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## Flipped classroom and the history of Mathematics: Fourier's legacy and calculus in perspective

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### ABSTRACT

**Objective:** The present work aims to strengthen the dialogue between mathematics in higher education and the history of mathematics, as a way of promoting the construction of interfaces between these fields of knowledge, listing as its theme the work of Jean-Baptiste Joseph Fourier (1768-1830), which was established in 1822 with the publication of the work *Théorie analytique de la chaleur*. **Method:** A qualitative approach, of a biographical nature and with exploratory objectives involving students of the Differential and Integral Calculus discipline in an Engineering course at a University Center in São Paulo. Through activities and literature review, we adopted the Flipped Classroom teaching methodology and an approach from a “history” perspective, anchored by Grattan-Guinness. **Results:** We consider the legacy of the character Fourier, considered fundamental in the discussion about the redefinition of the concept of function, by extrapolating mathematical concepts that were rooted in academic society in the 1800s. The protagonism of students and the time gained in the classroom for discussions about the Fourier Series content proved to be a result of the methodology adopted in permanent dialogue with the historical approach. The theoretical framework brought by the students – represented by the majority of bibliographic references used in this report, brings signs of maturity, favoring possibilities for developments in future didactic experiments. **Conclusion:** Currently, the countless applications of Fourier's studies are present, explicitly or implicitly, in several areas within the scope of engineering, reaching our lives in an inexorable way, often transcending the mere application of technology.

### KEYWORDS

University education. History of Mathematics. Fourier series. Flipped classroom. Active methodologies.

## Sala de aula invertida e história da Matemática: legado de Fourier e cálculo em perspectiva

### RESUMO

**Objetivo:** O presente trabalho objetiva estreitar o diálogo entre a matemática no ensino superior e a história da matemática, como forma de promover a construção de interfaces entre estes campos de saberes, elencando como temática o trabalho de Jean-Baptiste Joseph Fourier (1768-1830), que consagrou-se em 1822 com a publicação da obra *Théorie analytique de la chaleur*. **Método:** Uma abordagem qualitativa, de cunho biográfico e com objetivos exploratórios envolvendo discentes da disciplina Cálculo Diferencial e Integral em um curso de Engenharia em um Centro Universitário em São Paulo. Por meio de atividades e revisão de literaturas, adotamos a metodologia de ensino de Sala de Aula Invertida e uma abordagem sob o viés da “história”, ancoradas por Grattan-Guinness. **Resultados:** Consideramos o legado do personagem Fourier, tido como fundamental na discussão sobre a redefinição do conceito de função, ao extrapolar conceitos matemáticos que estavam enraizados na sociedade acadêmica nos oitocentos. O protagonismo dos estudantes e o tempo ganho em sala de aula para discussões acerca do conteúdo Séries de Fourier, revelou-se decorrente da metodologia adotada em permanente diálogo com a abordagem histórica. O arcabouço teórico trazido pelos estudantes – representados pela maioria das referências bibliográficas utilizadas neste relato, traz indícios de amadurecimento, favorecendo possibilidades de desdobramentos em experimentos didáticos futuros. **Conclusão:** Atualmente, as inúmeras aplicações dos estudos de Fourier encontram-se presentes, explícita ou implicitamente, em diversas áreas no âmbito das engenharias, alcançando as nossas vidas de forma inexorável, transcendendo muitas vezes, a mera aplicação de tecnologia.

### PALAVRAS-CHAVE

Ensino superior. História da Matemática. Séries de Fourier. Sala de aula invertida. Metodologias ativas.

## El aula invertida y la historia de las Matemáticas: el legado de Fourier y el cálculo en perspectiva

### RESUMEN

**Objetivo:** El presente trabajo tiene como objetivo fortalecer el diálogo entre las matemáticas en la educación superior y la historia de las matemáticas, como una forma de promover la construcción de interfaces entre estos campos del conocimiento, teniendo como tema la obra de Jean-Baptiste Joseph Fourier (1768 -1830), que se estableció en 1822 con la publicación de la obra *Théorie analytique de la chaleur*. **Método:** Un enfoque cualitativo, de carácter biográfico y con objetivos exploratorios que involucra a estudiantes de la disciplina de Cálculo Diferencial e Integral de un curso de Ingeniería en un Centro Universitario de São Paulo. A través de actividades y revisión de literatura, adoptamos la metodología de enseñanza Flipped Classroom y un enfoque desde una perspectiva de “historia”, anclada en Grattan-Guinness. **Resultados:** Consideramos el legado del personaje Fourier, considerado fundamental en la discusión sobre la redefinición del concepto de función, al extrapolar conceptos matemáticos que estaban arraigados en la sociedad académica del siglo XIX. El protagonismo de los estudiantes y el tiempo ganado en el aula para discusiones sobre los contenidos de la Serie Fourier resultaron ser resultado de la metodología adoptada en diálogo permanente con el enfoque histórico. El marco teórico aportado por los estudiantes – representado por la mayoría de las referencias bibliográficas utilizadas en este informe, trae signos de madurez, favoreciendo posibilidades de desarrollo en futuras experiencias didácticas. **Conclusión:** Actualmente, las innumerables aplicaciones de los estudios de Fourier están presentes, explícita o implícitamente, en varias áreas dentro del ámbito de la ingeniería, llegando a nuestras vidas de manera inexorable, trascendiendo muchas veces a la mera aplicación de la tecnología.

### PALABRAS CLAVE

Enseñanza superior. Historia de las Matemáticas. Series de Fourier. Aula invertida. Metodologías activas.

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## 1 Introduction

In recent decades, there has been a growing debate surrounding the teaching and learning of mathematics, particularly regarding the use of alternative methodologies in education. Some teachers and researchers have attempted to integrate the practice of science teaching with the history and philosophy of science, with a focus that extends beyond primary education.

D'Ambrosio (2011) emphasizes that philosophy and history should be given priority in teaching mathematics, even at the expense of in-depth technical understanding. This is because digital media now allows for the contemplation of mathematical concepts. All technical terms are explained when first used, and the language is clear, objective, and value-neutral. The text follows a conventional academic structure and maintains a formal register. The sentences and paragraphs create a logical flow of information with causal connections between statements. The text is free from grammatical errors, spelling mistakes, and punctuation errors. No changes in content have been made. D'Ambrosio argues that the teaching of knowledge in the present depends on understanding its origins, motivations for development, and what can be learned from the past. The presence of this knowledge in school curricula is justified by a vision of the future (D'Ambrosio, 2011, p. 3).

Matthews (1995, p. 165) emphasizes the lack of understanding in science classrooms, where formulas and equations are often recited without comprehension. To address the crisis in contemporary science teaching, the scholar suggests a more dialogical approach to scientific concepts, linking them to the history and philosophy of science.

Additionally, during historical investigations, teachers have a responsibility to reveal to students the essential human nature of science. When considering the history of mathematics and its concepts, it is important to note that they primarily emerged as a result of human needs. It is also important to recognize that the constitution of this knowledge underwent a process of re-signification that lasted several centuries, culminating in the way we present and recognize these mathematical concepts today (Godoy; Leite, 2021, p.3).

Faced with these perspectives, specifically in engineering courses and throughout the process of building knowledge in academic teaching, we see that the first contact with the work of Jean-Baptiste Joseph Fourier (1768-1830), for example, occurs mainly with the presentation of the series of functions, which were fundamental to the development of the law of heat propagation in the 18th century. Nowadays, these discoveries have proved to be of considerable importance not only in the field of mathematics, but also in modern physics, due to the countless possibilities of applications that arise from such concepts, surpassing the initial proposal of the scholar at the time of their development. However, the approach to this important mathematical concept and its developments are often presented in a bland manner in the classroom, and disconnected from relevant aspects in relation to its historical construction.

With these perspectives in mind, we will present in this paper some results of a theoretical scientific initiation, which reverberated in classroom practice, made possible through some dynamics, in the interstice of an academic term in 2018 at a University Center in the state of São Paulo. Specifically developed in a class of future engineers, we sought to establish a dialog between the teaching of mathematics and the history of mathematics, as a way of sparking discussions and promoting the construction of interfaces between these fields of knowledge.

To this end, we chose as our theme the work of Fourier, a figure considered fundamental in the discussion on redefining the concept of function, by extrapolating mathematical conceptions that were rooted in academic society at the beginning of the 19th century, based on his studies on the propagation of heat. We base this work on the idea that the production of meanings of mathematical concepts is closely related to the professional context of the engineering course, in this case, and from this perspective, we pay attention to highlighting the investigation of some historical scenarios and paths taken, both by Fourier and by other scientists who succeeded him, through the historical-mathematical perspective, entering into other sciences that supported and boosted the emergence of a range of applications. This first stage of the work is related to theoretical development, the fruit of Scientific Initiation.

In the second stage, we will examine the classroom experiment, a result of the first stage. Although the authors did not conduct a specific study on it, the experiment indicated possibilities for teachers who need to teach the content covered in the initial research and see the feasibility of dialogue between the history of science and active learning methodologies. The decision to facilitate the understanding of complex concepts for numerous students justified our choice.

Our didactic proposal is based on the idea that classes dealing with mathematical content should be preceded by a historical-mathematical analysis and reflection on the development of Fourier's studies on the propagation of heat. We will discuss these procedures in detail below.

**2 Theoretical foundation**

One premise of this work is to approach the history of science in teaching cautiously, avoiding a distorted view of the past and distancing ourselves from anachronism. As Matthews (1998) suggests, it is desirable for students to develop a less naive view that considers the complexity of the scientific enterprise (cited in Schimiedecke & Porto, 2014, p. 234).

To achieve the proposed objective of our scientific initiation, we analyzed the studies of Ivor Grattan-Guinness (2004), a renowned mathematical historian of the 19th century who critiqued Fourier's first monograph on the propagation of heat presented in 1807 during the theoretical stage of our didactic experiment.

As a means of understanding the historical context in which Fourier developed his work and its consequences, we highlight the relevance of Pifer's (2015) work. This author's theoretical contributions were used in the first phase of the research and their work was suggested as a reference to aid the teaching methodology in the second phase, utilizing the SAI Active Methodology - Inverted Classroom. The book aims to analyze the conceptual and epistemological foundations of the theory of heat conduction, as demonstrated in Fourier's studies.

Regarding the active SAI methodology, in addition to Bergmann and Sams (2012), Weiderpass (2003) helped us with the scope of Fourier's work and Doria (2010), who pointed out the range of applications of these studies today, especially about the use of Fourier transforms in signal studies in electrical engineering and quantum computing.

We also took a critical approach to the historical and social moment in which Fourier developed his studies on the propagation of heat, based on Roque (2012), without presenting a “modern” view of the facts. Although we focused on contemporary historiography, we also used the traditional work by Boyer (1996), as it was the main source the students used in their individual research, since we used the SAI methodology.

It is important to consider that the didactic experiment is linked to the possibility of approaching content whose syllabus indicated study material and, in the case of Fourier Series, the book by Loreto et al. (2012) helped in the treatment of this mathematical content. We will go into some specifics of how the classes were conducted at an appropriate time in this presentation.

3 Methodological procedures

This was also a biographical study with exploratory objectives, considering what Gil (2002) pointed out, which was to develop the work based on previously produced documents, as opposed to documentary research, which uses materials that have not yet received an analytical treatment. The exploratory objectives are linked to the purpose of examining a little-studied topic, in this case, Fourier series, in its historical aspects, with pedagogical potential.

Considering the singularities of historical research, the qualitative approach was linked to the methodological foundation of mathematics historian Grattan-Guinness (2004), who refers to two approaches to historical research: “history”, which emphasizes history as such, and “inheritance”, which emphasizes inheritance from the perspective of historical legacy, and through cumulative processes.

To distinguish between the two types, the scholar starts with some fundamental questions. In the first case, in general terms, he seeks to reflect on when “history” asks "what happened in the past?' [...] 'why?' [...] 'what didn't happen in the past? and why not?'" (Gratthan-Guinness, 2004, p. 164). From an “inheritance” perspective, the crux of the question refers to: "how did we get here?'" (Gratthan-Guinness, 2004, p. 165).

Regarding these approaches, while defending the legitimacy of both approaches, the scholar highlights problematic aspects of confusing them, either by taking "inheritance" as "history", or "history" as "inheritance". Regarding the philosophical difference between the two approaches, the scholar highlights "The philosophical difference is that heirs tend to focus on knowledge (theorems as such, and so on), while historians also look for motivations, causes and understanding in a more general sense" (Gratthan-Guinness, 2004, p. 165). Having said that, and given the necessary explanation of this method, in our theoretical work we have adopted a predominantly "historical" approach, as we believe that analysis from the perspective of "heritage" requires more in-depth studies of the subject, which would be beyond our scope.

In the second stage of the work, the central object of this presentation, through the reverberation of theoretical research into classroom experience, we adopted the Inverted Classroom (IAS) as a teaching methodology, as we believe that this pedagogical practice favors the formation of more autonomous and questioning students, inviting them to (re)signify their role in the classroom. To this end, we draw on the work of Bergmann and Sams (2012), which, although based on the implementation of the inverted classroom in their American high school subjects, has recently become a pedagogical alternative in many universities, since this student search for knowledge, in addition to the exercise of freedom, favors the search for their future identity in the professional sphere.

In addition, given that the didactic experience emerged from an offshoot of the Scientific Initiation project, which was theoretical in nature, the use of SAI seemed more relevant as a way of optimizing time in the classroom, a factor that often hinders bolder practices. It also evokes the need to review the positions and responsibilities of the teacher and student throughout the teaching and learning process, as Felcher et al (2021) point out, alluding to Bergmann and Samns (2018).

As for the actual lessons listed in the course syllabus, it should be noted that the experiment took place over two weeks with two weekly meetings of 2 hours each, preceded by a lesson before the first week, as a way of stimulating the readings that were to follow. In a total of 9 hours of lessons, in the week leading up to the experiment, specifically in the second half of the lesson, a broad historical focus on the whole panorama that was taking shape in the scientific milieu at the end of the 18th century was initiated, asking as a dynamo some questions that we will go into more specifically in our results and discussions.

4 Paths and the "mathematics" of Fourier

The study of series began in the 18th century with the works of Jean Le Rond D'Alembert, Leonhard Euler, and Daniel Bernoulli. They were primarily concerned with solving the problem of vibrating strings and aimed to determine the mathematical law at a given time *t* based on the movement of the string. During the 19th century, the concept of function was extensively debated. Fourier was particularly notable in this regard, as he connected the definition of function to a physical problem: the study of heat propagation (Roque, 2012).

Fourier was born in Auxerre, France, and was orphaned at the age of eight. Despite his humble origins, he was appointed to the chair of mathematics at the military school run by the Benedictines, where he received his education. Although he was appointed to a chair at the Polytechnic School in Paris, he chose to accompany Napoleon on his expedition to Egypt due to his political involvement. He served as the governor of Lower Egypt in 1798 and returned to France in 1801. During his time as the mayor of Grenoble, he focused on his experiments with heat conduction in bars, as noted by Nunes (2002).

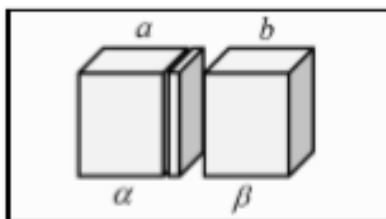
In 1807, Fourier submitted a paper on heat propagation to the Paris Academy of Sciences. In this work, he developed his initial concepts on representing functions through trigonometric series. Although he attempted to demonstrate that, any function could be represented by the sum of a trigonometric series, it was not until 1822 that he gained fame with the publication of his work, *Théorie analytique de la chaleur*, as Roque (2012) recalls.

Although Fourier studied under prominent physicist-mathematicians of his time, including Joseph-Louis de Lagrange (1736-1813), Pierre Simon Laplace (1749-1827), and Gaspard Monge (1746-1818), the first two criticized his work for its lack of rigor. Fourier believed that a function of one variable, whether continuous or discontinuous, could be expanded into a series of sines of multiples of the variable. Lagrange was the main critic of Fourier's demonstration because he had previously dealt with this problem.

Since the 18th century, the Newtonian model has been a reference point on the European continent, influencing and stimulating countless scholars in their search for understanding, implications, and the development of mathematical research methods. During this period, a process began that would culminate in the replacement of the geometric synthesis method with algebraic analysis. This made it possible to describe physical phenomena using differential equations, which became the primary tool for researching natural phenomena. Pifer (2015) notes this shift in his work. These discussions and concerns about the redefinition of the concept of function, as well as issues such as the redefinition of 'number', led to the emergence of a movement in the mathematical community that questioned its foundations (Roque, 2012).

In this context, and at the mercy of the prevailing epistemology, Fourier began his investigations into the propagation of heat in solid bodies between 1802 and 1803, with the aim of formulating a law for the conduction of heat, as Pifer (2015, p.69) points out. With the aim of obtaining a mathematical law that would make it possible to indicate the relationship between the distribution of temperatures in a solid body as a function of time, Fourier first developed a simplified model, using only two discrete bodies of the same mass  $m$ . These bodies were separated and had different initial temperatures  $a$  and  $b$ , and Fourier started from the premise that there was no dissipation of heat into the environment, as Pifer (2015) points out.

Figure 1: Model of heat conduction between two discrete bodies



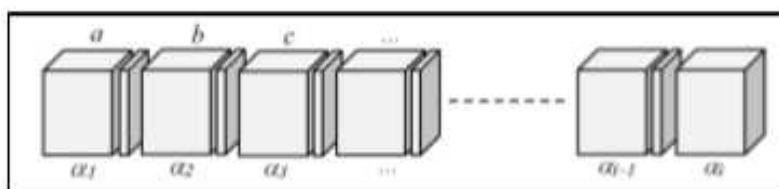
Source: Pifer (2015, p. 70)

According to Pifer (2015), the exchange of heat between these two bodies took place through a thin layer of mass  $dm$ , which moved from the warmer body  $a$  towards the colder body  $b$ , transmitting heat to it instantly. After contact, the layer returned to its original body, and this sequence was repeated until the temperature between the bodies equilibrated. In this way, Fourier obtained the following first-order linear differential equations, the solution of which provided the temperatures  $\alpha$  and  $\beta$  of each body as a function of time:

$$\alpha = \frac{1}{2}(a + b) + \frac{1}{2}(a - b)e^{-\frac{k}{m}t} \quad \beta = \frac{1}{2}(a + b) - \frac{1}{2}(a - b)e^{-\frac{k}{m}t} \quad (4.1)$$

However, his main goal was to obtain a corresponding solution for continuous bodies through a previous study. To this end, he postulated that the discrete n-bodies tended to infinity (Pifer, 2015):

Figure 2: Model containing discrete n-bodies



Source: Pifer (2015, p. 71)

The scholar even managed to find a general solution for the conduction of heat, presenting the calculation of temperature for two and three bodies, but failed to complete the fundamental analysis, which was his goal, claiming that:

The analysis we employ can be used to determine the laws of heat propagation in bodies of a few dimensions. But this transition from the solution of a finite number of bodies to an infinitesimal solution (if we may say so) requires complicated calculations (Fourier, 1822 *apud* Pifer, 2015, p.72).

After obtaining these findings, Fourier abandoned his approach to discrete bodies. However, in the latter half of 1804, he resumed his investigations into the propagation of heat in continuous bodies after being influenced by the work of French physicist Jean Baptiste Biot (1774-1862), who had studied heat diffusion in solid bodies (Pifer, 2015).

According to Pifer (2015), the studies were influenced by Biot and based on Newton's law of cooling. The approach applied different conductivity coefficients to the internal and



external propagation of heat. From these studies, Fourier arrived at a new equation (4.2) that expresses the amount of heat transmitted between the molecules of a solid body in three dimensions.

$$\frac{dv}{dt} = k \left( \frac{d^2v}{dx^2} + \frac{d^2v}{dy^2} + \frac{d^2v}{dz^2} \right) - hv \tag{4.2}$$

In which:  
 hv = refers to the heat given off to the medium.

Regarding the importance of these results, it is noted that:

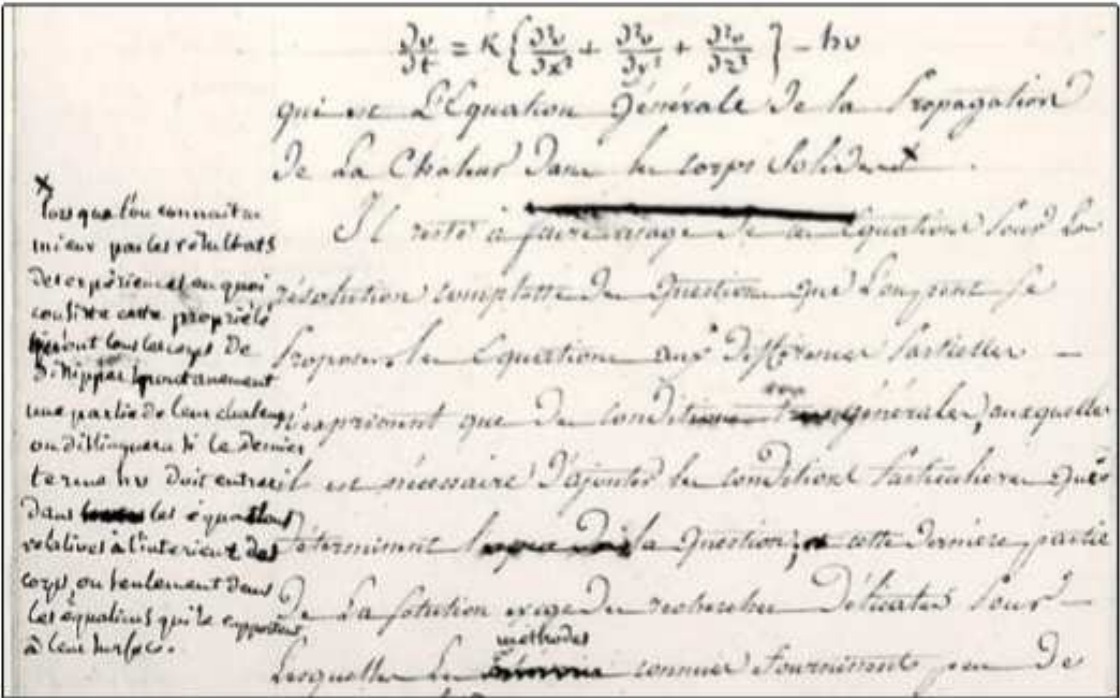
They were the first to attempt a quantitative study of heat propagation in a continuous body by translating it into differential equations. They also studied the movement of heat in a non-stationary state for the first time (Pifer, 2015, p.76).

However, these works presented restrictions in relation to some physical aspects and, faced with this obstacle, Fourier points out that:

When experimental results provide a deeper understanding of the property that allows all bodies to spontaneously dissipate heat, we will know whether to include the term 'hv' in equations related to the interior of bodies or only in equations related to their surfaces. This statement was made by Fourier in 1822 and cited by Pifer in 2015 (p.76).

In the manuscript (fig. 3), Fourier discusses the equation for the propagation of heat in three dimensions for the non-stationary state, and in the left margin, there is a note about the doubt of the studies regarding the results of his equation. This note was transcribed by Grattan-Guinness and Ravetz (1972, p. 111), according to Pifer (2015).

Figure 3: Extract from the 1805 manuscript



Source: Pifer (2015, p. 7).

Another limitation, in the field of physics, was the lack of knowledge of heat flow in certain specific conditions, among others, which prevented Fourier's studies from advancing. Pifer, referring to Herivel, points out that:

These first two approaches to the problem of the propagation of heat in solid bodies were recorded in a collection of manuscripts, dated 1805, which was probably prepared intending to be published, which did not happen (Herivel, 1975 apud Pifer, 2015, p. 69).

Despite these obstacles, the mathematician persevered with his research. He incorporated and articulated previously absent physical concepts, which allowed him to obtain new results. At the end of 1807, he submitted a new work entitled '*Mémoire sur la propagation de la chaleur*' to the Academy of Sciences of the *Institut de France in Paris*.

Fourier presented the main physical concepts used in the development of the theory and demonstrated the determination and solution of the heat propagation equation in a stationary bar. He also considered bodies with other geometries. However, according to Pifer (2015), Fourier showed the development of the general equation of heat propagation in a solid by applying it to a cube with finite dimensions.

Roque (2012, p. 318) notes that Fourier claimed any function can be expressed as a sum of a trigonometric series. Fourier solved the equation for each component of the sine curve with different wavelengths and combined them.

He believed this method could solve any problem, even sudden temperature changes. However, only Euler and Lagrange acknowledged this possibility. They did so only for particular functions. As a result, mathematicians who evaluated his work did not show enthusiasm, nor did they openly criticize him for his supposed 'lack of rigor,' as Roque (2012) points out.

Also in 1810, Fourier revised his previous work, adding only two new sections and aiming for the prize offered by the *Institut de France*, which, despite being awarded, was not published. On this study, the examining committee made up of Lagrange, Laplace, Etienne Louis Malus (1775-1812), René Just Haüy (1743-1822) and Adrien-Marie Legendre (1752-1833), would add:

This work contains the true differential equations of heat transmission, both inside bodies and on the surface, and the novelty of the subject, together with its importance, made the committee crown this work, noting, however, that the way in which the author arrives at his equations is not without its difficulties and that his analysis to integrate them still leaves something to be desired, both in terms of generality and rigor (Darboux, 1890 apud Pifer, 2015, p.84, emphasis added).

Fourier Transforms are the result of his equations. They calculate the amplitudes and frequencies of sinusoidal curves that make up a 'signal' varying in time.

Weiderpass (2003) defines a periodic signal as a signal that can be broken down into

sinusoidal or cosine components, each with multiple frequencies that are multiples of the fundamental frequency, which is the inverse of the signal's period. If present, a continuous component is also included in the spectrum of the signal, which consists of the continuous component, the fundamental component, and its harmonics.

The Fourier analysis of a signal characterizes its components by distributing its power or energy among its harmonic components. This can be done using a computer and the discrete Fourier Transform (Orsini apud Weiderpass, 2003, p.2). The Fourier transform of a signal decomposes it into complex exponential functions at different frequencies, allowing us to obtain its frequency components. Equation (4.3) shows how this is done.

$$X(v) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi v t} dt \tag{4.3}$$

In which:  
 t = represents time in (s),  
 v = the frequency in Hertz (HZ),  
 x = the signal in time  
 X = the signal in the frequency domain.

Nowadays, Fourier transforms have numerous applications in our daily lives. They are used to analyze earthquake vibration signals, obtain the structure of DNA from X-ray images, and process signals in telecommunications. In this article, we present some examples of the applications of Fourier transforms as pointed out by students in our classes.

Fourier's work has a wide range of applications in electrical engineering education, particularly in signal studies of electronic devices. One such application is determining harmonic distortion, or non-linear distortion, in various systems. This phenomenon, observed in MOS transistors, corresponds to the existence of signals with frequencies that are multiples of the original signal applied to the input. According to Doria (2010), the distortion in the system is dependent on the operating point and the amplitude of the input signal due to the non-linear transfer of the device used.

According to Doria (2010), applying a sinusoidal signal with an angular frequency of  $\omega = 2\pi f^1$  to the input of a MOS transistor will result in a non-linear transfer characteristic that produces a harmonic output signal with frequencies that are multiples of  $\omega$ , thereby modifying the original signal.

Thus, the Fourier series or Fourier transform can be used to determine harmonic distortion in various systems, which is a fundamental finding for Automation and Control engineers. This topic is studied by students who have entered the specifics presented, beyond the usual decontextualized classroom calculations. In systems with non-periodic signals, the fast Fourier transform is necessary, while Fourier series are used for periodic signals, as highlighted by Doria (2010).

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<sup>1</sup> The term f corresponds to the angular frequency of the signal.

## 5 Results and discussions

Felcher et al. (2021) describe the Flipped Classroom methodology as the possibility of completing classroom tasks at home. This approach resulted in more comprehensive research by students concerning Fourier series and reverberation. This paper summarizes the main class discussions based on text references explored by the students and suggested by the instructor.

As previously stated, we held two weekly meetings lasting 2 hours each, with a 1-hour lesson preceding the first meeting, for a total of four meetings. During this introductory class, we provided an overview of the history of mathematics in the late 1700s and early 1900s, a pivotal moment as discussed in the previous chapter, where we reflected on the following questions: It is interesting to note that although function calculus is typically taught before integral and differential calculus in high school, the concepts actually arose in reverse order. How is this possible?

Based on these questions, we directed the rest of the meetings, offering texts from our finalized theoretical research with a historical focus. We suggested that they research sources on preliminary ideas about some types of functions and their characteristics. The language used is clear, objective, and value-neutral, with a formal register and precise word choice. The text follows a logical structure with causal connections between statements and avoids biased language. It is free from grammatical errors, spelling mistakes, and punctuation errors. No changes in content were made.

Regarding classroom procedures, in the first meeting after the request for reading and research, we discussed the historical context that influenced the development of Fourier's series. We had dynamic discussions about the maturation of function definitions to better understand the scholar's work and its implications. This was done superficially. This difficulty in selecting reliable texts and sources for the next stage is a common issue among students. Our goal was to introduce them to the history of mathematics in the early 19th century, and only after this initial exposure did the students begin to share their impressions and suggestions in subsequent meetings.

Furthermore, considering that our goal was to help students understand Calculus III in a course for future engineers, before we got into the formal concepts of the Series of Functions content, we suggested in all the previous classes, except for the first of the four meetings, as mentioned, that the students research the character of Fourier and the scope of his work. We presented the findings of our initial research in the previous section of this report, and the results reported by the students throughout this section.

It was only in the second meeting, in the second class specifically, that we entered the field of Fourier's series of functions and the mathematical concepts involved, which was favored by the alignment of the first meeting, in which discussions emerged about the path taken by Fourier in the construction of such concepts, and concerns of contemporaries about mathematical knowledge of that period. The textbook used in the course refers to the material

by Loreto et al. (2012), which presents the content related to the course syllabus. In all the meetings, the discussions were conducted in the form of debates between groups of students, with freedom in this formation.

In these discussions, in which we highlight the third and fourth meetings, it was possible to investigate, based on the students' greater maturity, some historical scenarios and paths taken by both Fourier and other scientists who succeeded him, through the historical-mathematical perspective, going into other sciences that supported and boosted the emergence of a range of applications that arose from this.

It was also observed that the students sought to understand the implications of the difficulties faced by Fourier in validating his work, particularly in relation to the mention of the committee that analyzed and referenced it in 1810. The committee's rejection, pointing out that his analysis "left something to be desired", due to its supposed lack of rigor, revealed to us the underlying interest in establishing the mathematical edifice on more "solid" ground at the beginning of the 19th century. This process would inflate from that period onwards, and erupt in mathematical concepts to the present day.

These events have also led to reflections on the controversies that arise in the history of science, where missed opportunities or delayed discoveries are often the result of prevailing epistemologies and the influence of authority.

Grattan-Guinness notes that the historical events referred to are assuming that knowledge has been constructed through the usual sequences ( $N_0, N_1, N_2, \dots$ ), but acknowledges that this may not always be the case (Grattan-Guinness, 2004). Furthermore, it should be noted that a path leading from  $N_0$  to  $N_1$  may give the impression of being based on a deterministic perspective, with oversimplified generalizations based on cause-and-effect relationships. It is important to avoid such subjective evaluations and instead present a clear and objective analysis.

During class discussions, we considered the circumstances under which a path taken to validate a theory or discovery could be classified as determined or not. We also discussed whether these paths developed slowly or intermittently, as numerous factors are interwoven throughout such processes. This enriched our understanding of the controversies surrounding these topics.

Regarding the students' theoretical feedback, it is worth noting that many of them brought documents that added information about the subject from the second meeting onwards. Additionally, the fact that a significant number of students had previously read traditional historiography on the history of mathematics positively contributed to the class discussion. This is because when they encountered other positions based on contemporary historiography, it allowed for a more critical reflection. They realized that history can always be rewritten, given the differences that emerged when they briefly glimpsed aspects of the historiography of the sciences.

Joseph Fourier's work emerged during the Industrial Revolution, a crucial period in modern history that influenced countless aspects of everyday life. This uniqueness sparked heated discussions. It is important to note that this evaluation is subjective and should be clearly marked as such. Some students referred to contemporary historians who aim to reevaluate this movement from various perspectives. They understand that changes usually occur gradually, and using the term 'revolution' is incorrect.

Despite the historiographical controversies, it is evident that the late 18th and early 19th centuries were marked by the emergence of innovative ideas and a desire for new mechanisms, as noted by Costa (2014). This period represented a significant time for technological, economic, and social advancements that had been developing in Europe for centuries. Pifer (2015, p. 17) notes that these movements enabled the creation of a new scenario characterized by novel research procedures, the discovery of new phenomena, and, most importantly, the development of new theories to explain them.

The debate between tradition and modernity, which was significant throughout the eighteenth and nineteenth centuries, had broad repercussions in mathematics. One of the most relevant repercussions of our work was the gradual replacement of the geometric synthesis method by algebraic analysis, as suggested by Moura (2017, p.18). During the transition process, the notion of function underwent a series of redefinitions, driven by the concept of rigor at the time. As a result, functions became the primary focus of mathematical study during this period, largely due to the development of calculus, which had been evolving since the 1600s.

Regarding this change, Roque (2012, p. 271) notes that mathematics was transformed into its object, becoming the law of variation or function, according to Jaques Hadamard. Mathematics being ceased to be just a number and was enriched by new methods.

The changes were not limited to mathematics but also impacted physics. The study of physical phenomena, such as heat, light, gases, electricity, and magnetism, became more systematic and less experimental (Pifer, 2015). The scholar also notes that the use of 'qualitative elements,' such as 'subtle fluids,' enabled the prediction, rather than just the understanding, of physical phenomena (Pifer, 2015, p.18). During this period, the description of physical phenomena underwent a gradual change. Pifer (2015, p.19) notes that this was achieved through the use of analogous differential equations. This allowed for physical explanations to be replaced by the deduction and resolution of equations that describe such phenomena, even when their origins were unknown.

In this context, and amidst this buzz, it is evident that the current perception of mathematics is closely linked to the events of the 19th century. This is particularly true in relation to the history of analysis and infinitesimal calculus, with emphasis on the notion of function, as noted by Roque (2012, p. 343). The scholar notes that Fourier's work, among countless others, would become fundamental in discussions about the notion of function in that

period due to a physical problem in the sphere of heat propagation. This work would serve as a guiding thread that would energize what would follow, in many respects, up to the present day.

It is important to note that most works in the History of Science that aim to highlight Fourier's works are not thoroughly explored from a Physics perspective, as pointed out by Pifer (2015). Therefore, the choice of this work is particularly relevant as it can also facilitate interdisciplinary approaches in Physics education.

Additionally, it is recognized that the formalism of quantum physics since the 20th century is also connected to the Fourier series. This mathematical concept was developed by a scholar in the early 19th century and is currently being applied in studies related to computing. The work of this scholar has had a significant impact on modern research. The Calculus classes initially took a historical approach to Fourier's work. However, the students' feedback in the last meeting enriched the classes with knowledge that emerges in contemporary science. This feedback, based on reviews of the classes, proposed material, and material brought in by the students, is presented here.

Thus, we engaged in discussions about the mechanistic conceptions that were strongly rooted in 19th century European scientific society. All physical phenomena in nature are described by laws with a high degree of determinism. This is in line with our proposal to provoke reflection.

In general, during this last meeting, we discussed the emergence of the quantum theory at the beginning of the 20th century. This theory arose due to the need to explain certain physical phenomena, particularly the behavior of radiation emitted by black bodies, which classical physics, until then, consolidated, could not satisfactorily explain. From this point on, Quantum Mechanics would begin to be developed, becoming a mathematical framework capable of describing various physical phenomena on an atomic scale (Marquezino, 2006).

This new theory completely opposed the notions of determinism and reality present in classical mechanics, making predictions in probabilistic terms and describing the world essentially from indeterministic perspectives, as pointed out by Pessoa Jr. (2003). Discussions have revolved around inquiries regarding the description of an unobserved reality, in contrast to the positivist view, based on the premise that the role of science is limited to the description of observable phenomena, avoiding any speculation about mechanisms that are beyond experimental verification.

Regarding quantum computing, it is important to note that its emergence occurred during a period of prevalence of a positive view of science. This fact challenged the assumptions that supported quantum theory, given the scarcity of empirical evidence supporting, among other points, the possibility of the existence of infinite parallel universes. Despite the emergence of various attempts to interpret quantum theory in recent decades, many of these interpretations remain within a positive perspective.

These new interpretations have primarily relied on the notion of 'information', which has in a way ensured a non-emblematic definition. However, despite the correctness of the mathematical theory regarding the concept of 'information', there are still many problems, particularly in the philosophical sphere, related to this concept, as pointed out by Pessoa Jr. (2003). Questions such as 'What language is this?' and 'What is the origin of computing?' were raised in class, considering that we may need a theory that requires more foundation.

Regarding reflections on quantum computing, the students in our class expressed that the current level of computational development is closely related to the philosophical movements that emerged in the sciences, particularly in mathematics, during the second half of the 19th century. It is important to note that this is their subjective evaluation and should be clearly marked as such. These considerations confirm the importance of exploring the proposed theme.

Despite numerous controversies surrounding the philosophical and conceptual aspects of quantum mechanics, which were not the focus of our didactic approach, these debates led the students to consider the significance of quantum algorithms for computing development, particularly in terms of time-saving when solving problems compared to classical analogues. The use of the Quantum Fourier Transform (QFT) in the quantum algorithm has been shown to be faster than the classical algorithm. This reveals that there is still much potential for further Fourier studies.

6 Final considerations

In this brief experience report, we observed that the intervention of the historical approach adopted in the classroom in an engineering course in Calculus III met our objectives, notably due to the choice to use the Inverted Classroom methodology, as far as the students' protagonism was verified and the time gained in the classroom for discussions on the subject was expanded. Subsequently, when the mathematical content involved in Fourier series was taught, there were no difficulties in terms of time management, which, we believe, was mainly due to the motivation shown by the students.

As for the theoretical framework brought by the students - represented by the majority of the bibliographical references used in this article, this involvement revealed an indication of the maturity of the students in the class, favoring future research. However, it is important to consider that this receptivity may have been mainly favorable because the group of students were not in the first periods of their undergraduate studies, since this active and autonomous participation, a fundamental aspect of the process, is not always understood and assumed by students in general.

Moreover, as the purpose of our classes was to discuss the relevance and scope of Fourier's discoveries, and the discussions gave rise to reflections in the field of quantum physics, we even chose to discuss this topic in class, but we tried not to dwell on deeper philosophical analyses and discussions. In this respect, we draw on the emphasis given by Bergmann and Sams (2012, p. 54-55), when referring to situations in which the teacher must be



able to admit when he or she does not have the answer to all the students' questions and must be ready to assume this. These would also be opportunities for the teacher to be a student leader, guiding and collaborating in this vast world of information, not having all the answers.

In turn, we believe it is pertinent to point out that, although the treatment given by Fourier in his work dates back to the beginning of the 19th century, its scope goes beyond the time barrier, given that quantum theory appeared almost a century later. Situations like this allow us to infer that many discoveries in the field of mathematics do not always have immediate applications. In fact, this fact is notorious.

As this is a development of research, which until then had been theoretical and at the level of scientific initiation, we understand that much still needs to be added to this classroom experiment, as it was not programmed at the level of academic research. Thus, our presentation, reflections, and directions signal a possibility to be adapted and contemplated with further meetings, theoretical research, etc. The results of our experiment provide a glimpse of the possibility of didactic projects to be promoted in higher education engineering courses. Hence, the use of the unprecedented SAI methodology emerges as a way of optimizing the experiment.

No less importantly, we believe that in the classroom experience, the proposals for reflections on episodes that discussed the formalism of quantum physics in relation to Fourier series, starting from the historical approach to the interlocution with some aspects of computing, favored, above all, the possibility of humanizing the sciences, bringing them closer to personal, ethical, cultural and political interests and, thus, decapsulating the student's critical and reflective thinking.

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