Reflections and knowledge of teachers who study the area of plane figures

Reflexões e conhecimentos evidenciados por professores que estudam área de figuras planas

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Abstract

This study aims to discuss and understand the process of (re)significance of teachers’ professional knowledge based on discussions and reflections carried out by a group of teachers who teach mathematics for the initial years of the elementary education. The teachers studied how to calculate the area of plane figures using squared arrays. This study is based on Zeichner’s and Serrazina’s research to discuss the reflection on the practice and, in Ball, Thames, and Phelps, to analyse the (re) significance of the knowledge of the professionals involved. The data analysed were collected during the group meetings at the school where the participants taught. Through the analyses of the discussions held, we could observe that the teachers expanded their common and specialised content knowledge, mainly in relation to the strategies to calculate area, and started using reconfiguration of the figures and formulas of area more assertively. Mutual support allowed them to identify their own needs and then modify their analyses of the strategies for area calculation to be adopted.

Keywords: Study group, Teaching professional knowledge, Plane figure area

Resumo

Apresenta-se, neste artigo, um estudo cujo objetivo é discutir e compreender o processo de (re)significação dos conhecimentos profissionais dos seus participantes concernente às discussões e às reflexões explicitadas por um grupo de professores que leciona matemática para os anos iniciais do Ensino Fundamental. Os professores estudaram o cálculo de área de figuras planas por meio da utilização de quadriculados. Fundamenta-se nos estudos de Zeichner e Serrazina para discutir a reflexão sobre a prática e, em Ball, Thames e Phelps, para analisar a (re)significação dos conhecimentos dos profissionais envolvidos. Os dados analisados foram coletados durante os encontros do grupo na própria escola na qual os participantes lecionavam. Por meio das análises das discussões ocorridas, foi possível observar que os professores ampliaram o conhecimento comum e especializado do conteúdo, especialmente em relação às estratégias de cálculo da área, e ainda passaram a utilizar reconfiguração das figuras e as fórmulas de área com maior facilidade. O apoio mútuo permitiu que eles identificassem suas próprias necessidades e, então, modificassem suas análises acerca das estratégias para o cálculo de área a serem adotadas.

Palavras-chave: Grupo de estudos, Conhecimento profissional docente, Área de figuras planas

Submetido em: 18/06/2018 – Aceito em: 24/08/2020 – Publicado em: 18/12/2020

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Introduction

In this article, we present an analysis of discussions and reflections that occurred in the school environment with the participation of a group of studies of teachers who teach mathematics for the initial years of elementary school. The participants of this research studied and reflected in a shared way on the theme area of flat figures and their teaching for the initial years of elementary school, to discuss and understand the process of (re)significance of their professional knowledge.

The relevance of the topic in the mathematics curriculum is notorious both for its practical application and for the understanding of other mathematical concepts. This fact is explicitly considered in the Brazilian curricular guidelines, such as in the National Curricular Parameters – PCN – (Brasil, 1997) and in the National Common Curricular Base – BNCC – (Brasil, 2018), for example. In them, the idea of an area appears when both the skills of the thematic unit (or block) Grandezas e Medidas (quantities and measures) and those related to Números (or Números e Operações - numbers and operations) are described. In these two documents, the development of this idea is expected to introduce the meanings not only of multiplication but also of the rectangular configuration of multiplication, to understand the idea of measurement from and figures drawn on grids, and comparison of perimeters and areas of two figures without the use of formulas.

In this context, referenced in Gimeno Sacristan (2000), we consider the relevance of the teachers’ role in the curriculum guidelines, since they are fundamental in at least four of the six levels of the curriculum development. Teacher shapes and implements the curriculum, and often assesses the effects produced. Analysing the curricular indications for the teaching of the area of plane figures, we consider it important for the teachers’ professional performance that they understand that the calculation of the area in squared arrays presupposes much more than “counting squares” or using the empirical compensation square by square.

Besides looking at the curriculum documents, we also rely on research that discusses the teaching and learning processes of the theme and the teachers’ and prospective teachers’ knowledge of plane figure areas. Clements and Stephan (2004) discuss issues related to the understanding of the concept investigated here. The authors assessed types of tasks that may favour the understanding of the area of plane figures and concluded that the recurrent practice of simple counting of units to find an area “is a recipe for disaster” (p.20). They consider that, for teaching the theme, teachers should take into account that students must:

(a) construct the idea of units of measurement (including developing a sense of measurement for standard units; for example, finding common objects in the
Like Clements and Stephan (2004), we understand that the calculation of an area is quite complex and, therefore, it is necessary that the professional who will teach it also knows, among other skills, how to calculate the measure of the area, analysing the figure as a whole, by gathering of figures, the equivalence of areas, and the composition and decomposition of figures.

Regarding the knowledge of future teachers who teach mathematics for the initial years related to the calculation of the area of plane figures, we are guided by investigations such as those of Baturo and Nason (1996), Garcia Silva et al. (2013) and Gomes et al. (2018).

Baturo and Nason (1996) assessed the prospective teachers’ knowledge of the measurement of areas through individual interviews during the resolution of eight tasks created for the research. In proposing such tasks, the authors sought to identify concrete and computational knowledge of the concept of area, and found limitations in the repertoire of the concrete knowledge, since the participants had difficulties in relating the rules or formulas with concrete experiences. On the other hand, Garcia Silva et al. (2013) and Gomes et al. (2018), when investigating the knowledge of experienced teachers who taught mathematics for the initial years, found that the strategies used by the participants of a continuing education process were focused mainly on the counting and reorganisation of small squares, which, in our view, could be a limiting factor to the teaching of the theme.

Thus, considering the complexity inherent to the construction of teaching knowledge, we believe it is relevant to analyse the (re)significance of teachers’ professional knowledge of the intuitive models for calculating the area in squared arrays and their teaching, from work with a group of teachers who teach, study, and reflect on the area of plane figures in the school where they work. As in this article we will analyse the attention given to this topic in the work carried out in a study group, we will base our analysis on investigations that discuss issues related to this type of continuing education, on professional teaching knowledge, and on reflection on the practice that we will present below.

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6 a) construct the idea of measurement units (including development of a measurement sense for standard units; for example, finding common objects in the environment that have a unit measure); (b) have many experiences covering quantities with appropriate measurement units and counting those units; (c) spatially structure the object they are to measure (e.g., linking counting by groups to the structure of rectangular arrays; building two-dimensional concepts); and eventually (often in the intermediate grades), (d) construct the inverse relationship between the size of a unit and the number of units used in a particular measurement; and (e) construct two-dimensional space and corresponding multiplicative relations.
Theoretical Framework

To analyse moments of reflection on practices collectively, we will rely on Zeichner (1993, 2008). Agreeing with this author, we understand that it is essential that teachers take a reflective attitude towards the social conditions that influence teaching and how it actually happens within the classroom. We believe that the constitution of study groups in the school where the participants teach can potentiate this attitude. It is also necessary to analyse what would be the teachers’ practical theories, such as the one described by Zeichner (1993), for whom “teachers are always theorising, as they are confronted with several pedagogical problems, such as the difference between their expectations and the results” (p.21). Therefore, we share the author’s idea that it is fundamental to listen to what the teacher has to say, so that he/she, in partnership with his/her colleagues in the group, expresses his/her needs, and analyses his/her failures together with the group, supporting each other. In our view, this may favour a better understanding of the contents needed to teach and the possibilities of professional development.

The ideas presented by Zeichner (2008) are similar to the studies of Ball, Thames, and Phelps (2008), which will also support this investigation, when they state that the teacher’s knowledge base used for teaching is supported by a web of relationships.

Ball et al. (2008), based on Shulman (1986), detail, based on the analysis of the mathematical demands of teaching investigated by the group, an interpretation of the knowledge necessary for mathematics teachers. The authors separate and detail the specific content knowledge described by Shulman in three categories: Common Content Knowledge (CCK); Specialised Content Knowledge (SCK) and Horizon Content Knowledge (HCK). In this article, we will analyse especially two of these categories: the Common Content Knowledge and Specialised Content Knowledge, which will be described below.

Ball et al. (2008) state that the common content knowledge (CCK) refers to knowledge of mathematical content that all professionals who study mathematics, whether teachers or not, should know. This category involves the knowledge that leads the teacher to, for example, recognise efficient algorithms to solve specific situations or even develop calculation procedures, identify when the textbook gives an inadequate definition, be able to use terms and notations correctly when speaking or writing on the blackboard. According to the authors, it is important knowledge, but only it is not enough to carry out teaching, since it involves “the knowledge that teachers need to do the work that they assign their students?” (Ball et al., 2008, p. 395). This knowledge may be related to the teacher’s recognition of a mistake made by his/her student when calculating the area of any polygon, for example.

Specialised content knowledge (SCK) is one that requires a deeper knowledge of concepts and procedures from the teacher than it is required from the student. For the authors,
this knowledge is exclusive of the professional teacher – “distinctly mathematical knowledge, but it is not necessarily mathematical knowledge familiar to mathematicians” (Ball et al., 2008, p. 394) - and essential to the achievement of teaching, because of this, closely related to practice. According to the authors, the teacher must be able to recognise the different ways of expressing the resolution of a given problem situation. Therefore, the teacher’s gaze will be focused on analysing, identifying, and assessing students’ responses, confronting all kinds of students’ solutions, and discovering, from the point of view of mathematics, what students have done.

Regarding the relationship between the expansion of the knowledge base and the reflection on the practice, we rely on studies by Serrazina (1999, 2012). The author, when researching the reflection capacity of mathematics teachers, observed that the ability to reflect one’s own practice occurs when teachers gain self-confidence and are able to do it. This presupposes a high degree of awareness that helps them to recognise their failures and weaknesses and assume a strong desire to overcome them. (Serrazine, 1999, p. 163)

Regarding the knowledge intended for teaching, Serrazina (2012), supported by Ball’s studies, states that teachers who teach mathematics for the initial years need to “have a deep understanding of the kind of mathematics they teach that is not limited to tacit knowledge of the know-how type, but that translates into explicit knowledge” (p. 282) and, therefore, suggests that teachers themselves feel “involved in learning experiences so that they experience the knowledge and ‘personal experience’ of the processes and nature of mathematical activity” (p. 282, emphasis in the original). Regarding these experiences, we also consider that, within a group that studies the contents to be taught, the teacher will have more favourable conditions to reflect on the practice. That said, we believe that the study group is an environment conducive to these experiments.

Methodological procedures

The qualitative research presented here included the participation of four teachers who teach mathematics in the initial years in a private school, located in a city in the greater São Paulo - Brazil. To preserve their identities, they will be identified in the research by names of flowers: teachers Acácia, Violeta, Jasmim, and Watsonia. All of them completed higher education in pedagogy at different times: two of them have less than two years of professional experience, and the other two have worked for 14 and 32 years, respectively. After an analysis of their answers to a questionnaire we applied, which constituted an initial investigation into teachers’ concepts and knowledge related to the notion of area and its teaching, we conducted study sessions on the subject. We sought elements to investigate how

8 is distinctly mathematical knowledge but is not necessarily mathematical knowledge familiar to mathematicians (Ball et al., 2008, p. 394).
the expansion of the knowledge base for teaching on the area of plane figures occurs, through studies carried out in groups in the school.

We collected information during the study sessions through written registers of the resolution of activities by teachers and video recordings of the study sessions of the group for later transcription.

For this article, we analysed three of the study sessions in which all the teachers participated: in one session we requested the resolution of a question that dealt with the calculation of the area of three figures associated with a squared array and, in the others, these questions were discussed from the analysis of the resolutions presented by the teachers.

**Data presentation and analysis**

The meetings dedicated to the analysis and reflections were guided by the assessment of the data collected through the protocols that contained the resolution of the participants to an initial questionnaire.

In the first working meeting, explanations were requested on the form of resolution they adopted for the areas of the figures given. Following, we will analyse and discuss the registers of resolutions and reflections that took place during the formative process.

**Initial questionnaire: analysis of the resolutions presented by the teachers**

The first question proposed in the initial questionnaire contained three figures to have their areas calculated.

![Figure 1 - Situation presented in the initial questionnaire](source: Research Collection)

In the resumption of the questions, teachers Jasmim and Violeta stated that, at first, they could not remember what was an area and/or determine the area measurement of the figures presented.

Teacher Acácia and Teacher Watsonia rearranged the shapes, causing, in figure (A), each square to change place until forming a larger figure “without holes” (teacher Watsonia). In figure (B), they regrouped the triangles to form a square and thus obtain a larger closed figure that, in their conceptions, would be better to determine the area of the figure. Figure (C) was more complex for teachers, as they had to rearrange the “halves” to form a small square, to then obtain a larger figure “without holes” (teacher Watsonia), to then obtain the total area of the shape. They also realised in the resolution of figure (C) that
it would continue with a “hole,” described thus by teacher Watsonia, and that they could make the total area minus 1 (square).

Teacher Acácìa, in turn, only placed the numerical value for each of the results, preferring not to mention any unit of measurement.
Analysing the strategy used by teacher Acácia, we observed that she joined parts of the figures to form little squares and, after that, she counted. This strategy was also found by Garcia Silva et al. (2013). These data were used to foster the meanings and reflections of the group in the formative context.

The group reflects on their resolutions

As we found, through the analysis of the initial questionnaire, that teachers Acácia and Watsonia rearranged the figure, grouped the triangles, and relocated the shapes to determine the area measurement, we asked them to explain to the other participants what conceptions they used when calculating the area of the figures. The presentation of the teachers led us to confirm that the strategies they adopted at the beginning of our research were exclusively the process of counting the “squares,” as we can see in the following images (Figure 4).

In another figure, they again demonstrated that they were using the counting and composition of “small squares” to determine the area measurement (Figures 5 and 6), as detected in the work of Garcia Silva et al. (2013).
At this point, we observed that teachers Violeta and Jasmim broadened their knowledge, since they had not been able to calculate the area of these figures, when they answered the initial questionnaire:

Teacher Violeta: *Guys, I didn’t really understand what the area was* (referring to the questions present in the diagnostic phase).
Teacher Jasmim: *I had no idea it was just counting the boxes.*
Researcher: *To calculate the area, you just count the squares, is that so?*
Teacher Acácia: *We counted because it was in the squared array.*
Teacher Jasmim: *That’s it, I want to know how many squares fits there.*
Researcher: *Yes, so the area is linked to the measure, don’t you think?*
Teacher Violeta: *The worst thing is that we use it in everyday life, we measure the floor area, for example. There we do this, we count how many tiles fit on the floor of the room, for example.*
Teacher Jasmim: *Actually, we knew how to measure, but we didn’t know what it was.*
At this moment, in the initial sentences of teachers Violeta, Jasmim, and Acácia that refer to the counting of squares, we noticed evidence of the expansion of common knowledge. When asked about the idea of measurement, teachers Violeta’s and Jasmim’s manifestation indicate the awareness of the need to move towards the specialised concept of area as a measure, especially when they relate this measure to everyday situations, especially when teacher Jasmim says: “The worst thing is that we use it in everyday life, we measure the floor area, for example. There we do this, we count how many tiles fit on the floor of the room, for example.” In this context, we consider that a greater understanding of area calculation procedures evidences the development of the these teachers’ common content knowledge, since the discussion generated on the idea of measurement may favour the expansion of the specialised knowledge of this content since, according to Ball et al. (2008), such understanding could favour the teaching.

However, they still had a strong tendency to use only square counting as a strategy. We present now three new figures for which counting squares would not be a good strategy. We aimed to identify whether participants would adopt new tactics to determine the given area.

![Figure 7](source: Research Collection)

When analysing the first figure (Figure 7), the teachers still tried to “complete the squares,” so that they could “reorganise” the figure, but realised that, for some cases, this would not be as immediate as in previous situations.

Teacher Acácia began resolving the question by counting the squares, but at a given moment, she estimated the compensation to arrive at a rectangle (Figure 8). At the end, she declared: “The approximate measure of the area of this figure is between 179 and 180. No, no, I believe it is 180, because is it more possible that 180 small squares are filled” (teacher Acácia).
Teacher Violeta, as well as Teacher Acácia, tried to complete and count the squares, but likewise she realised that she would not be able to compensate the squares in some cases (Figure 9). She also estimated the area of the figure was 179 “squares.”

Teacher Watsonia, in turn, began to realise that the figure could be divided into rectangles, starting a process of “macro” analysis of the figure. We observed that she began to analyse other resolution possibilities to obtain the given area (Figure 10). Despite her new perceptions, she was unable to reach the result of the requested area and, like the other teachers, estimated the measure as 180 “squares.” The fourth participant of our research, teacher Jasmim, did not participate in the analysis of these figures.
Noting that teacher Watsonia had found a “new” path to area analysis, we encouraged participants to analyze the figure as a whole, and even to divide it into rectangles and triangles to facilitate the determination of the area measurement (Figure 11).

When they realized that there was an easier method to determine the area measurement of the figures, participants began the process of decomposing the figures into triangles, squares, and rectangles to calculate the area of the parts and, subsequently, grouping them to identify the area of the whole figure. So, to analyze the participants’ understanding of this “new” process, we presented the other two figures (Figure 12), with the same objective as the previous question.
During the formative sessions, we observed that our participants built a “new” knowledge, and it was not only about the method of determining the area measurement. Part of the group found out that each figure could be analysed by composing and decomposing figures, and not just counting the little squares that were completely filled by the figure and estimating the other parts.

Acácia, for example, began the process by counting the squares, but soon realised that this procedure would only help her estimate the measure of the area, and would not indicate the exact measure. Then, she tried to form a rectangle of dimensions 6u x 4u, and discovered that there was another larger one that could contain the given triangle. Then, our participant scaled the rectangle measurements to 6u x 7u, which were exactly the measurements of the base by the height of the triangle. She also noticed that the area found was exactly twice the area of the triangle she wanted to find, so she divided this measure by two. We believe that at this point in the formative process, a representation of the square and triangle area formula began to make sense to the participant.

When analysing the second figure, Acácia felt more secure to present her calculations. She realised that the figure could be divided into three triangles and a square, which, from her point of view, “facilitates the calculation of the area when we divide the figure into parts, calculate the areas of these parts, and then add their results” (teacher Acácia).

The teacher also noted that the formulas accelerated the calculation process to obtain the area, and stated: “Now the results are obtained faster, because we no longer need to count the small squares... this changes the whole view I had about the formula and what it really represents” (teacher Acácia). We also could observe that, at this moment, teacher Acacia shows mastery of the common knowledge of the rectangular configuration as an important element to calculate the area of the rectangle through the formula, and specialised knowledge of the process of decomposition of the rectangle to arrive at the formula of the triangle and, more generally, to calculate the area of the figure.
Teacher Watsonia stated and wrote at the beginning of the discussion that she did not know how to solve the given situation, but, encouraged by the group, she was able, in the initial attempt, to understand that the triangle, in the first figure, could be contained in a rectangle of $6u \times 7u$, and also recognised that this area was exactly twice the area measurement of the triangle, so she divided this value by two. As well as teacher Acácia, she formalised this information through the formulas of area of the rectangle and triangle (Figure 14).
The third participant, teacher Violeta, despite being encouraged by the group, found it the analysis of the questions requested very challenging (Figure 15).

![Figure 15 - Protocol presented by teacher Violeta](image)

To make teacher Violeta more comfortable, and so that she and the other participants could build and strengthen new knowledge through the perception of rectangles, we asked them to explain how they had managed to reach the result (Figure 16).

![Figure 16 - Teachers’ explanation of how they used formulas to determine area measurement](image)

After the other two participants’ explanation, teacher Violeta was able to understand and help in the process of separating and reconfiguring the figure to obtain the total area, a
relevant process for teaching – specialised content knowledge, as described by Ball et al. (2008). At that time, they discussed this other strategy of calculating the area measurement:

Teacher Acácia: You’ve noticed that it’s easier to look at the area when I turn it into rectangles.
Teacher Violet: True, but you have to look out of the picture, I was making a mistake.
Teacher Watsonia: Rectangle is easy, you just see how many rows of squares I have.
Teacher Acácia: True, base times height. And when there’s a triangle, I can also turn it into a rectangle.
Teacher Violet: I never thought to look at the formula that way.
Teacher Jasmim: Today I learned another way to look at the area [referring to the calculation of the area in squared array].

At the beginning of the dialogue, when teachers Acácia and Violeta consider the triangle as part of the rectangle, they show evidence of the expansion of specialised knowledge of mathematics to teach area of plane figures, as described by Ball et al. (2008, p.394), since knowledge about different forms of composition of areas is, as the authors say, essential to the teaching, so “[...] closely related to practice.” Teachers Acácia and Violeta resorted to what Duval (1994) calls diving when they report having observed that it becomes “easier” to calculate the area “when I transform it into rectangles,” or when they observe the strategy of “looking out of the picture.” Also, they, at that moment, realised the importance of forming a study group within the school unit: “I would not have this perception by myself, I am learning a lot in our meetings” (teacher Violeta).

Like us, Etcheverria (2008) believes that the formation of study groups constituted within the school unit can favour the professional development of the participants.

From participating in the study groups, the educator can, in the exchange of ideas and experiences on the subject under study and discussion, to (re)think his/her work routine, challenging him/herself to qualify the teaching process. (p. 23)

Furthermore, the author also emphasises the reflection that each teacher should have on their teaching practice:

Considering the report and reflection on the practices, we can know the methodology and the resources the teacher used to achieve the objectives he/she has set him/herself. When the teachers reflect on the teaching-learning process that provides opportunities and their role in it, they manage to understand it, interpret situations related to it and also develops the necessary autonomy to rebuild it. (p. 23)

Given all of the above in the formative session described here, we can infer that the common and specialised knowledge of determining the area of non-rectangular figures, according to Ball et al. (2008), at first, was not dominated by the group we investigated. However, in this very first session, we found that there were advances in determining areas of polygons and that, in addition to the study, collaboration and trust leveraged the (re)construction of this knowledge.

9 “[...] closely related to practice” (Ball et al., 2008, p. 394)
Some conclusions and final considerations

Through the analysis of this result from the point of view of Ball et al. (2008), we identified that the mastery of the common knowledge of the square counting expanded to the specialised content, mainly the ability of participants to move through the construction of the concept of area, from the counting of squares to the reconfiguration of figures or diving operation, and to the identification, in practice, of the corresponding area formulas. We believe that the mastery of this common and specialised knowledge will probably contribute likewise to the teachers’ knowledge base for the teaching of the area of plane figures. In this context, it could, for example, favour the expansion of knowledge of content and teaching as described by Ball et al. (2008), which would require the professional to be clear on why to propose calculations of plane figures that lead the student to experience strategies that go beyond counting squares.

We also believe that the experiences lived by teachers deepened their tacit knowledge of the mobilisation of resources associated with reconfiguration to calculate the area in squared arrays to explicit knowledge of the subject and, in this sense, we consider, as Serrazina (2012), that, possibly, this fact contributed to improving the teachers’ knowledge base and, consequently, their ability to teach.

Based on Zeichner, we were also able to observe how relevant it was to listen and reflect together with the teachers on the teaching of the content area of plane figures. It was through mutual support that they identified their own needs and modified their analysis of the strategies to be adopted. Also, in the same way as the author, we could verify that this process helped to improve the understanding of this group on the subject studied and enabled them to expand their professional development.

However, we emphasise how important it is that the formation of study groups assume a permanent character of activity and focus on the several topics addressed throughout schooling, to constitute a fruitful space for discussion and reflection in the school environment.

References


