



Narratives and Public Policies: understandings about the Maths Hubs¹

Narrativas e Políticas Públicas: compreensões sobre o *Maths Hubs*

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Abstract

This paper aims to explore the potentialities of a narrative for the understanding of an English public policy of teacher training, called Maths Hubs. The narrative was constituted from an interview with a participant of one of the Maths Hubs, according to the methodological parameters of Oral History. The analysis was undertaken through dialogues between different oral sources (the interview granted specifically for this research and video lessons available on the project's website), and written sources (training materials, official documents and relevant literature). The results point to the potential of the narratives to create gaps that can be filled by other written sources, as well as to fill in the gaps that exist in the official documents, in addition, bring insights about the public policy in question that can promote reflections for those who are dedicated to its study, implementation and evaluation.

Keywords: Mathematic Education, Teacher Training, Oral History, Teaching for Mastery.

Resumo

O objetivo desse artigo é explorar as potencialidades de uma narrativa para a compreensão de uma política pública inglesa de formação de professores, denominada *Maths Hubs*. A narrativa foi constituída a partir de uma entrevista com um participante de um dos *Maths Hubs*, de acordo com os parâmetros metodológicos da História Oral. A análise se deu por meio do diálogo entre diferentes fontes orais (a entrevista concedida especificamente para essa pesquisa e vídeos de aulas disponíveis no sítio eletrônico do projeto), e escritas (materiais de formação, documentos oficiais e literatura pertinente). Os resultados apontam para a potencialidade que a leitura de narrativas possui de criar lacunas que podem ser preenchidas, ainda que parcialmente, por outras fontes escritas, bem como de preencher, também parcialmente, as lacunas que existem nos documentos oficiais, além disso, trazem compreensões sobre a política pública em questão que se espera possam servir de reflexão para aqueles que se dedicam ao seu estudo, implementação e avaliação.

Palavras-chave: Educação Matemática, Formação de Professores, História Oral, Teaching for Mastery.

Introduction

The Research Group on Oral History and Mathematics Education (GHOEM), has been dedicating itself since 2002 to the possibility of using Oral History as a methodological resource. Unlike other forms of qualitative research that use interviews, by following the

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methodological parameters of Oral History, the researcher intentionally creates historical sources, which can be used by other researchers.

Since its creation, this methodological option has been used to understand different issues in the area of Mathematics Education, generally in the Brazilian context, from a broad spectrum project called Mapping the Formation and Performance of Mathematics Teachers in Brazil. According to Garnica et al. (2011), this is a broad-spectrum project:

- (a) the long duration (the project was initiated in the early 2000s and has no plans to close); (b) the variety of geographic and cultural spaces considered (the numerous subprojects linked to this mapping project seek to weave narratives on the formation and performance of mathematics teachers from different regions and diverse socio-cultural shades [...]) (c) the option to focus on various moments in the History of Education and Mathematics Education (with emphasis on current events, given the central - but not exclusive - option of the Oral History method); (d) the study and adoption of different techniques of narrative composition; (e) the themeization of various aspects of the educational process (practices of teacher training and performance, policies - public or not - of school organization, architectural spaces, school materials, etc. are studied); (f) the use of various sources, which implies the care with the survey, recovery and study of written, oral and pictorial collections, for example, and (g) the participation, in the project, of researchers at different levels of training (undergraduate, master's, doctorate, postdoctoral) (pp. 240-241).

Some of these projects deal with the creation, implementation and evaluation of public policies for teacher training, one of the many facets of teacher training in Brazil, including Bagio (2014), Costa (2016) and Major (2018). There is also the work of Menjívar (2018) that goes beyond Brazilian borders, using Oral History to establish dialogues between the CFD - Teacher Training Course in El Salvador and Gestar II - Management of School Learning in Brazil.

In the same direction of these works, this article seeks to broaden the understanding of a public policy of teacher training based on a narrative constituted according to the methodological assumptions of Oral History. On the other hand, it takes England as its work field, another reality.

Methodology

The methodology used in this study is Oral History, as it has been understood within the GHOEM, and theorized by its members. According to Garnica (2007) Oral History is "a method that emphasizes the importance of memory, orality, testimonies, the lives of people deemed essential [...] to understand the 'objects' that the investigations intend to focus on (p. 8).

Unlike other qualitative methodologies, Oral History has as its objective the constitution of historical sources from orality. Such intentionality brings to the research a series of care with and about the sources produced. According to Garnica (2011), choosing to conduct research within the principles of Oral History means:

... opt for specific ways to (a) make research questions arise, (b) search for information and record memories - narratives - that allow us to deal with these issues; (c) take care of these records in an ethical way and work them according to specific procedures, making them public at the end of this process; (d) to analyze the data arsenal according to theoretical perspectives in tune with some previously established principles; and (e) to seek to create alternative narrative forms to those usually in force in the academic environment, constituting the work produced in this area more as fields of experimentation than as reasoning for certainties (pp. 5-6).

In the case of this research, the objective is to weave understanding about a large scale English public policy of training mathematics teachers - Maths Hubs - considering the context in which this takes place, which is England. To do so, we seek to interview a leading teacher from one of the Working Groups, as well as observations from teacher meetings, videos and written materials available on the official Maths Hubs website. The interview took place after the approval of the Ethics Committee³ of the University College of London and the signing of the copyright assignment, as has been understood by the Oral History Society⁴, in addition to the possibility for the interviewee to delete or alter elements of the interview that they considered inappropriate.

In order to fulfill the objectives of the research, dialogues are established with pertinent literature, noting that the reading of the constituted narrative creates a series of gaps, which can only be filled, even if always partially, by resorting to other written documents, whether training materials, pertinent literature and also other sources of data, videos and observations made. On the other hand, when analyzing the written documents, one notices gaps that can only be filled, even if partially, by listening to the participants.

Methodological Procedures

The research reported here was carried out as part of the researcher's post-doctorate, which began in February 2018, in the city of London, England, whose interest turned to the exploration of Oral History to understand public policies in contexts different from the Brazilian one. Because it was a public policy aimed at training teachers exclusively in Maths, and especially because of the scale to be achieved, Maths Hubs was the public policy chosen.

From April, through the project's website⁵, the researcher learned about the dates of the training meetings for the facilitators, leaders of Maths Hubs. On May 3rd, the researcher observed one of these meetings, at Russel School. In the same meeting, the trainer, Mastery Specialist, Nicki Ashton, informed about the meetings for teachers at Burntood School coordinated by the facilitator, leader of the Working Group, Daniel Lewis. On May 10 and June 14 the researcher observed the meetings for teachers. In September of the same year, teacher Daniel Lewis was contacted by email and invited to participate in this survey by giving an interview that took place on November 1 and lasted 56 minutes.

In addition to the data from the observations and the interview, a significant amount

³ Data protection registration number: Z6364106/2018/05/173.

⁴ http://www.ohs.org.uk/ethics/ohs_recording_agreement.pdf.

⁵ <http://www.mathshubs.org.uk>.

of training materials were consulted, as well as video lessons⁶ were watched. Official documents from the Department for Education⁷ in England served as sources to understand the English educational context and statistics. In the next section, the dialogue between Professor Daniel Lewis's narrative and the other sources will be established with a view to understanding Maths Hubs' public policy.

One narrative, multiple understandings

On November 1st, at 3pm, the researcher arrives at Burntwood School, a medium-sized school in southwest London. At 3:15, the interview with professor Daniel Lewis, who has 10 years of profession, begins.

After signing the concession agreement for the research, the interview begins with the researcher having chips with questions on the table that can be freely chosen by the interviewee. These questions deal with professional life, participation in the Maths Hubs program and its perceptions.

Maths Hubs' target audience: being a teacher in England

Originally I wanted to join the Royal NAVY (British Naval Warfare Force), but I found out I couldn't because I had asthma. I ended up doing my master's degree in Financial Mathematics and worked for a year, actually nine months, in a bank and then I left. When I was at University, we were offered an opportunity to work in schools as an assistant professor. For a small salary, but, for me who was 21 years old, it was good. And I really liked being an assistant teacher and working in schools. So I started my PGCSE - Post Graduate Certificate in Secondary Education at Kings College of London in 2008, 2009. I was working on my first PGCSE placement school for three and a half years. As I didn't have many opportunities for PGCSE - Continuous Professional Developing, I decided to go to a teaching agency, but my partner was working at this school and said it was a good place to work. I was offered an opportunity to finish the academic year and I have been here since 2013. One of the reasons to stay here are the CPD opportunities: training I do at home and external courses. (Interview with Daniel Lewis)

In trying to understand the teacher's account it is necessary to know, at least in general terms, the educational system in England, in particular with regard to teacher training. The initial formation of the mathematics teacher in England is quite diffuse, there are several possible itineraries, and it is not possible to trace an equivalent to the Brazilian formative journey. To start the formation in universities, it is necessary to have obtained minimum grades in English, Mathematics and Science in the GCSE - General Certificate of Secondary Education (equivalent to the ENEM - National High School Examination), to enroll in the processes of the universities, which evaluate the candidate's curriculum, accepting or not. This was the path of the professor, who opted for Financial Mathematics and, when he

⁶ These are classes filmed and commented by experts and teachers.

⁷ <https://www.gov.uk/government/organisations/department-for-education>.

decided to become a professor, sought the PGCSE - Post Graduate Certificate in Secondary Education at a renowned English institution, Kings College. This is a two-year course, in which it is necessary to present a study, and work in a school (placement school), guided by the University's teachers. The PGCSE enables the teacher to work in the Secondary School (11 to 16 years old) and also in the A's Levels (16 to 18 years old), advanced studies in subjects to choose the student who can help them to enter the University. Although this has been the teacher's path, it is important to consider that PGCSE is not necessary to start working as a teacher at Primary School (5 to 11 years old), it is necessary to pass an evaluation with the Department for Education, in addition to presenting good grades at GCSE.

As can be seen in the teacher's report, something that led him to reconsider the workplace were the opportunities for CPD - Continuous Professional Developing, opportunities for Professional Development. As we will see in the sequence, there are CPD sessions that have been used for the development of Maths Hubs public policy that will be presented in the next section.

Besides understanding the narrative, itself, we have elements to draw a mental image of Maths Hubs target audience. Teachers who do not have a specific training in Mathematics as we have conceived it in Brazil and have hours of activity for their professional development.

Operationalization of Maths Hubs

Here at this school, there is the Student Department. This department does several interventions and assists us. There are a variety of organizations that offer PCD courses on these interventions. One of them was Maths Hubs. In the first course I attended, we learned a lot of things to use in our classes. For example: how to elaborate questions for the students, and I really enjoyed it. I could see how to use that knowledge directly in my class. A few months later I received an e-mail from my supervisor, who put me in touch with Maths Hubs to start a new group. I then took a course on teacher training. I had no idea what would happen. But, as I had really enjoyed the first course I took, I wanted to get involved. And so I formally became part of Maths Hubs as a Work Group facilitator. (Interview with Daniel Lewis)

According to the Maths Hubs website, each Maths Hubs is a partnership, led locally by a school or college with the objective of assisting in the professional development of math teachers based on the concept of Teaching for Mastery, which will be addressed below.

Math Hubs is coordinated by National Centre for Excellence in the Teaching of Mathematics (NCETM), and funded by the UK Department of Education which provides financial grants to cover operating expenses. The project also opens up the possibility of establishing other partnerships with universities and institutions linked to education.

Although several parallel projects can be seen, the main axis of action is the Teaching for Mastery Programme. This is the project to which the professor refers. In it, two teachers from each school, called facilitators, participate in a group called the Research Teachers Group, which meets regularly. This group is composed of six or seven schools and

coordinated by a Mastery Specialist. The facilitators, in turn, meet with the teachers of their Maths Hubs, composed of teachers from the same school and from nearby schools. The program is held every two years, the first year the work is guided by the Mastery Specialist and the other year it develops more independently.

We can see how the Maths Hubs work in the following organization chart:



Figure 1: Maths Hubs Organization Chart

Source: Author's elaboration

The Mastery Specialist receives specific training and is linked to NCETM. Although it did not occur in the teacher's speech, according to the Maths Hubs website, each of the school facilitators is selected by a process involving NCETM, the Department for Education and the National College for Teaching and Leadership. The teachers, on the other hand, participate in the program by adherence and on a voluntary basis.

Maths Hubs started its activities in 2014 and currently has 35 Maths Hubs that involve two thousand schools and intends to involve ten thousand, half of the schools in England, by the end of 2023. Maths Hubs is present in all English cities and includes all levels of education.

Origins and Design of Maths Hubs Education: Teaching for Mastery and the Five Great Ideas

I had heard about a teacher training course based on the ideas of Shanghai and I had read about Math Education in Singapore. This course was about bar models for working reasons, fractions and percentages. It was the first course I took at Maths Hubs.

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When I started my workgroup I went to a training moment in Birmingham. First we had a presentation that gave us a vague idea of how to implement the Mastery Approach. [...] On the second day we had a brief presentation of the Five Great Ideas, and an interesting PCD session observing practices. Good talks and good discussions with other Maths Hubs. (Interview with Daniel Lewis)

The reason why ideas based in Shanghai and Singapore have been part of the teacher's repertoire of memories is the fact that the English curriculum changed in 2014 and was influenced by the curriculum in Singapore, Shanghai and Hong Kong. The choice of this aspect is due to the success obtained by South Asia in the Programme for International Student Assessment (PISA).

There is a massive and constant investment for the Mastery Approach to be adopted by all schools in England. This influence is announced on official websites, as can be seen in the following quote from the Department for Education (2016).

The South Asian Mastery Approach to mathematics teaching is about to become a standard in England's elementary school, thanks to a major expansion announced today by School Minister Nick Gibb.

With the help of up to 41 million pounds of funding, more than 8,000 elementary school - half the total number in England - will receive support to adopt the approach, which is used by some of the world's leading math experts, including Shanghai, Singapore and Hong Kong. (Department for Education, 2016)

The concept of Teaching for Mastery is based on five great ideas that can be visualized in the diagram below:

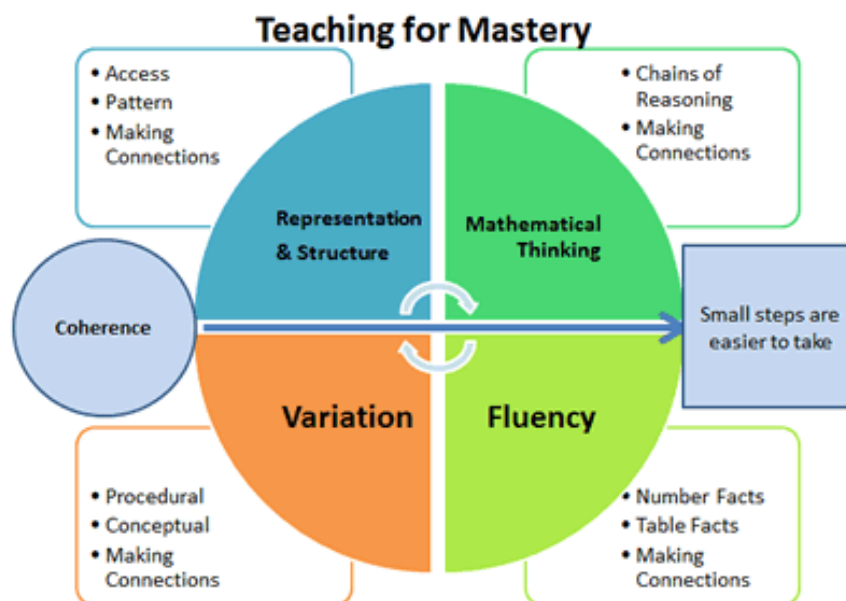


Figure 2: Five Big Ideas

Source: NCETM (2018a)

From elements of the narrative, the ideas will be deepened. The order in which the ideas will be addressed here refers to the emphasis with which they have been treated in the narrative.

Variation: procedural and conceptual

Variation is one of the key concepts of Teaching for Mastery. It is something that should be done with care, avoiding mechanical repetition and encouraging mathematical thinking. Let's see how the Variation is understood in the teacher's narrative and what are the gaps to be filled by other sources and literature.

I believe that Variation is one of the ideas I most understand from Mastery Approach. This idea was part of the work I needed to do between two modules of a course I took. Certainly I use more variations than before. This completely changed the way I presented the examples and the questions. And it made me think more deeply about how to change from issue to issue. I used to run, and put the questions on the board. Now I think: how can I change that? What effects can they have [...]. And it seems to me that the students are realizing things in greater depth. So I think: How can I vary my questions? I think about how to amplify their level of difficulty, much more than I did before. In CPD meetings, I try to make the Working Group think more deeply about how they can demand more of their students to reach deeper levels of understanding. (Interview with Daniel Lewis)

As already presented, the concept of Teaching for Mastery has its origins in China. The interest of researchers in teaching and learning mathematics in China is not recent and is largely due to the high rates achieved by Chinese students in international evaluations. This interest has prompted Chinese researchers to end up compiling research results and theoretical discussions in *How Chinese Learn Mathematics: Perspectives from Insiders* (Lianghuo et al., 2004). As with other concepts, the idea of variation is seen in the dialogue with Western learning theories. Gu et al. (2004), thus present the ideas of conceptual and procedural variation:

Conceptual variation aims to help students understand concepts from multiple perspectives while procedural variation helps students establish internal connections between what they already know and new learning objects. These two aspects are critical and complementary. In fact, both types of variation are based on analysis of experiences and experiments with mathematics teaching in China and can be understood and interpreted by certain Western theories, such as Dienes' mathematical learning theory (Dienes, 1973), Marton's theory of variation (Bowden & Marton, 1998; Marton & Booth, 1997), and scaffolding theory (Bruner, 1985; Wood, Bruner & Ross, 1976). (p. 333)

In order to help implement the ideas of Teaching for Mastery, NCETM (2018a) has maintained a series of publications in which it seeks to bring such ideas to the attention of teachers. The idea of variation is thus presented:

Teaching with conceptual variation involves the comparison of static models and images of a mathematical concept that enable students to compare and identify what is equal and what is different about models and images, revealing what is essential and what is not essential to the concept. For example, multiple examples of different triangles allow students to generalize that for a shape to be a triangle it must be a

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closed shape with three straight sides and three vertices (i.e. the essential characteristics of a triangle). The non-essential characteristics can be the size of the sides, the angles and the orientation in relation to the horizontal line. Teaching with procedural variation [...] involves teaching a mathematical procedure in which the procedure is gradually 'unfolded' into a carefully chosen succession of steps, and allows the child to determine "what procedures have remained the same" and "what procedures have changed" at each step. This allows students to identify the variant and invariant characteristics of the processes, seeing connections between the steps and leading to a generalization that can be applied in all situations where the process is used.

In addition to these publications that broaden the theoretical discussion, NCTEM has developed a wide range of resources that are available on the Maths Hubs website. These are written and video materials (lessons and classroom case studies). One of the main resources is a set of 8 books: 6 books for years 1 to 6 (Primary School), one book for Secondary School and one book called Calculation Guide. Reading these materials makes it possible to understand how the conceptual and procedural variation is viewed according to Teaching for Mastery, as seen in the sequence:

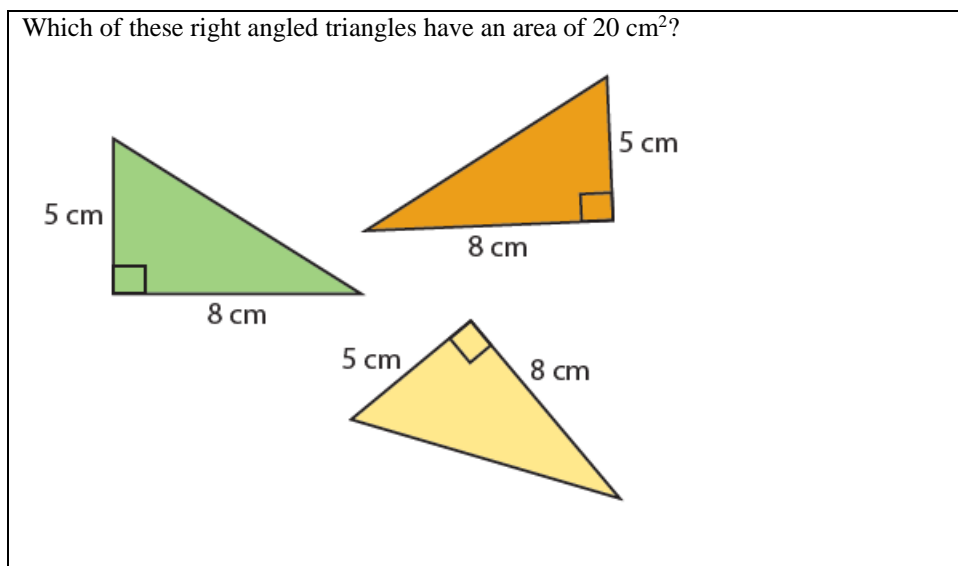


Figure 3: Example of Using Conceptual Variation

Source: Askew et al. (2015a, Year 6, p. 32)

Ramesh is exploring two sequence-generating rules.
 Rule A is: 'Start at 2, and then add on 5, and another 5, and another 5, and so on.'
 Rule B is: 'Write out the numbers that are in the five times table, and then subtract 2 from each number.'
 What's the same and what's different about the sequences generated by these two rules?

Figure 4: Example of Using Procedural Variation

Source: Askew et al. (2015a, Year 6, p. 27)

In the figure 3, the intention is to work with the concept of rectangle area, the student must then decide what is essential and what is not essential for rectangle triangles to have the

same area, in this case the emphasis is on the fact that color and position are not essential characteristics.

By constructing the sequences of the figure 4, one obtains the sequence generated by Rule A, 2, 7, 12, 17... and the sequence generated by Rule B, 3, 8, 13, 18... Students are encouraged to note that, although the numbers 2 and 5 are the same, with varying procedures the sequence is changed, although the difference between the terms remains the same.

Representations and Structure

In the Teaching for Mastery, the representations used should expose the mathematical structure so that students can gradually solve problems without support from the representations. This idea is thus perceived by the teacher:

The Great Idea of Representations, of visual representations, is one of the important aspects of Maths Hubs Program. One of the reasons I really got interested in the bar modeling course, my first course at Maths Hubs, is because I heard about how visual representations can really help students to understand, especially topics involving reasons, much faster, much more... [...]

One of the things I really like are the representations for algebra. These representations are very rich. This was completely new to me. It was really rare to use any visual representation, outside of Geometry or Graphics, in school, college or university. Algebra representations are really new to me. (Interview with Daniel

Gu, Huang and Marton (2004) refer to the work on Bruner's representations (1964) and on Freudenthal's Mathematization process (Freudenthal, 1990) to base the Great Idea of Representations present in Teaching for Mastery. For the authors, the process of Mathematization consists in starting from an Inactive (concrete) representation, moving on to an Iconic (semi-concrete) representation, to reach a Symbolic (abstract) representation. The inverse process, which is to start from a symbolic representation to reach a concrete representation, is called Looking for Meanings.

As an example, the authors present a situation where a work is developed with children who have 7 beans to be distributed equally in 3 plates. The Mathematization process would involve a first stage (Inactive) where the child distributes manipulable beans on plates, the second (Iconic) involves the representation of the beans by drawings and/or the mental distribution of the beans and the third (symbolic) is the representation of the situation through the arithmetic expression $7 \div 3 = 2 \text{ rest } 1$. The inverse process, called Searching for Meanings would require the child to recreate the context from the arithmetic expression. Figure 3 illustrates the situation.

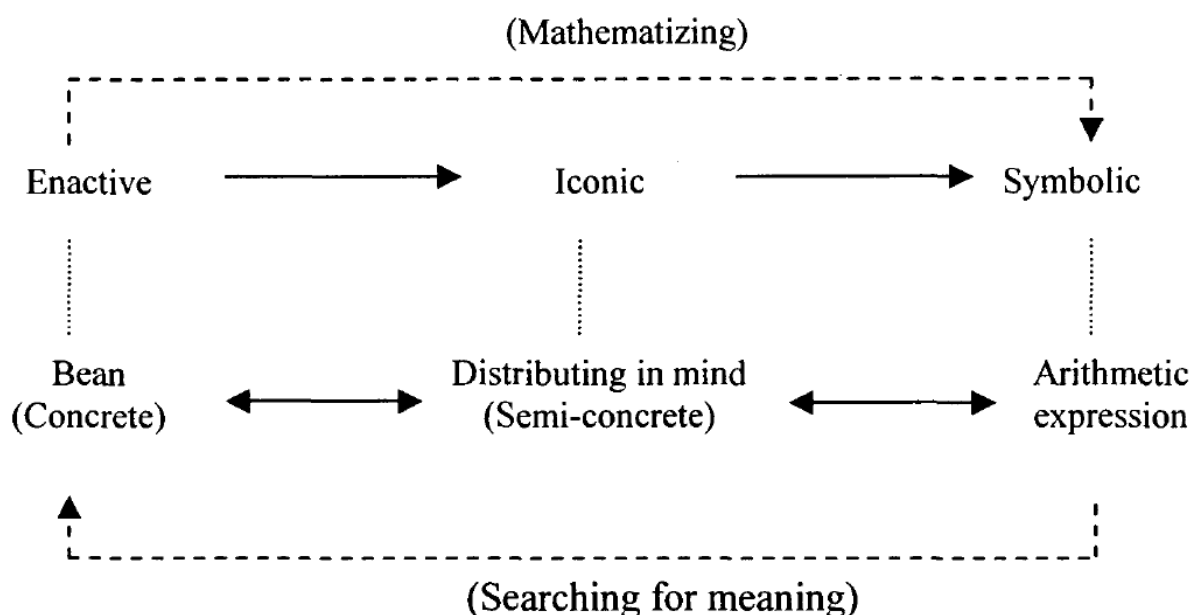


Figure 5: Distributing Beans and Representations

Source: Gu et al. (2004, p. 331)

The teacher makes several references to the use of the bar model. Such representation, located in the Iconic level is used for the work with several concepts (basic operations with integers and decimal numbers, ratio, percentage, fraction, etc.) in the training materials made available by NCETM, and also in the video classes. One of the highlights in the teacher's narrative refers to the use of the bar model for the work with reasons. The chart below shows an indication of the resolution of a problem involving reasons with the use of the bar model.

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Sam and Tom have stickers in the ratio of 2 to 3. Together they have 25 stickers. How many stickers do they each have?

Solution:

Together Sam and Tom have 25 stickers:



As the ratio is 2 to 3, we need five parts. The bar is divided into five parts, two of which represent the amount of stickers that Sam has and three the amount of stickers that Tom has.

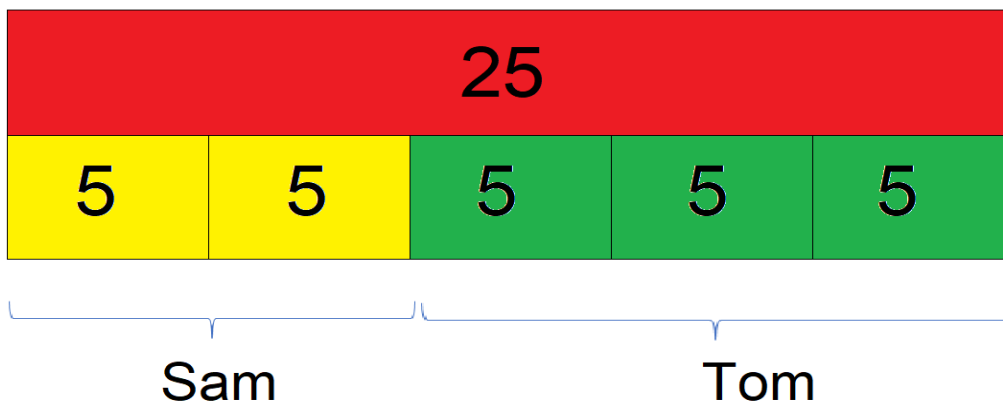


Figure 6: Example of using the bar model

Source: NCETM (2018b)

The algebra representations were also a highlight in the teacher's narrative. We observe their use in the example below:

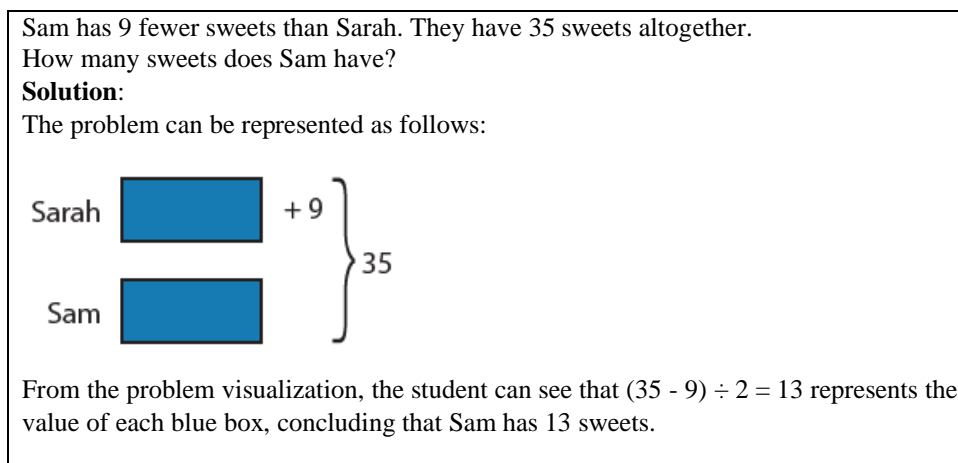


Figure 7: Example of using a bar model

Source: NCETM (2018b)

The Meaning Search process is often completed by asking the student to build a word problem from a given bar model or arithmetic expression. As exemplified below:

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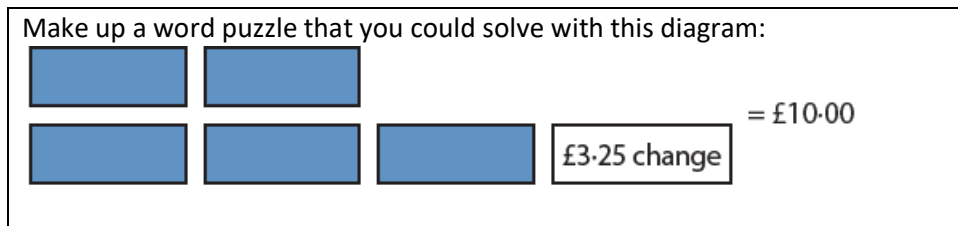


Figure 8: Example of the Process of Searching for Meanings from a Bar Model

Source: Askew et al. (2015a, Year 6, p. 23)

The representations occur from year one (5 to 6 years) and are also indicated for the Secondary. However, the teacher reports difficulties for their use at later ages and how the process of Searching for Meanings occurs.

When we say this to the students they think that drawing diagrams is childish, they don't realize the potentiality of diagrams. Except when we work with a Geometry problem or with a problem about Graphs where you have to have the figure. But when it involves reason or another subject it is more difficult. For example, in year nine, when I introduce reason I say: - We can draw a diagram. And they say: - No. Do I have to do that? And you have to show them that the understanding they had was weak, very weak. So the students need to explain what the diagram is showing, what each number of the diagram means, what each number means. And you notice a very, very fast growth, isn't it? (Interview with Daniel Lewis)

The potential of these representations for Brazilian education is something that still needs to be studied in more depth. Studies on the use of bar models in Brazil are rare, among them we can mention Cintra (2017), Góis (2014) and Queiroz (2014), Masters dissertations defended at the Federal University of São Carlos and guided by Professor PhD Yuriko Yamamoto Baldin.

Coherence

In Teaching for Mastery, the idea of Coherence indicates that new ideas must be connected to concepts already understood and mastered. The steps taken must be small to ensure mastery and connections. In the teacher's narrative, one can advance this understanding.

The idea of Coherence is one of the things I really care about. Through the Maths Hubs Program, I started to think about how I can keep the focus of my class more restricted in order to achieve a deeper learning on the same point. It is really a challenge to develop activities in this sense.

With this I can analyze how to make connections between different areas of mathematics, right? Because if I keep the focus, it is better to make the connections. The idea is that the students acquire a mastery over basic aspects. Currently I can make connections much better than before and I try to help my students to start making those connections too. And they are starting, they are getting used to it. In addition, some procedures have become more automatic.

[...] I'll give you a good example. I was working with a mixed group of students. And the other day we were working with division, by the mental method. We used the whole class with this and we were exploring. We realized that we need time to explore multiple methods to do things. And we need to give the students time to

explain what they are doing. They have to explain what they are doing to anyone. They like it, because they feel able to talk, to explain. So they have to explain, because some calculations have the same result, how we can modify a division so that it is easier to do. This is exactly understanding the work with numbers. (Interview with Daniel Lewis)

The idea of Coherence implies considering that the steps taken between the concepts should be small and that during the classes the focus should be maintained. It is a construct that finds little relation with western theories and, as can be observed in the narrative, it is a counterpoint in the teaching process commonly carried out in England, which, among other traditions, kept the focus on the rigorous fulfillment of an extensive curriculum⁸ focused on large scale tests. The principles of Coherence are thus described by NCETM:

The Mastery Approach includes the belief that all children are able to understand and do mathematics, giving themselves enough time.

Teaching for Mastery Approach is a set of pedagogical practices that keeps the class working together on the same topic, while aiming for all students to master the curriculum topics with great depth and understanding. The challenge is more the deep understanding than the presentation of a new topic. Teaching is focused and rigorous, to ensure learning is sustainable over time. Long-term learning gaps should be avoided by rapid teacher intervention. More time is spent teaching the topics to ensure learning. Well-planned classes promote a conceptual journey through mathematics, engaging students to reason and think mathematically. (Askew et al, 2015, Year 6, p. 6)

The citation notes the call for all learners to be working on the same topic. It is an attempt to change the English tradition of dividing students into groups according to skills even in the same room. In addition, there are references to the need for constant revision to ensure sustainable learning. Such ideas are represented in the following scheme that represents the teaching standard in China:

⁸ The curriculum was changed in 2014.

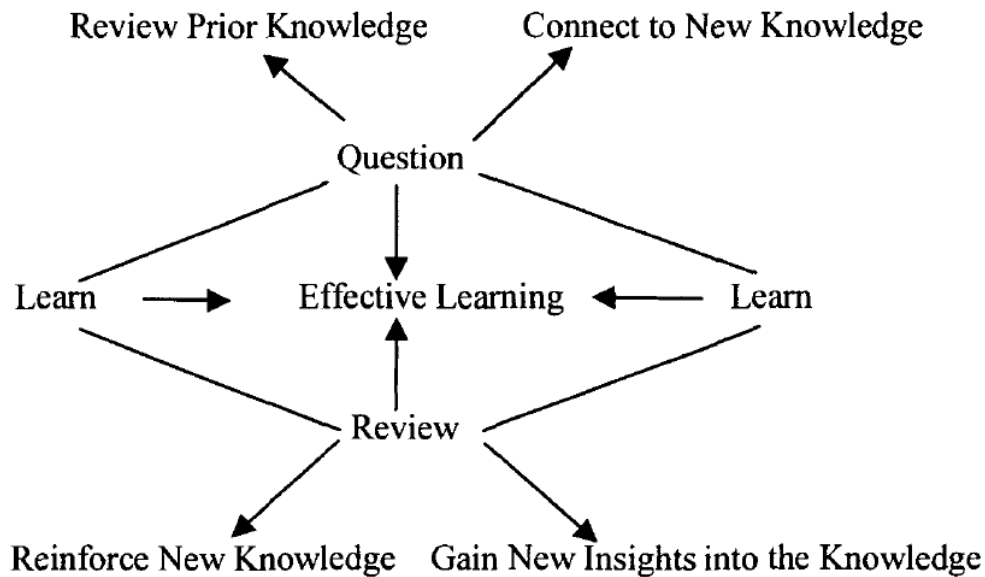


Figure 9: Teaching Standard in China

Source: Na (2004, p. 466)

According to An (2004) Mathematics Education in China has followed this pattern of teaching and learning for centuries. To help students build knowledge, teachers ask questions where students need to review their previous knowledge and build connections with new knowledge. In addition, teachers ask questions to help students think mathematically and gain new insights into knowledge.

Fluency

Fluency is understood as speed and efficiency in remembering facts and procedures and the flexibility to move between different contexts and mathematical representations. By referring to fluency the teacher also reports the use of stem sentences, which will be treated in sequence.

I understand stem sentences as a set of definitions that I can refer to at all times. For example: - If I have a power division of the same base, what do we do with the powers? - Look at the definition. - You have to subtract the powers. An example: - (xy/xw) . - Are the bases the same? - Yes, they are. - What operation is it? - Division. - What do I do with the powers? - Subtract. And that's done several times, repeating, repeating. And if someone does something wrong, you can go back to the setting, automatically. If that's what I understand, I like it, because you have a reference. You are giving the students a reference. (Interview with Daniel Lewis)

Stem sentences are statements that are intended to help students develop the ability with language and mathematical structure and acquire fluency in procedures. An example of a stem sentence obtained in one of the observations made by the researcher is shown in the following table:

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_____ and _____ are factors of _____ because _____ x _____ = _____
An example of the use of this stem sentence is:
4 and 6 are factors of 24 because $4 \times 6 = 24$.

Figure 10: Example stem sentence

Source: Author's Training Material

Students must repeat the sentence as it stands. There is a set with a wide variety of sentences that cover a good part of the curriculum. Besides the aspect related to remembering aspects, it is important to emphasize that the concept of fluency goes beyond that, because it requires the student to pass between different contexts and representations. Below is an example that requires the transit between two representations:

Solve calculations using a place value grid and equipment alongside a column method to demonstrate understanding.

Hundreds place	Tens place	Ones place

325 + 247 ----- -----

Figure 11: Example of transit between different representations

Source: Askew et al. (2015c, Year 3, p. 15)

It is understood that fluency goes beyond the memorization of basic facts, but it does not dispense with it. In the video lesson of year 3⁹ one can observe the teacher Sue Nattall working with multiplications in which it is remarkable the repetition of sentences and the correction of language.

Mathematical Thinking

The Great Idea of Mathematical Thinking is thus presented in the training materials: if the ideas to be taught are to be understood in depth, the students cannot receive information passively but participate actively: thinking, reasoning and discussing with others. In the following excerpt, the teacher refers to the development of mathematical articulation and the need to ask students to explain and discuss what they are doing:

⁹ <https://www.ncetm.org.uk/resources/48211>

The level of articulation, the ability to explain things has been visibly expanded. They no longer use poor mathematical terminology; they use precise mathematical terminology. They correct each other. They are increasingly better at explaining things. Because once only in a few lessons were they asked to explain, and now in most lessons they are asked: explain it. And they feel happy about this, because if one group did not give a good explanation, the other group says that the explanation is not good enough. (Interview with Daniel Lewis)

It is observed that the development of Mathematical Thought goes through much of the teacher's narrative, as well as the need to build connections between different concepts and is present in the training materials. One of the resources used is Captain Conjecture. The Captain makes statements that students must agree or not and explains the reasoning used, as illustrated below:

Captain Conjecture says that a rectangle is a regular shape because it has four right angles.
Do you agree?
Explain your reasoning:

Figure 12: Example of development of mathematical thinking

Source: Askew et al. (2015b, *Year 4*, p. 25)

The questions "What if...?" "Why?" and the request to explain the reasoning used is constant and recommended recurrently.

About Maths Hubs potentialities

In addition to the aspects related to the principles of Teaching for Mastery, the teacher presents, in his narrative, aspects about the differences between Maths Hubs and other courses he has taken:

It was the first course I took at Maths Hubs. And the reason I really liked it is that... in terms of similarities most of the courses I took were good, because we have a lot of people working together, working with math at the same time, just like Maths Hubs. But in the case of this course I could see things that I would use directly in my classroom. For example, if I compare it to a course I had taken some time before, it was about Problem Solving. The problems were very interesting, but nothing that could really be used in the classroom. (Interview with Daniel Lewis)

What I really like about these CPD courses is "how I can use this in my classroom". I participated in an event called Math Teachers Session on Saturdays, maybe at UCL - University College London. The same as I have already commented, but I did not see how to use that in my classroom. They never discuss how to use it in the classroom. However, all Maths Hubs CPD that I have gone, we have discussions, we create activities. (Interview with Daniel Lewis)

The lack of reference in practice is also one of the negative observations that other teachers make in the Brazilian context, as can be seen in Major (2018). Maths Hubs, by instigating teachers to create activities based on the principles of Teaching for Mastery, brings the perception of a formative path of immediate utility. The same is noticed when the teacher reports the activities that lead to the Working Group he coordinates:

In the Working Group that I lead, since it is a group that aims to implement

something, it is not exactly a course. I do it together with the CPD sessions. I present what is variation, procedural variation, conceptual variation, and give some related examples. But the main part is making people discover and create things for themselves. Create a set of issues that use Variation, conceptual variations, procedural variations, formal definitions, and share that. [...] We can see that given a particular topic people are discovering different things. But the key is to justify what they are doing, right? I'm doing it because I have this goal. What do I want with my set of questions? So we come back, we share the activities, we look at the activities. (Interview with Daniel Lewis)

In the same excerpt it is realized that the sharing of knowledge produced in meetings is something to be pursued. In the two sessions in which the researcher participated, at the end of the session, the facilitator sent to all the ideas of lesson plans that were built in the meeting.

The teacher still refers to the construction of a network that was made possible by the work at *Maths Hubs*.

One of the teachers who used to participate in my Work Group is now doing the Mastery Specialist Program. So, with these people, we started networking. A network with other teachers and we go sometimes to other CPD courses. You start to build a great network with other people, with other professionals. (Interview with Daniel Lewis)

The creation of a network is something fundamental in the professional development of the teacher. A network of professional relationships can only be built with a regularity of medium and long term contacts and collective work.

Final Considerations

This article sought to explore the potentialities of a narrative constituted according to the methodological assumptions of Oral History with a view to broadening the understanding of an English public policy of teacher training, Maths Hubs. To this end, an interview was conducted with a participant of the mentioned public policy and dialogues were established with the relevant literature and training materials available on the program's website.

In order to understand how a public policy of teacher training occurs, other methodologies could have been used, the potential of using a narrative is highlighted in this article. Naturally, when reading a narrative, we create gaps that invite dialogue with literature, with other materials written or not. The dialogue exercise was carried out with a series of written materials available on the program's website, official documents, and pertinent literature. These dialogues have broadened the understanding of the conceptions of Mathematical Education present in the approach to politics, filling, even if always partially, some gaps glimpsed by the researcher. Thus, themes such as teacher training in England, and the ideas of the Mastery Approach, were addressed and discussed. Such topics, because they address general aspects of teacher training and, more specifically, the teaching and learning of mathematics, can contribute as much to those interested in the area of teacher training who teach mathematics as in teaching and learning.

In another direction, it is natural that when we look at written policy documents we see gaps. How do the courses take place, what are the limitations, what are the potentialities beyond those presented? In this sense, we observe that the narrative helps us realize that adherence to politics comes from the immediate applicability of the knowledge that circulates during moments of training. The demand for practical aspects in formative processes also occurs in Brazilian works, such as in Major (2018). It is necessary to take a close look at this demand, but careful, because the practical appeal can hide the complexity of the teaching and learning process that, in the long and medium term, can bring frustrations. In addition to this knowledge, the narrative highlights the potential of public policy in the creation of networks, which helps to reduce the isolation characteristic of the teacher's profession.

From a methodological point of view, we highlight a particular reading of the use of narratives for research on teacher training. This mobilization joins the work of other researchers who have been using narratives in Mathematics Education. To the interested reader, we highlight the work of Marinéia dos Santos Silva, "What can narratives in Brazilian Math Education", in which the author interviews eight researchers, mostly leaders, of the research groups that use narratives in studies and research in Math Education. The results of the research undertaken here corroborate the researcher's considerations when she points out that

The power of working with narratives in teacher training processes is linked to a politics of narratively, which includes an ethical dimension, because it respects the subjects' worldview; aesthetics, in the sense that it proposes a style of writing; and politics, because it concerns the processes of subjectivation and personal empowerment. (Silva, 2020, p. 21)

As far as teacher training is concerned, and particularly public policies for teacher training, this work joins efforts to reflect on and problematize these processes. In this sense, for more than a decade, the researcher Dario Fiorentini (2008) has published the article "The Research and Practice of Mathematics Teacher Training in the Face of Public Policies in Brazil. The researcher, at that time, emphasized that although he recognizes that researchers in math education have participated in actions in isolation, there are

The need for SBEM to mobilize the community of mathematical educators, trying to establish partnerships with other scientific entities and similar institutions in order to participate and intervene with responsibility and commitment in the conception and management of educational policies in Brazil. (p. 43)

We observe that even today SBEM does not have such protagonism, aggravated today by the tension with foundations of different natures. We observe that Maths Hubs is assumed as a public policy of teacher training, fully assumed by NCETM. Protected the characteristics of each country and each entity, such protagonism is seen as desirable and possible in the Brazilian context, because, despite the continental dimension of our country, we have a highly capillary SBEM and several experiences at the national level, such as, for example, the National Pact for Literacy in the Right Age that reached almost all the 5570 Brazilian municipalities.

There is no doubt that other gaps were created by the reading of the narrative and have not been addressed here, as well as there are gaps glimpsed in the documents that would require other interviews with different actors of the process. This article was presented from the perspective of a researcher, limited by his previous readings and experiences and conducted according to his interests that are expected to be shared by other researchers and those interested in public policy for teacher training and that the knowledge shared here may benefit.

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Referências

- An, S. (2004). Capturing the chinese way of teaching: the learning-questioning and learning-reviewing instructional model. In F. Lianghuo, W. Ngai-Ying, C. Jinfa, & L. Shiqi (Eds.), *How chinese learn mathematics: Perspectives from insiders* (pp. 462-482). World Scientific Publishing Co. Pte. Ltd.
- Askew, M., Bishop, S., Christie, C., Eaton, S., Griffin, P., Morgan, D., & Wilne, R. (2015a). *Teaching form mastery: questions, tasks and activities to support assessment – Year 6*. Oxford University Press.: <https://www.ncetm.org.uk/resources/46689>.
- Askew, M., Bishop, S., Christie, C., Eaton, S., Griffin, P., & Morgan, D. (2015b). *Teaching form Mastery: questions, tasks and activities to support assessment – Year 4*. Oxford University Press. <https://www.ncetm.org.uk/resources/46689>.
- Askew, M., Bishop, S., Christie, C.; Eaton, S., Griffin, P., & Morgan, D. (2015c). *Teaching form Mastery: questions, tasks and activities to support assessment – Year 3*. Oxford University Press. <https://www.ncetm.org.uk/resources/46689>.
- Bagio, V. A. (2014). *Da escrita à implementação das DCE/PR de Matemática: um retrato feito a cinco vezes e a milhares de mãos*. [Dissertação de Mestrado em Educação em Ciências e em Matemática]. Universidade Federal do Paraná. <http://www.exatas.ufpr.br/portal/ppgecm/dissertacoes/>
- Bowden, J., & Marton, F. (1998). *The university of learning*. Kogan Page.
- Bruner, J. S. (1964). Some theorem on instruction illustrated with reference to mathematics. In E. R. Hilgard (Ed.), *Theories of learning and instruction* (pp. 306- 335). University of Chicago Press.
- Bruner, J. S. (1985). Vygotsky: A historical and conceptual perspective. In J. V. Wertsch (Ed.), *Culture, communication and cognition: Vygotsky perspective* (pp. 21-34). Cambridge University Press.
- Cintra, C. C. (2017). *Um estudo do ensino de frações para 7º ano: desde um diagnóstico à aprendizagem mediada por Modelo de Barras*. [Dissertação de Mestrado Profissional]. Universidade Federal de São Carlos.

- Costa, E. X. C. (2016). *Narrativas de professores alfabetizadores sobre o PNAIC de Alfabetização Matemática*. [Dissertação de Mestrado em Educação em Ciências e em Matemática]. Universidade Federal do Paraná. <http://www.exatas.ufpr.br/portal/ppgecm/dissertacoes/>
- Department for Education - UK. (2016). *South Asian method of teaching maths to be rolled out in schools*. <https://www.gov.uk/government/news/south-asian-method-of-teaching-maths-to-be-rolled-out-in-schools>.
- Dienes, Z. P. (1973). A theory of mathematics learning. In F. J., Crosswhite, J. L. Highins, A. R. Osborne, & R. J. Shunway (Eds.), *Teaching mathematics: Psychological foundation* (pp. 137- 148). Charles A. Jones Publishing Company.
- Florentini, D. (2008). A pesquisa e as práticas de formação de professores de Matemática em face das Políticas Públicas no Brasil. *Bolema*, 21(29), 43-70. <https://www.redalyc.org/pdf/2912/291221870004.pdf>
- Freudenthal, H. (1990). *Revisiting mathematics education: China lectures*. Kluwer Academic Publishers.
- Garnica, A. V. M. (2007). *Manual de História Oral em Educação Matemática: outros usos, outros abusos*. SNHMat-SBHMat.
- Garnica, A. V. M. (2011). História Oral e História da Educação Matemática: considerações sobre um método. *Anais do I Congresso IberoAmericano de História da Educação Matemática* (pp. 1-12). Covilhã: Portugal. http://www.apm.pt/files/177852_C32_4dd79e66be182.pdf.
- Garnica, A. V. M., Fernandes, D. N., & Silva, H. (2011). Entre a amnésia e a vontade de nada esquecer: notas sobre regimes de historicidade e história oral. *Bolema*, 25(41), 212-250. <https://repositorio.unesp.br/handle/11449/73000>.
- Góis, R. C. (2014). *O efeito do material concreto e do modelo de barras no processo de aprendizagem significativa do conteúdo curricular de frações pelos alunos de 7º ano do ensino fundamental*. [Dissertação de Mestrado em Ensino de Ciências Exatas]. Universidade Federal de São Carlos.
- Gu, L., Huang, R., & Marton, F. (2004). Teaching with variation: A chinese way of promoting effective mathematics learning. In F. Lianghuo, M. Ngai-Ying, C. Jinfa, & L. Shiqi (Eds.) *How Chinese learn mathematics: Perspectives from insiders* (pp. 309-347). World Scientific Publishing Co. Pte. Ltd.
- Lianghuo, F., Ngai-Ying, W., Jinfa, C., & Shiqi, L. (Eds.) (2004) *How chinese learn mathematics: Perspectives from insiders*. World Scientific Publishing Co. Pte. Ltd.
- Major, M. C. (2018). *Histórias sobre a formação continuada de professores de matemática dos anos finais do EF da RME de Curitiba*. [Dissertação de Mestrado em Educação em Ciências e em Matemática]. Universidade Federal do Paraná. <http://www.exatas.ufpr.br/portal/ppgecm/dissertacoes/>
- Marton, F., & Booth, S. (1997). *Learning and awareness*. Lawrence Erlbaum Associates, Publishers.
- Menjívar, J. C. C. (2018). *CFD e Gestar II: compreensões sobre a formação continuada de professores que ensinam matemática a partir de um estudo comparado*. [Dissertação de

Mestrado em Educação em Ciências e em Matemática]. Universidade Federal do Paraná. <http://www.exatas.ufpr.br/portal/ppgecm/dissertacoes/>

National Centre for Excellence in the Teaching of Mathematics (NCETM). (2018a). *Teaching for Mastery*. www.ncetm.org.uk/resources/50042.

National Centre for Excellence in the Teaching of Mathematics (NCETM). (2018b). *Primary Magazine*, 74. www.ncetm.org.uk/resources/46773.

Queiroz, J. M. S. (2014). *Resolução de problemas de pré-álgebra e álgebra para Fundamental II do Ensino Básico com auxílio do Modelo de Barras*. [Dissertação de Mestrado em Ensino de Ciências Exatas]. Universidade Federal de São Carlos.

Silva, M. S. (2020). *O que podem as narrativas na Educação Matemática brasileira*. [Tese de Doutorado em Educação Matemática]. Universidade Estadual Paulista. <https://repositorio.unesp.br/handle/11449/193252?show=full>

Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, 17(2), 89-100.