep structure of the informal concept of number, but as a 'conceptual explication' claiming that the informal concept of number can be replaced by the concept of extension to construe a scientifically more respectable concept of number.[19] To be materially adequate, it suffices that the *definiens* satisfies (H).

In the literature, the background problem that (H) does not achieve a sharp delimitation of extension of the concept of number has only received little attention.[20] I call this the 'logical problem' because, in § 74 of *Grundlagen*, Frege claims that the sharp delimitation of concepts with regard to their extension is the only requirement that concepts must satisfy to be acceptable from the point of view of logic:

> All that can be demanded of a concept from the point of view of logic and with an eye to rigour of proof is only that the limits to its application should be sharp, that it should be determined, with regard to every object whether it falls under that concept or not. (Frege 1884, § 74)

In a footnote, he adds that to define an object in terms of a concept under which it falls, it is necessary first to show two things:

1. that some object falls under this concept;
2. that only one object falls under it. (Frege 1884, § 74)

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[19] See Quine 1960, §§ 53 and 54.

[20] In Heck's account of the Caesar problem, for instance, the logical problem does not play an essential role; see Heck 2005, 2011, pp. 13 ff. and 2012, chap. 4. An exception is Bentzen 2019, section 5.

As we have seen, Frege rejects the inductive definition (I) just because it does not justify the definite article in 'the number that belongs to the concept F'. It must be rejected because we cannot show that only one objects falls under the concept of being the number that belongs to the concept F.

Moreover, in § 68 of *Grundlagen*, Frege explicitly says that his first and second heuristic attempt to define the numbers are unsatisfactory precisely because they do not provide us with a concept of number "with sharp limits to its application":

> Seeing that we cannot by these methods obtain any concept of direction with sharp limits to its application, nor therefore, for the same reason, any satisfactory concept of Number either, let us try another way. (Frege 1884, § 69)

The alternative method Frege proposes is the explicit definition of number by (N). There can thus be no doubt that Frege considers the logical problem to be the crux of the Caesar problem.

However, in *Grundlagen*, Frege does not explain why logic demands the sharp delimitation of concepts. This is made explicit by him only in the second volume of *Grundgesetze* (1903). At the beginning of Part III, he sets out his criteria of adequacy for definitions. The first criterion is the following Principle of Completeness:

> A definition of a concept (a possible predicate) must be complete; it has to determine unambiguously for every object whether it falls under the concept or not (whether the predicate can be applied to it truly). Thus, there must be no object for which, after the definition, it remains doubtful whether it falls under the concept, even though it may not always be possible, for us humans, with our deficient

> knowledge, to decide the question. Figuratively, we can also express it like this: a concept must have sharp boundaries. [...] Logic cannot recognize such concept-like constructions [i.e. concepts without sharp boundaries] as concepts; it is impossible to formulate exact laws concerning them. The law of excluded middle is in fact just the requirement, in another form, that concepts have sharp boundaries. Any object Δ either falls under the concept Φ or it does not fall under it: *tertium non datur*. (Frege 1903, § 56)

Obviously, Frege derives the requirement of the sharp delimitation of concepts from the principle of bivalence: every sentence must be either true or false. According to his explanations, a concept F is sharply delimited if for all objects x: either x falls under F or x does not fall under F. Analogously, a relation R is sharply delimited if for all objects x and y: either x and y stand in the relation R or they do not stand in the relation R. The law of excluded middle is a generalization of such requirements. It demands that for all p: either p or not p, where 'p' is a variable for which proper names of a truth value can be substituted. Natural language is logically defective because it violates the law of excluded middle. In an ideal language for science, every sentence must be either true or false. To overcome the referential indeterminacy established by the Caesar problem, we must accordingly close all truth value gaps. This is exactly Frege's strategy to solve the Caesar problem in *Grundgesetze*. To fix the truth values of mixed identity statements like (T), he identifies the True with the value-range 'ἐ — (ε)'. This stipulation delimits more closely both the extension of the concept of truth value and the extension of the concept of value-range.[21]

[21] See Frege 1893, § 10.

The deeper problem with (H) is that proper names like 'the number of Jupiter's moons' are *ambiguous* as long as we limit their semantic interpretation to (H). To see this, consider again the simpler definition (D). On its natural reading, (D) is a definition of the concept of having the same direction. The Caesar problem arises because we consider (D) as a definition of the concept of direction itself. Frege explains this approach as follows:

> The judgement 'line *a* is parallel to line *b*', or using symbols, '*a*//*b*', can be taken as an identity. If we do this, we obtain the concept of direction, and say: 'the direction of line *a* is identical with the direction of line *b*'. Thus we replace the symbol // by the more generic symbol =, through removing what is specific to the content of the former and dividing it between *a* and *b*. We carve up the content in a new way different from the original way, and this yields us a new concept. (Frege 1884, § 64)

This procedure works as follows. We already know the content of 'Line *a* is parallel to line *b*'. To obtain the concept of direction, we must first reformulate this sentence as an identity statement, 'The direction of line *a* = the direction of line *b*'. The content of the two-place predicate 'Line x is parallel to line y', which is the relation *line x and line y are parallel*, is identical to the content of the two-place predicate 'The direction of line x = the direction of line y', which is the relation *line x has the same direction as line y*.[22] Parallelism is the identity of direction. The next step is to decompose the relation of parallelism into the more generic relation of identity and the concept of direction. The relation of identity

[22] The notion of content *in Grundlagen* is ambiguous. In our context, it can be identified with the notion of reference.

is already known to us. To obtain the concept of direction, we must finally read (D) as a definition whose *definiendum* is the function *the direction of line x*. So defined, the direction of line x is the value of the function that maps two lines to the same object if and only if they are parallel. Put vividly, the direction of a line is what this line has in common with all and only those lines to which it is parallel.

The permutation argument already shows that there are various functions satisfying this condition that do not always have the same value for the same argument. Some of them map the Earth's axis to Julius Caesar and others to the number 5. Hence, the uniqueness condition expressed by the definite article in 'the direction of line a' is not fulfilled. To make this explicit, we can define the direction of the Earth's axis as follows:

(D_E) The direction of the Earth's axis = the value of that function that maps two lines to the same object if and only if the lines are parallel, for the Earth's axis as argument.

(D_E) reveals that contextual definitions like (D) and (H) do not well-define their *definienda*. This is the core of the Caesar problem, as Nathan Salmón has recently shown.[23]

In Frege's view, ambiguous proper names lack a referent. The definite description 'the book written by Kant', for instance, does not ambiguously refer to all books written by Kant, but it does not refer to any object at all, because the value of the function *the book written by x* is not well-defined for authors of more than one book.[24] This view implies that

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[23] See Salmón 2018, pp. 1652-1653.
[24] See Frege 1893, § 11.

sentences like 'The direction of the Earth's axis is influenced by the Moon' are neither true nor false.[25]

According to Russell's alternative theory, which Quine uses for the 'elimination' of singular terms in his ideal language for science, definite descriptions are not proper names, but syncategorematic expressions that have meaning only in the context of a sentence.[26] According to (D_E), the definite description 'the direction of the Earth's axis' can be paraphrased as 'the x such that x is the value of the function that maps two lines to the same object if and only if the lines are parallel, for the Earth's axis as argument'. This is a complex definite description containing the definite description of second order 'the function that maps two lines to the same object if and only if the lines are parallel'. Following Russell's approach, we can paraphrase 'The direction of the Earth's axis is influenced by the Moon' as 'There is one and only one object x such that there is one and only one function f such that f maps two lines to the same object if and only if they are parallel, and x is the value of f for the Earth as argument, and x is influenced by the Moon'. The latter sentence is false because the uniqueness condition concerning the function f is not fulfilled. This consequence will be important for the criticism of Quine's proxy-function argument I shall make.

2. Quine's Proxy-Function Argument

Quine's proxy-function argument is the core of his theory of ontological reduction. It is designed to show that we can replace the domain of a theory with any other domain as long

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[25] See, for instance, Frege 1997, p. 157.
[26] See Quine 1948 and Quine 1960, § 38.

as we preserve its structure. This 'global structuralism' is a generalization of the structuralist view in mathematics that there are various reductive definitions of number that are equally satisfactory, although they are incompatible with each other, because they assign different extensions to their *definientia*.

To reduce the original objects of a theory to new ones, we must show that all sentences of the theory that presuppose the existence of the old objects can be reformulated in such a way that they presuppose only the existence of the new ones. To this end, we must specify a proxy function that correlates the old objects with the new ones one-to-one. It allows us to reinterpret the terms of the theory in a systematic way such that they no longer refer to the original objects, but to their proxies.[27] In order to reduce dogs to their space-time regions, for instance, we can paraphrase 'Fido is a dog' as 'The space-time region of Fido is the space-time region of a dog', where *the space-time region of x* is a proxy function mapping physical objects one-to-one to their space-time regions. On the intended reading, the latter sentence does not presuppose the existence of dogs, but only the existence of their space-time regions. Quine describes this method in a more general way as follows:[28]

> All that is needed […] is a rule whereby a unique object of the supposedly new sort is assigned to each of the old objects. I call such a rule a proxy function. Then, instead of predicating a general term '*P*' of an old object *x*, saying that

[27] See Quine 1964, p. 214.

[28] The first versions of the argument are sketched in Quine 1964, 1968a, and 1976. The mature version is to be found in Quine 1981, 1983, 1984, 1990, 1992, 1995, and 2019. I shall focus on the version of the argument in Quine 1981, which is the most elaborated one.

x is a *P*, we reinterpret *x* as a new object and say that it is the *f* of a *P*, where '*f*' expresses the proxy function. Instead of saying that *x* is a dog, we say that *x* is the lifelong filament of space-time taken up by a dog. Or, really, we just adhere to the old term '*P*', 'dog', and reinterpret it as '*f* of a *P*', 'place-time of a dog'. (Quine 1981, p. 19)

It is hard to see how Quine's global structuralism can be made compatible with his realism about physical objects.[29] Happily, we can ignore this problem because it is not relevant to our discussion.

The question arises how such paraphrases can be justified with regard to their material adequacy. Why is it sufficient that they preserve the structure of the original extensions? The answer is given by Quine's behaviorist conception of meaning and reference. He rejects Frege's assumption that the reference of our terms is determined by the senses we grasp in as a mentalist myth.[30] To account for the public and social character of language, he argues, we must assume that the reference of terms is determined in a public and social way, by our publicly observable verbal behavior. To lay down the intended interpretation of 'dog', we have only visible gestures like the pointing to a dog and the uttering of linguistic expressions at our disposal. Technically speaking, this means that we can fix the extensions of terms only by their ostensive definition.[31] To establish the intended

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[29] For a detailed discussion of this tension, see Hylton 2000, pp. 298-299, Hylton 2004, pp. 144-145, Hylton 2007, pp. 317-323, Gregory 2019 and Janssen-Lauret 2019.

[30] Quine 1968a, p. 27.

[31] Quine does not use the term 'ostensive definition' *expressis verbis*. But the ostensive method of teaching the meaning of a term that is described by him in his theory of language acquisition consists

interpretation of 'dog', we must point to a typical dog and stipulate that the object we are pointing to counts as a dog, and that everything similar to that object also counts as a dog, and that nothing else counts as a dog. However, such definitions do not really connect the word 'dog' with dogs. Rather, they connect the sentence 'This is a dog' in a *holophrastic way* with patterns of stimulations.[32] From the behaviorist point of view, language is connected to the world only in virtue of our dispositions to accept a sentence as true or to reject it as false when our receptors are triggered by appropriate patterns of stimulations. It is for this reason that Quine considers sentences to be the primary units of language. He adopts a behaviorist context principle according to which the reference of the term 'dog' is mainly determined by the stimulus meaning of the sentence 'This is a dog'.[33]

However, whenever we point to a dog, we also point to the space-time region of a dog. Consequently, the observations that speak for the truth of 'This is a dog' also speak for the truth of 'This is the space-time region of a dog'.[34] The two sentences are empirically equivalent. Hence, we can reformulate a theory that speaks about physical objects as a theory that speaks about space-time regions without changing the empirical evidence for that theory.[35] Moreover, the original formulation and the reformulations

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basically in the ostensive definition of that term. See, for instance, Quine 1968a, pp. 30-54.

[32] See Quine 1992, p. 8.

[33] I am simplifying here. For a more detailed account, see Hylton 2000 and 2004.

[34] Cf. the parallel argument in Quine 1968a, p. 32.

[35] See Quine 1981, p. 21, Quine 1990, p. 12, Quine 1992, p. 9, and Quine 1995, p. 295, for instance.

are also behaviorally equivalent in the sense that they are connected in exactly the same way with our verbal dispositions. Given Quine's behaviorist assumption that there is no semantic difference between two speakers without a corresponding difference in their verbal behavior, it finally follows that the two formulations are also semantically equivalent:[36]

> The apparent change is twofold and sweeping. The original objects have been supplanted and the general terms reinterpreted. [...] Yet verbal behavior proceeds undisturbed, warranted by the same observations as before and elicited by the same observations. Nothing has really changed. (Quine 1981, p. 19)

From the behaviorist point of view, the paraphrases are hence materially adequate. They preserve not only the structure of the extensions, but also the (stimulus) meanings of all sentences of a theory. The paraphrases consequently show that it does not matter to a physical theory which objects we choose for it; we can always supplant the original objects without changing that theory in any substantial way.

From this Quine derives two conclusions that are intimately connected. The first is the thesis of the indeterminacy of reference:

> The conclusion I draw is the inscrutability of reference. To say what objects someone is talking about is to say no more than how we propose to translate his terms into ours; we are free to vary the decision with a proxy function. (Quine 1981, pp. 19-20)

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[36] Cf. the characterization of Quine's linguistic behaviorism in Quine 1960, p. 78, Quine 1968a, p. 27, and Quine 1981, p. 19.

The second conclusion is his 'global structuralism', according to which it does not matter to a theory which ontology we choose for it: we can reduce its objects to any other objects as long as we preserve their structure:

> Structure is what matters to a theory, and not the choice of its objects. F.P. Ramsey urged this point fifty years ago, arguing along other lines, and in a vague way it had been a persistent theme also in Russell's *Analysis of Matter*. But Ramsey and Russell were talking only of what they called theoretical objects, as opposed to observable objects. I extend the doctrine to objects generally, for I see all objects as theoretical. This is a consequence of taking seriously the insight that I traced back from Bentham—namely, the semantic primacy of sentences. It is occasion sentences, not terms, that are to be seen as conditioned to stimulations. […] The objects, or values of variables, serve merely as indices along the way, and we may permute or supplant them as we please as long as the sentence-to-sentence structure is preserved. (Quine 1981, p. 20)

The main point of Quine's global structuralism is that the empirical evidence for a theory does not depend on the choice of it objects. We can replace physical objects with abstract ones without changing the empirical implications of physics, for instance.[37]

[37] For a detailed reconstruction of this epistemological thesis, see Janssen-Lauret 2019. It is not clear whether Quine also claims that the truth of a theory does not depend on the choice of its objects. A positive answer is suggested in Quine 2019, p. 82, and a negative one in Quine 1981a, p. 21. For a detailed discussion of this question, see Greimann 2021.

3. The Problem with Quine's Argument from Frege's Perspective

We have seen that the referential indeterminacy established by Frege's Caesar problem consists in the ambiguity of definite descriptions whose semantic interpretation is limited to contextual definitions like

(D) The direction of line x = the direction of line y if and only if x and y are parallel.

Thus, the definite description 'the direction of the Earth's axis' is ambiguous, when it is defined exclusively by (D), because there are different objects fulfilling the condition for being the direction of the Earth's axis that is contained in (D). As a consequence, sentences like 'The direction of the Earth's axis is influenced by the Moon' are either false or neither true nor false, because they contain definite descriptions whose uniqueness presupposition is not fulfilled.

The same problem arises in Quine's system. Under the behaviorist interpretation, the reference of 'Fido' is basically determined by the ostensive definition of 'Fido'. To be Fido means to be the object to which we point when our utterance of the sentence 'This is Fido' is true. This term is ambiguous, because the condition for being the referent of 'Fido' is satisfied not only by Fido, but also by his proxies. It can be compared to the term 'The book written by Kant', which suffers from exactly the same kind of referential indeterminacy. Consequently, 'Fido is a dog' is not true, even on the assumption that Fido is actually a dog. Under the Fregean reading of 'Fido' as a singular term, 'Fido is a dog' is neither true nor false, because the reference of 'Fido' is not well-defined, and under the Russellean reading of 'Fido is a dog' as 'There is one and only one x such that x is Fido and that x is a dog', this sentence is false, because the uniqueness condition that there is exactly one object that is Fido is not fulfilled. Given the Russellean reading, the proxy-function argument consequently implies that almost all sentences containing the term 'Fido' are false.[38]

Quine is committed to the Russellean treatment of singular terms in his ideal language for science. To make the ontological commitments of science more transparent, he reduces singular terms to definite descriptions and the latter to predicates and quantifiers by means of Russell's method. This 'elimination of singular terms' is an integral part of his regimentation of language. It is the cornerstone of his criterion of ontological commitment.[39] Consequently, the proxy-function argument implies that almost all sentences of our overall theory of the world are false, when we formulate it in Quine's canonical notation. They contain definite descriptions whose uniqueness conditions are not fulfilled. This problem has never been discussed in the literature, as far as I can see.

However, in contrast to Frege, Quine does not draw from the proxy-function argument the conclusion that our terms are ambiguous. The referential indeterminacy Quine has in mind is of a different kind: it consists in the existence of different interpretations of our terms that are equally correct, but incompatible. His thesis of referential indeterminacy can be regarded as a generalization of the structuralist thesis in mathematics that there are various reductive definitions of number that are equally correct, but mutually incompatible. Thus, Frege's definition and von Neumann's are both correct definitions of number, according to the structuralist

[38] See Quine 1948, pp. 7-8.

[39] See Quine 1948 and Quine 1960, § 38, for instance.

criterion of material adequacy, but they are also incompatible because they identify the same numbers with different sets. To take this into account, we must relativize the reference of '5' to definitions of numbers. Relative to Frege's definition, '5' refers to the set of all sets with 5 elements, whereas, relative to von Neumann's definition, '5' refers to the set of all numbers that are smaller than 5. Quine generalizes this relativization by applying it also to concrete terms like 'Fido' and 'dog'.[40] Relative to the homophonic translation of 'dog' as 'dog', 'dog' refers to dogs, and relative to the translation of 'dog' as 'space-time region of a dog', 'dog' refers to space-time regions of dogs. We can thus interpret 'dog' in various different ways that are equally correct, though they are incompatible. This kind of referential indeterminacy consists in the referential relativity of terms: they have a well-defined extension only relative to a given translation into a background language.[41]

Although Quine does not explicitly address the problem of ambiguity, one might consider his relativization of reference to be an implicit solution. Let us assume that

(M) 'Fido' refers to the space-time region of Fido

is a metalinguistic stipulation laying down that 'Fido' refers to the space-time region of Fido in the object language. We can then say that, relative to (M), the reference of 'Fido' is fixed. This solution corresponds to Frege's solution of the parallel problem that axiom (V) does not determine the reference of the proper names of a value-range completely. His stipulation (T), according to which '$\dot{\varepsilon} — (\varepsilon)$' refers to the True, determines the reference of these names more closely.

[40] See Quine 1960, § 7, and Quine 1968a, pp. 47-48, for instance.
[41] See Quine 1968a, pp. 46-51, and Quine 1990, pp. 51-52.

In 'Ontological Relativity' (1968), Quine assumes that the translation of 'Fido' into a background language actually fixes the reference of this term.[42] It has the same effect as the metalinguistic stipulation (M). However, in contrast to Frege, Quine cannot regard such stipulations as a part of the semantic interpretation of 'Fido'. According to his semantic behaviorism, the reference of terms is determined exclusively by our verbal dispositions. Consequently, he cannot consistently introduce further parameters on which the reference of terms depends. This is an important difference between Frege and Quine. To overcome the referential indeterminacy of the proper names of a value-range, Frege can add semantic stipulations to (V) that make their interpretation more complete. Quine, on the other hand, is committed to the view that the ambiguity of terms cannot be removed at all. Although 'Fido' does have a fixed reference relative to (M), this term remains ambiguous, because (M) cannot be considered as a part of the semantic interpretation of 'Fido'.

Moreover, the assumption that the translation of 'Fido' into the metalanguage somehow eliminates the ambiguity of this term presupposes that the terms of the metalanguage have a well-defined extension independently of their translation into a meta-metalanguage. This presupposition is again incompatible with Quine's semantic behaviorism because, as Quine admits, the thesis of the indeterminacy of reference also applies to the metalanguage.[43]

There are some texts in which he suggests that the translation of 'Fido' into a background language fixes the reference of 'Fido' only in a relative way.[44] In this case, the

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[42] See, for instance, Quine 1968a, pp. 47-49.
[43] See Quine 1968a, p. 47.
[44] See Quine 1990, p. 52 and Quine 1990a, p. 6.

translation of 'Fido' as 'the space-time region of Fido' lays down that the reference of 'Fido' in the object language is identical to the reference of 'the space-time region of Fido' in the metalanguage, but it leaves open to which object 'Fido' in the object language refers, because the reference of 'the space-time region of Fido' in the metalanguage is also indeterminate. Again, the relativization does not solve the problem of the ambiguity of 'Fido', because it does not add anything to the semantic interpretation of this term.

Moreover, the relativization of reference does not solve the more serious problem that almost all sentences affirmed in our overall theory of the world are false when they are formulated in Quine's canonical notation. *Ex hypothesi*, the semantic interpretation of the ideal language for science does not allow us to specify what the objects of this theory are supposed to be. The variables always refer to all objects, because we cannot restrict the domain of the theory. Consequently, the uniqueness presuppositions of the definite descriptions are not satisfied.

But there are three other strategies that Quine might use to solve the problem. The first is to treat 'Fido' neither as a definite description nor as a singular term, but as an indexical term whose reference depends on the context of utterance. This approach is suggested by Quine's claim that the problem of the indeterminacy of reference arises only in contexts in which we do not 'rock the boat', that is, in which we do not consider alternative translations of 'Fido'.[45] In a context of utterance in which 'Fido' is translated homophonically as 'Fido', 'Fido' refers to Fido, and in a context of utterance in which this term is translated as 'the space-time region of Fido', it refers to the space-time region of Fido, and so on. But, in this case, 'Fido' and 'dog' are

[45] See, for instance, Quine 1981, p. 20.

indexical terms like 'here' and 'today'. They do not constitute genuine cases of referential indeterminacy in Quine's sense because they cannot be interpreted in different ways that are equally correct, but incompatible. In a given context of utterance, there is only one correct interpretation of indexical terms like here' and 'today'. The referential indeterminacy Quine has in mind does surely not consist in the context dependence of the reference of indexical terms.

The second strategy is to consider 'Fido' as a variable bound by an invisible existential quantifier. It is suggested by Quine's explanation of his global structuralism in terms of Ramsey's method of defining theoretical terms.[46] This method can be roughly described as follow.[47] We presuppose that the observational terms of a theory T are categorematic expressions whose semantic interpretation is already fixed. Their meanings may be identified with their stimulus meanings, for instance. By contrast, we treat the theoretical terms as syncategorematic expressions that are implicitly defined by the so-called Ramsey sentence of T. Let C be the conjunction of the axioms of T. C is a single sentence that can be considered as a complete formulation of T. Let $C[x_1, ..., x_n]$ be the open sentence that results from replacing all theoretical terms $t_1, ..., t_n$ that occur in C by the variables $x_1, ..., x_n$, respectively. The Ramsey sentence of T is the existential sentence $\exists x_1, ..., \exists x_n\ C[x_1, ..., x_n]$. To make its point explicit, call an n-tuple that satisfies $C[x_1, ..., x_n]$ under the intended interpretation of all observational terms a 'realization' of T.[48] The Ramsey sentence of T then claims that T has at least one realization, without saying which

[46] See, for instance, Quine 2019, p. 82, and Quine 1992, p. 5.

[47] In what follows, I am referring in large part to the classical account of Ramsey's method in David Lewis 1970.

[48] This is the terminology used in Lewis (1970, p. 430).

realization this is. In contrast to the original formulation of T, the Ramsey sentence leaves open what the objects of T are. In the recently published *Kant Lectures*, Quine calls this feature 'Ramsey's anonymization' of the objects of a theory. It reveals, in his view, the structuralist character of all theoretical notions, including the physical ones:

> This structuralistic character not only of class talk but of theoretical terms generally must have been appreciated long since and many times. In a vague way it was a recurrent theme in Russell's Analysis of Matter. Ramsey's anonymization is an effective way of driving it home. It is already implicit in the recognition of sentences as primary in semantics: in the recognition that words depend for their meaning on sentences. (Quine 2019, p. 82)

Note that T and the Ramsey sentence of T have exactly the same empirical consequences. The reformulation of T by its Ramsey sentence preserves the stimulus meanings of all sentences of T. From Quine's behaviorist point of view, the Ramsey sentence can hence be regarded as a materially correct reformulation of T. It makes explicit that the choice of objects does not matter to T.

However, Quine's thesis of the indeterminacy of reference presupposes that 'Fido' is a singular term. If we consider 'Fido' as a bound variable, it refers to all object, and not to a specific one. To make both interpretations compatible, we must consider 'Fido' as a variable for which there is one and only one value satisfying it. This is the basic idea behind Ramsey's method of defining theoretical terms. Let x_1 be the variable representing 'Fido' in the Ramsey sentence of our theory T. We can then define Fido as the first element of the n-tupel that uniquely satisfies $C[x_1, ..., x_n]$:[49]

(F) Fido = the x_1 such that $\exists x_2, ..., \exists x_n\ C[x_1, ..., x_n]\ \&\ \forall y_1, ..., \forall y_n(C[y_1, ..., y_n] \rightarrow x_1 = y_1\ \&\ ...\ \&\ x_n = y_n$.

Obviously, Ramsey's method works only on the condition that $C[x_1, ..., x_n]$ has a unique realization. If $C[x_1, ..., x_n]$ has more than one realization, almost all sentences containing 'Fido' are false, on the Russellean reading of the definite description in (F), and they are neither true nor false, on the Fregean reading. [50] According to the proxy-function argument, the Ramsey sentence of our overall theory of the world has multiple realizations. It consequently implies, again, that almost all sentences of this theory are either false or neither true nor false.

To avoid this consequence, Quine must treat 'Fido' as a bound variable that cannot be simultaneously regarded as a singular term. In this case, the proxy-function argument still implies global structuralism, but it does not imply the thesis of referential indeterminacy. The reason is that the Ramsey sentence of our overall theory of the world does not contain any expressions whose reference is not fixed. The bound variables in '$\exists x_1, ..., \exists x_n\ C[x_1, ..., x_n]$' refer to all objects (of the appropriate type) of all realizations. The same applies to 'Fido', considered as a variable bound by an invisible existential quantifier.

The third strategy finally is suggested by the verificationist theory of truth and meaning that Quine

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49 Cf. Lewis 1970, pp. 429, 437-438. I am simplifying Lewis' account here.

50 This point is also stressed in Lewis 1970, p. 438.

defends in some of his writings.[51] Its main doctrine is that the meaning of a sentence depends only on the observations that speak for or against its truth.[52] The meaning of observation sentences can thus be identified with their stimulus meaning. Since there are no observations speaking directly for or against a theoretical sentence, such sentences have meaning only in an indirect way, namely in virtue of the stimulus meanings of the observation sentences they imply. However, theoretical sentences do not imply observation sentences in isolation, but only together with other theoretical sentences. For this reason, a single theoretical sentence does not have meaning on its own, but only in the context of larger sets of sentences. This holistic thesis distinguishes Quine's version of the verificationist theory of meaning from the versions defended in the Vienna Circle.[53]

Since there are no observations speaking for or against a single word, a singular term has meaning only in the context of the theoretical sentences in which it occurs. The meaning of 'Fido' consists in the contribution it makes to the meanings of the theoretical sentences in which it occurs.[54] According to this context principle, 'Fido' is connected to the world only in virtue of its occurrence in sentences that are connected to the world in virtue of our verbal behavior.

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[51] Davidson (1990, pp. 73-78) distinguishes between two theories of meaning and truth that are defended by Quine, a 'proximal' and a 'distal' theory. The 'proximal' theory is a verificationist approach according to which the truth of sentences depends exclusively on our sensory stimulations (observations).

[52] In 'Epistemology Naturalized', for instance, Quine holds with Peirce that 'the meaning of a sentence turns purely on what would count as evidence for its truth' (1968b, p. 80 f.).

[53] See Quine 1968b, p. 79.

[54] The meanings of words are 'abstractions from the truth-conditions of sentences that contain them' (Quine 1981, p. 69).

The assignment of objects to the bound variables may help us to learn language, but it is not a part of its semantic interpretation. [55] The truth conditions of quantified sentences also depend exclusively on the stimulus meanings of the observation sentences they imply. The primary bearers of reference are observation sentences, and not bound variables, from the verificationist point of view.

The problem of ambiguity does not arise, because 'Fido' is a purely syncategorematic term. The truth of sentences in which 'Fido' occurs does not depend on the existence and the uniqueness of a referent for 'Fido', but only on the observations that speak for or against their truth. Hence, the verificationist approach does not imply that almost all sentences of our overall theory of the world are either false or neither true nor false.

However, this solution of the problem is clearly incompatible with the proxy-function argument. The ontological reductions that are designed to establish global structuralism presuppose that theories have ontological commitments. According to the verificationist approach, on the other hand, theories are ontologically neutral. Observation sentences do not have any ontological presuppositions because they are connected to the world only in holophrastic way, and theoretical sentences do not have any ontological presuppositions, either, because their truth depends only on the truth of the observation sentences they imply. From the verificationist point of view, theories do not have an ontology at all.

The conclusion to be drawn is that Quine's proxy-function argument is a *non sequitur*, from the point of view of

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[55] In his theory of language learning, Quine assumes that the 'positing' of physical objects is an important step of learning the theoretical part of language; see Quine 1973, part III, for instance.

Fregean semantics. It does not show that there are different interpretations of our terms that are equally correct, but only that these terms are ambiguous. Moreover, the assumption that the reference of terms is determined exclusively by our speech dispositions has the absurd consequence that almost all of our sentences are either false or neither true nor false, because they contain definite descriptions whose uniqueness presupposition is not fulfilled. The proxy-function argument must therefore be regarded as a *reductio ad absurdum* of the behaviorist premise that the reference of terms is determined only by our linguistic behavior.[56]

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