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Number line estimation, working memory and quantitative reasoning: relations in math achievement[1]

Estimativa numérica, memória de trabalho e raciocínio quantitativo: relações no desempenho matemático

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Abstract

While evidence shows number estimation and working memory are important factors in math achievement, little is known about the relationship between number estimation and working memory. Therefore, the aim of this study is to investigate the relationships between number estimation, working memory and quantitative reasoning. To do so, 143 3rd and 4th graders from two public schools in Porto Alegre/RS were assessed. The results reveal significant relationships between the three measures, with the highest levels of correlation being between quantitative reasoning and number line estimation and between the central executive and episodic buffer, which are components of the working memory, and additive reasoning. The results suggest a direct association between ability in number line estimation, working memory and math skills in quantitative reasoning.

Keywords: Number Line Estimation; Working Memory; Quantitative Reasoning.

Resumo

Há evidências de que tanto a estimativa numérica quanto a memória de trabalho são habilidades importantes para o desempenho em matemática, porém pouco se sabe sobre a relação entre estimativa numérica e memória de trabalho. Assim, pretendeu-se neste estudo verificar as relações entre as capacidades de: estimativa numérica, memória de trabalho e raciocínio quantitativo. Para isso, 143 crianças de 3.º e 4.º anos do Ensino Fundamental de duas escolas municipais de Porto Alegre/RS foram avaliadas. Verificaram-se relações significativas entre as três medidas, sendo que os maiores níveis de correlação foram entre raciocínio quantitativo e estimativa numérica e entre os componentes executivo central e *buffer* episódico da memória de trabalho com o raciocínio aditivo. Os resultados indicam associação direta entre as medidas avaliadas.

Palavras-chave: Estimativa Numérica; Memória de Trabalho; Raciocínio Quantitativo.

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Introduction

The development of numerical skills is an important research topic in the areas of Mathematics, Education and Psychology. Evidence shows good performance in mathematical skills from the early school years is predictive of later mathematical achievement (Ashcraft & Moore, 2012; Booth & Siegler, 2006; Moore & Ashcraft, 2015; Siegler & Booth, 2004). Studies have shown that, from Early Childhood Education, children are already able to understand the relationship between numerical quantities and to use them in different contexts (Moore & Ashcraft, 2015). However, they often fail to understand an important aspect of the numerical system, namely, that the whole numbers, when represented along a number line, are evenly spaced (Booth & Siegler, 2008; Dehaene, 1997; Siegler & Opfer, 2003).

In the mental representation of numerical magnitudes, the distances placed by children between the smaller numbers are very different from the distances between the larger numbers. For example, studies of number line estimation in children showed they left bigger distances between the numbers 10 and 20 than between 80 and 90 (Booth & Siegler, 2006; Siegler & Booth, 2004). The tool most widely used to assess that capacity is the number line estimation task, which consists of a horizontal line, delimited by 0 on the left and 100 on the right, on which the child is asked to estimate, that is, mark the location of a given number on the number line. This type of task assesses the ability to estimate magnitudes on a number line and has been widely used to determine children's understanding of numerical magnitudes and ordering (Laski & Siegler, 2007; Link, Nuerk, & Moeller, 2014; Moore & Ashcraft, 2015).

Another important cognitive resource for mathematics achievement is the working memory (Gathercole & Alloway, 2004; Raghubar, Barnes, & Hecht, 2010), which refers to the combination of short-term storage and information processing necessary for complex mental activities, such as solving mathematical problems (Baddeley, 2011; Gathercole & Alloway, 2004). A widely accepted model proposed by Baddeley (2000) indicates the working memory has four components: the central executive, which lacks storage capacity and is responsible for processing information; the phonological loop and visuospatial sketchpad that are responsible for the temporary storage of information and support the central executive; and the episodic buffer, which comprises a form of storage capable of supporting the recall of a sequence and of associating phonological and spatial information together with other types of information (Baddeley, 2000).

Studies investigating the relations between working memory and arithmetic achievement indicate that the episodic buffer helps in numerical representation, since this component can encode different information (visual, spatial or phonological) and perform the conversion between one form and another, which contributes to mental calculation and influences performance in mathematics (Leung, 2011; Towse & Houston-Price, 2001). Regarding the other components of the working memory, it can be said that in an arithmetic problem solving activity, for example, the central executive is responsible for retrieving

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information about the operation to be used at the same time as the other two subsystems store the numbers involved in the calculation (Corso & Dorneles, 2012).

Evidence from the literature demonstrates that both number line estimation (Laski & Siegler, 2007; Link et al., 2014; Moore & Ashcraft, 2015) and working memory (Andersson, 2008; Andersson & Lyxell, 2007; Lee & Bull, 2016; Meyer, Salimpoor, Wu, Geary, & Menon, 2010) are related to mathematics achievement. Thus, in the present study, we investigated the existence of a relationship between working memory, mathematics achievement (specifically quantitative reasoning), and performance in number line estimation.

Quantitative reasoning refers to the understanding of the relations between quantities, whereby the numerical values used to represent the quantities are thought to become secondary. For this reason, quantitative reasoning is fundamental to the construction of the initial concepts of mathematics, allowing children to reason about the nature of the problems and the operative relations to be used (Nunes, Dorneles, Lin, & Rathgeb-Schnierer, 2016). In quantitative reasoning, the relations between quantities can be classified into additive and multiplicative reasoning. Additive reasoning is based on part-whole relations between quantities, while multiplicative reasoning is represented by the one-to-many correspondence relations and on the ratios established between the quantities (Nunes et al., 2016). In this study, three types of situations were used for additive reasoning: quantity composition, transformation and comparison; and three for multiplicative reasoning: direct and inverse relations between quantities, and product of measures. These situations are in accordance with Nunes and colleagues (2016).

With additive reasoning, in situations that involve the composition of quantities, it is necessary to add or separate the quantities to obtain the result (For example, “Arthur will keep 5 purple t-shirts and 4 green t-shirts in the closet. In all, how many t-shirts will he keep?”). In situations involving transformation, addition or subtraction operations can be performed to find the answer (For example, “Ana had some stickers. She played with her friends and won 3 stickers. Now she has 12 stickers. How many stickers did she have?”). In situations that involve comparison, an understanding of the words ‘more’ or ‘less’ is expected (For example, “Fábio has 13 books and Zeca has 8. Who has more books? How many more?”).

In multiplicative reasoning, situations that involve a direct relation between quantities can be solved by multiplication or division operations, depending on the unknown quantity (An example of a problem involving multiplication is, “Renato invited three friends to his birthday party. He wants to give five balloons to each friend. How many balloons does he need to buy?”; An example of a problem involving division, “Renato has 15 balloons. He will distribute them equally among his three friends. How many balloons will each receive?”). Situations involving an inverse relation between quantities can only be solved using division, even if one-to-many correspondence is used (“Antonio and Luis ride their bicycles, leaving from the same starting point, Antonio’s path was 200m and Luis’ path was 600m. They both

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arrived at the same place at the same time. Was their speed the same?”). For situations involving product of measures, a third quantity is formed from two other quantities (For example, “Luísa has 2 types of skirts and 3 types of blouses. How many different combinations of clothes can she make?”). Both types of reasoning are essential for understanding the four fundamental operations of mathematics (addition, subtraction, multiplication, and division) and their relations.

To the best of our knowledge, the relations between the ability to estimate and the working memory have not yet been a focus of study, despite the numerous studies that have verified the influence of working memory on mathematical performance. While few studies have considered number line estimation as a complementary task of cognitive skills and mental numerical representation (Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout, 2013; Xenidou-Dervou, van der Schoot, & van Lieshout, 2015), they do not specifically focus on relations between number line estimation and working memory.

Regarding the relations between number line estimation and mathematics achievement, one notable study assesses the development of the mental representation of the number line by children using number line estimation tasks, relating it to the performance in tasks involving numerical magnitude comparison and counting, carried out with preschool pupils up to the second year of school (Link et al., 2014). Based on these tasks, the estimation accuracy patterns were calculated and classified as indicators of the development of the mental representation of numbers (Laski & Siegler, 2007; Link et al., 2014). These patterns classified into estimation development models, were observed among children of different ages and between children and adults. Children were found to improve their performance in estimation according to age, school grade and experience with the type of task (Link et al., 2014).

There is evidence that performance in number line estimation is related to basic and complex numerical skills in arithmetic (Booth & Siegler, 2006; Laski & Siegler, 2007; Link et al., 2014; R. Siegler & Booth, 2004). A relation was also found between performance in number line estimation and the abilities to categorize and compare numerical magnitudes among young children in kindergarten through to the second school grade (Laski & Siegler, 2007). Among pupils up to the third year of school, a relation was observed between performance in number line estimation and mathematics achievement, since the participants who performed well at estimating on the number line also obtained good results in standardized mathematical performance assessment tests (Booth & Siegler, 2006).

A study by Xenidou-Dervou et al. (2015), focusing on how children solve addition problems, related working memory to different formats of additive problems: symbolic and non-symbolic. The researchers looked at how the children’s performance changed between the additive problem formats, the cognitive process underlying performance in each type of problem, and the relations between working memory components and performance in number line estimation tasks. As assessment measures, the authors used symbolic (using numeric symbols) and non-symbolic (using dots to represent quantities) addition tasks, both requiring

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approximate or exact answers, to evaluate each of the three components of working memory - corsi blocks to evaluate the visuospatial sketchpad, forward digit recall to evaluate the phonological loop and backward digit recall to evaluate the central executive - and number estimation tasks on the symbolic and non-symbolic number line. Some of the findings reported by the authors are pertinent to the present study. In the symbolic addition, performance in the working memory and in number line estimation tasks indicated that the children translate the symbolic information into a non-symbolic coding when approximate answers are required. For example, the symbolic information in the additive problem (the indicated number) is retained in the memory in its original (symbolic) encoding, while the central executive translates it into non-symbolic information (an approximate answer). However, when exact answers are required, the authors explain that children phonologically store the numerical information in the additive problem using symbolic encoding, with no translation being required between different types of information encoding (Xenidou-Dervou et al., 2015).

Another important finding from the same study concerns the relations between the representations on the number line and performance in addition problems: the more accurate the representations on the symbolic number line, the better the performance in exact addition problems. Therefore, based on the study, we can see that number line estimation presented a relation with working memory together with addition problems (Xenidou-Dervou et al., 2015).

The present study aimed to investigate the students' performance in working memory and quantitative reasoning tasks, and to investigate the relations between these abilities and two number line estimation tasks. We hypothesize that there is a relation between number line estimation and both the working memory and quantitative reasoning.

Method

Participants

The sample consisted of 143 3rd and 4th graders, aged between 8 and 11 years (M=9.8 years, SD=0.74), from two public schools in Porto Alegre – RS/Brazil. The two schools were chosen based on convenience, number of students and because they serve communities with similar socioeconomic status.

The exclusion criterion was a classification below the 50th percentile in Raven's Colored Progressive Matrices - Special Scale (Angelini, Alves, Custódio, Duarte, & Duarte, 1999). In this test, the 50th percentile is considered to represent the average intellectual level, with scores below this percentile being considered below average in the expected intellectual capacity for the age assessed. Therefore, students with scores below the 50th percentile were disregarded for the data analysis, which allows for a more reliable and homogeneous analysis of the data, without interference from performances resulting from a possible intellectual disability. Table 1 shows the sample characteristics.

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Table 1 - Sample characteristics

		N	%
Gender			
	Female	62	43,36
	Male	81	56,64
School grade			
	3rd	76	53,15
	4th	67	46,85
Age			
	8	22	15,38
	9	59	41,26
	10	54	37,76
	11	8	5,59

Source: Elaborated by the author

Below, the tasks used to assess each of the skills considered by this study, as well as the statistical analysis used, are described.

Number Estimation Assessment Task

Two types of tasks were used to assess the students' ability to estimate. The number-to-position task (NP) and the position-to-number task (PN), adapted from Siegler and Opfer (2003). In both tasks, number lines, marked only with 0 on the left end and 100 on the right end, were shown. Participants were asked to estimate the location of a number in the NP task, and, in the PN task, to estimate a number for a mark given on the line. There were 22 numbers to be estimated (2, 3, 5, 8, 12, 17, 21, 26, 34, 39, 42, 46, 54, 58, 61, 67, 73, 78, 82, 89, 92, 97), taken from Laski and Siegler (2007), which were presented in random order. The children received a notebook for each task, with a number line and a number or position to be estimated on each page, on which they marked or wrote down their estimates.

Working Memory Assessment Tasks

In this study, the components of working memory were evaluated using the Block Recall and Forward and Backward Digit Recall tasks, selected from Pickering and Gathercole (2001) and using the Rey Auditory Verbal Learning Test (RAVLT) (Malloy-Diniz, Fuentes, Abrantes, Lasmar, & Salgado, 2010). These tasks were applied individually.

The Block Recall task was applied to assess the visuospatial sketchpad. In this task, the child is presented with nine blocks fixed on a plastic base, where the faces facing the child are smooth and the faces facing the evaluator have a number between 1 and 9 on each block. To start the task, the evaluator indicates a sequence of blocks by placing the index finger on each of them, at a rate of one block per second. The test has nine levels, starting with the first level corresponding to sequences of only one block and continues, progressively

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increasing, until sequences comprising nine blocks. For each level there are six sequences that must be repeated correctly in the same order. The test is interrupted if the child makes three mistakes at the same level. Performance in this task is measured according to the number of sequences repeated correctly (Pickering & Gathercole, 2001).

To evaluate the phonological loop and central executive components, the Digit Recall Forward and Backward were used, respectively. There are nine levels in the forward task and six in the backward task. At each level there are six sequences of digits that gradually increase as the level changes, starting with sequences of one digit in the forward task and two in the backward task. The sequences in the forward task assess the phonological loop, that is, the capacity to store information from speech, while the sequences in backward task assess the central executive, that is, the ability to manipulate information. The test is interrupted when three consecutive incorrect answers are given at the same level (Pickering & Gathercole, 2001).

The episodic buffer was assessed using the Rey Auditory Verbal Learning Test (RAVLT). This test was chosen because it is used to assess this component of the working memory (Martins & Ortiz, 2009; Nobre et al., 2013), is a standardized test, with a translation into Portuguese and has already been applied in studies in Brazil (Martins & Ortiz, 2009).

The test consists of a list of 15 nouns (list A) that is read aloud to the participant five consecutive times, always in the same order, with a one-second interval between words. After each repetition, the participant is immediately asked to recall as many words as possible (the order in which they are remembered is not taken into account). After the fifth attempt, an interference list, also composed of 15 nouns (list B), is read to the participant, which he is similarly asked to recall. Then, the participant is asked to recall the words in list A without them being re-presented. After an interval of 20 minutes, during which the participant can do other activities that do not require verbal reasoning, the participant is asked to remember the words in list A (a delayed recall measure that assesses the episodic buffer) without the list being read for him. After this delayed recall, the recognition memory test is performed, in which a list of 50 words, containing the 15 words in list A, 15 in list B and another 20 phonetically or semantically similar words to those in lists A and B, are read to the participant. The participant is asked to identify whether each of the words belongs to list A, list B or neither of the lists (Malloy-Diniz et al., 2010). For this study, the RAVLT provides a measure of delayed recall, which is influenced by the episodic buffer since this recall does not require the examiner to read the list beforehand (Martins & Ortiz, 2009), all recalls are made verbally without the use of any written record.

Quantitative Reasoning Assessment Task

The quantitative reasoning assessment task, based on Nunes (2009), was applied to assess arithmetic reasoning in problem solving. The quantitative relations involved were additive reasoning (including composition of quantities, situations of transformation and comparison) and multiplicative reasoning (containing situations of direct and inverse relations between quantities and product of measures). The task comprised 18 problems, nine

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involving additive reasoning (three for quantity composition, three for transformation and three for comparison) and nine involving multiplicative reasoning (three for direct relation, three for inverse relation and three for product of measure). The task was applied collectively, in small groups of a maximum of 10 students, with each student individually resolving the issues, but at the same time as the others. Each participant received a notebook containing only the illustrations of the problems, one problem per page, and without any written information. Instructions were given orally by the evaluator, as this allowed students to expose different solution strategies and did not depend on their reading ability. After the students filled out the notebook with the answers in the indicated place, the evaluator collected the notebooks for later analysis.

Data analysis

Quantitative analysis was performed to compare the performance in the working memory and quantitative reasoning tasks with the performance in the number estimation tasks. First, the distribution of variables was analyzed using Shapiro-Wilk's normality test. To compare the performance between school grades and between genders in the working memory and quantitative reasoning tasks, the Mann-Whitney non-parametric test was conducted. Associations between performance in the number line estimation tasks and performance in the other tasks were analyzed using Spearman's correlation test, which was used to check the relation between performance and age.

The data on the children's performance in number line estimation were obtained by calculating the accuracy regarding each estimated number, that is, by calculating the percentage of absolute error for each estimate. This calculation is adapted from Siegler and Booth (2004) and represented by the following formula:

$$\left|\frac{Child's\ estimate - Target\ number}{Scale\ of\ Estimates\ (100)}\right|$$

So that, considering 70 the number to be estimated, if the mark made by the child corresponded to the number 60, according to the formula presented, the percentage of the absolute error would be 10%, which corresponds to the result of $\left|\frac{70-60}{100}\right|$. Therefore, the more accurate the child's estimate, the lower the value resulting from the formula, i.e. the percentage of absolute error will be close to zero.

Each child's performance in the working memory tasks was considered based on the total number of correct answers in each task applied. The Forward Digit Recall and Block Recall tasks have nine levels with six sequences each, totaling 54 points in each task. The Backward Digit Recall task, by contrast, has six levels each with six sequences, totaling 36 points. In the task used to measure the episodic buffer (RAVLT), the score was based on the number of words correctly evoked in the delayed recall, after the 20-minute interval.

In the quantitative reasoning test, the number of correct answers in relation to the total number questions and the number of correct answers per type of problem (additive reasoning and multiplicative reasoning) were considered.

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Results

Working Memory Performance

The assessment of working memory considered the phonological loop, visuospatial sketchpad, central executive and episodic buffer components contained in Baddeley's model (2000). The phonological loop (PL) was measured using the Forward Digit Recall, the visuospatial sketchpad (VSSP) was accessed using the Block Recall task, the central executive (CE) was assessed using the Backward Digit Recall task, and the episodic buffer (EB) was measured using the RAVLT A7, which corresponds to the delayed recall of the word evocation test (Rey Auditory Verbal Learning Test). Table 2 shows students' performances according to task type.

Table 2 – Achievement on WM tasks

WM Components	Mean (SD)	Median [p25; p75]
Phonological Loop	27,97 (5,32)	29 [24;31]
Visuospatial Sketchpad	23,53 (4,07)	24 [21;26]
Central Executive	10,10 (3,75)	10 [7;12]
Episodic Buffer	7,83 (2,88)	8 [6;10]

Source: Elaborated by the author

When relating the working memory scores with the variables gender, age and school grade, a significant difference was found between gender and phonological loop score ($U=1941$, $p<0.05$), suggesting girls ($M=29.29$, $SD=5.48$) performed better than boys ($M=26.96$, $SD=4.99$), and between school grade and episodic buffer score ($U=3294$, $p<0.05$), in which the 4th grade ($M=8.48$, $SD=2.96$) performed better than the 3rd grade ($M=7.25$, $SD=2.70$). In relation to age, a significant but weak correlation was only found with the the visuospatial sketchpad score ($rs=0.18$, $p<0.05$).

Quantitative Reasoning Performance

The quantitative reasoning assessment task assessed the children's ability to solve mathematical situations involving arithmetic reasoning in the four fundamental mathematical operations. The task results were computed by totaling the number of correct answers in the task and the number of correct answers in each type of problem (additive reasoning and multiplicative reasoning). In general, the students had an average of 10.55 correct answers ($SD=3.22$). When the scores were considered according to the type of problem, the students obtained better results in the additive reasoning ($M=6.03$, $SD=1.92$) than in the multiplicative reasoning ($M=4.51$, $SD=1.81$). When relating performance in the quantitative reasoning task with gender, age and school grade, there was a statistically significant difference with the school grade ($U=3256$, $p<0.05$) and a direct, but weak correlation with age ($rs=0.18$, $p<0.05$). These findings indicated there was an increase in the total number of correct answers from the 3rd to the 4th grade (3rd grade: $M=9.74$, $SD=3.32$; 4th grade: $M=11.46$, $SD=2.87$) and a significant increase ($U=3486.5$, $p<0.05$) between school grades only for the additive reasoning (3rd grade: $M=5.42$, $SD=2.06$; 4th grade: $M=6.73$, $SD=1.47$), as presented in Graph 1.

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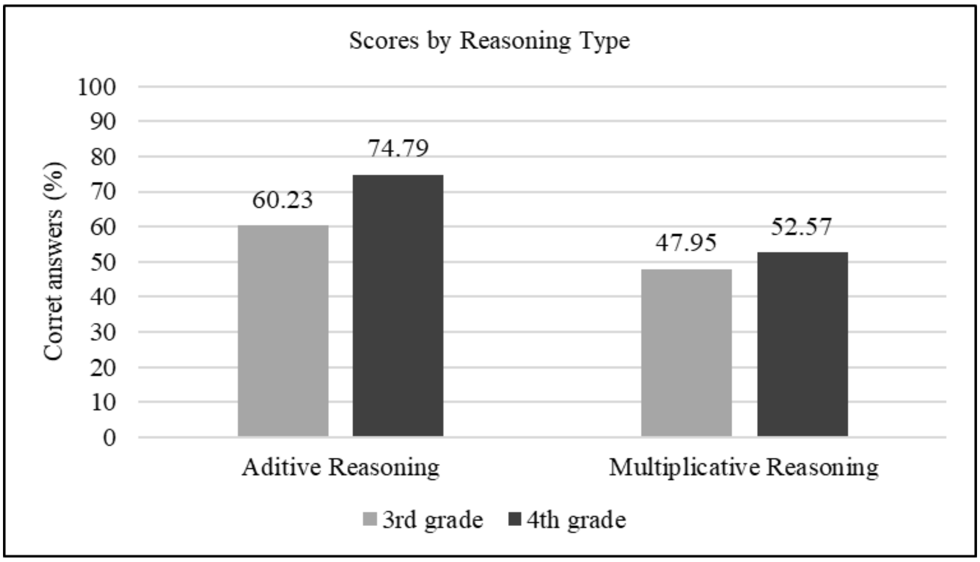
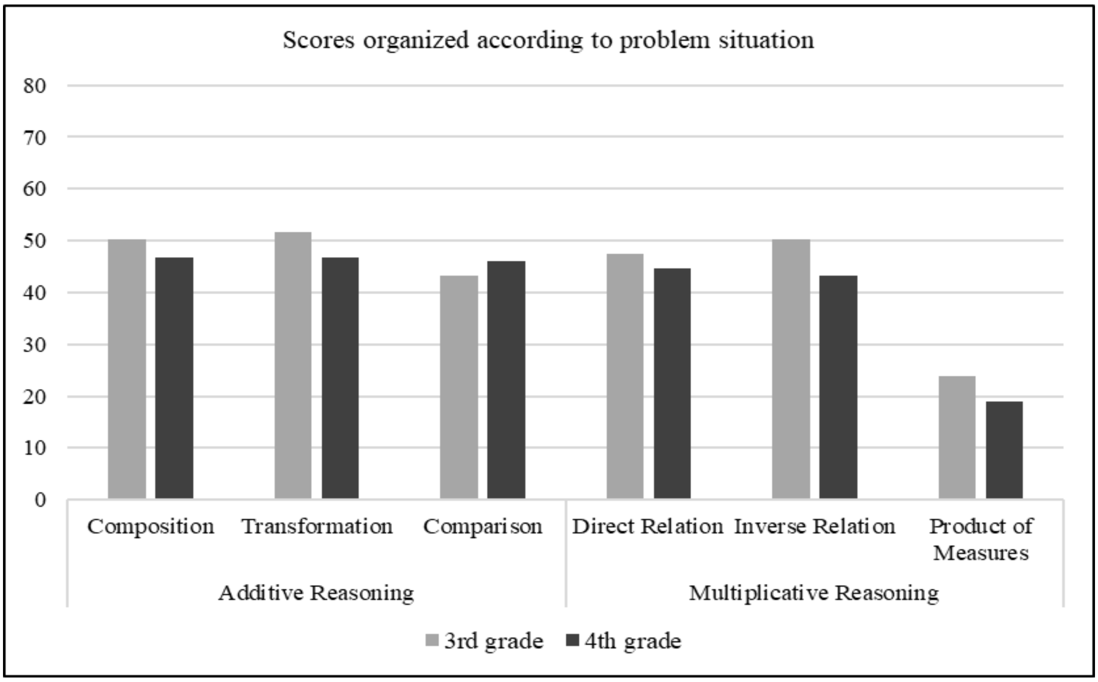
Graph 1 - Student Scores in the Quantitative Reasoning Assessment Task



Source: Elaborated by the author

When additive and multiplicative reasoning situations were analyzed (Graph 2), there was a higher percentage of correct answers in the situations of additive reasoning involving composition of quantities and transformation. In the multiplicative reasoning situations, the students had better results in problems involving direct and inverse relations.

Graph 2 - Scores organized according to problem situation



Source: Elaborated by the author

Descriptive analysis was carried out to identify which in problems the students gave the highest and lowest number of correct answers, in both additive and multiplicative reasoning (Chart 1).

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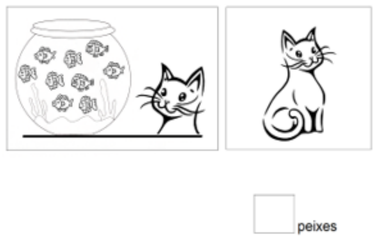
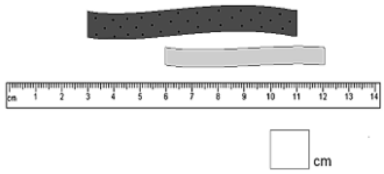
Chart 1 - Problems with highest and lowest number of correct answers in Quantitative Reasoning

Additive Reasoning	95,8% of correct answers		**Transformation:** There were 9 fish in the aquarium. The cat ate 3. How many fish are in the aquarium now? Write your answer in the space provided.
	11,9% of correct answers		**Comparison:** Luísa has two ribbons. Are any of them longer? If so, circle the longest ribbon. How many centimeters is this ribbon longer than the other? Write your answer in the space provided.
Multiplicative Reasoning	85,3% of correct answers		**Direct Relation:** The teacher wants to take her 16 students to the zoo. The teacher will drive a car and some mothers will have to drive other cars as well. Each car will fit 8 children. How many cars are needed for all children to go to the zoo?
	23,1% of correct answers		**Product of Measures:** Francisco has two shorts and three T-shirts. If he combines shorts and shirts differently, how many sets can he make?

Source: Adapted from Nunes (2009)

Thus, it was found that among the additive reasoning categories, the problem with the highest rates of correct answers (95.8%) was that involving simple transformation, which requires the direct application of the mathematical operation, while that with the least correct answers (11.9%) was a comparison problem. Among the multiplicative reasoning categories, the problem with the highest number of correct answers (85.3%) was that of a direct relation between the quantities involved, and the one with the lowest percentage of correct answers was one of product of measures (23.1%).

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Quantitative Reasoning Performance Measures

Correlation analyzes were performed to check the relations between number line estimation, working memory and quantitative reasoning. This revealed a weak but significant correlation of the scores in the VSSP (rs=-0.17, p<0.05), CE (rs=-0.19, p<0.05) and EB (rs=-0.19, p<0.05) tasks with performance in the NP number line estimation task, indicating a relation between working memory capacity, specifically the visuospatial sketchpad, central executive and episodic buffer components, and performance in NP number line estimation task. In the PN task, there was also a weak but significant correlation with the scores in the CE (rs=-0.26, p<0.01) and EB (rs=-0.22, p<0.01) tasks, indicating an relation of the central executive and episodic buffer components with performance in PN number line estimation task.

For the quantitative reasoning scores, moderate and significant correlations were found between general performance in the quantitative reasoning task both in the NP task (rs=-0.39, p<0.01) and the PN task (rs=-0.42, p<0.01). Likewise, there were also moderate and significant associations between performance by type of reasoning and performance in the two number line estimation tasks (Additive reasoning - NP: rs=-0.38, p<0.01; PN: rs=-0.39, p<0.01; Multiplicative reasoning - NP: rs=-0.29, p<0.01; PN: rs=-0.33, p<0.01).

The correlations between the ability to make numerical estimates, working memory and quantitative reasoning can be seen in Table 3.

Table 3 - Correlations between numerical estimation, working memory and quantitative reasoning

	Number Line Estimation		Working Memory				Quantitative Reasoning		
Variables	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1. NP	1								
2. PN	0,660**	1							
3. PL	-0,053	-0,091	1						
4. CE	-0,189*	-0,257**	0,427**	1					
5. VSSP	-0,174*	-0,021	0,063	0,088	1				
6. EB	-0,190*	-0,218**	0,062	0,292**	0,087	1			
7. AR	-0,378**	-0,394**	0,283**	0,361**	0,124	0,274**	1		
8. MR	-0,292**	-0,328**	0,143	0,169*	0,099	0,142	0,502**	1	
9. QR	-0,389**	-0,420**	0,239**	0,297**	0,131	0,251**	0,857**	0,864**	1

*p<0,05; **p<0,01

Subtitle: NP – Number-to-position task (Number Line Estimation); PN – Position-to-number task (Number Line Estimation); PL – Phonological Loop; CE – Central Executive; VSSP – Visuospatial sketchpad; EB – Episodic Buffer; AR – Additive Reasoning; MR – Multiplicative Reasoning; QR – Total score in Quantitative Reasoning.

Source: Elaborated by the author

These results reveal correlation between the NP estimation task and the visuospatial sketchpad, central executive and episodic buffer components of the working memory. While in the PN estimation task, the correlation was between the central executive and episodic buffer components. Therefore, it was decided to see whether these correlations are associated with high or low scores in the VSSP, CE, and EB tasks. To do so, the scores of the

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participants in these tasks were separated into tertiles, which were classified as high, moderate or low performance, as shown in Table 4.

Table 4 - Scores classified in tertiles in the Working Memory tasks

		NP Task		PN Task	
WM Tasks	**N**	**Mean (SD)**	**Median [p25; p75]**	**Mean (SD)**	**Median [p25; p75]**
VSSP Scores					
Low (scores ≤ 22)	52	0,103 (0,051)	0,091 [0,071; 0,126]	0,087 (0,047)	0,074 [0,055; 0,105]
Medium (23 ≤ scores ≤ 26)	58	0,103 (0,049)	0,092 [0,065; 0,123]	0,092 (0,059)	0,073 [0,051; 0,111]
High (scores ≥ 27)	33	0,084 (0,041)	0,074 [0,059; 0,096]	0,078 (0,035)	0,074 [0,052; 0,091]
CE Scores					
Low (scores ≤ 8)	62	0,111 (0,061)	0,094 [0,062; 0,141]	**0,098 (0,057)**	0,079 [0,064; 0,119]
Medium (9 ≤ scores ≤ 12)	46	0,092 (0,035)	0,087 [0,067; 0,115]	0,083 (0,045)	0,068 [0,053; 0,098]
High (scores ≥ 13)	35	0,086 (0,034)	0,078 [0,064; 0,112]	**0,073 (0,039)**	0,058 [0,049; 0,087]
EB Scores					
Low (scores ≤ 7)	52	**0,111 (0,059)**	0,093 [0,071; 0,127]	**0,095 (0,054)**	0,078 [0,058; 0,110]
Medium (8 ≤ scores ≤ 9)	52	0,098 (0,041)	0,092 [0,066; 0,123]	**0,091 (0,049)**	0,078 [0,057; 0,115]
High (scores ≥ 10)	39	**0,083 (0,039)**	0,079 [0,058; 0,097]	**0,070 (0,040)**	0,061 [0,048; 0,084]

Subtitle: VSSP Scores: visuospatial sketchpad task; CE Scores: central executive task; EB Scores: episodic buffer task.

Source: Elaborated by the author

From these results, a significant difference was found between the high and low scores in the CE and PN tasks (H(2)=23.69, p<0.05), suggesting the students who obtained higher scores in the central executive component, performed better in the PN number line estimation task. Also, significant difference between the high and low scores in the EB task was seen in the NP task (H(2)=21.20, p<0.05), suggesting students with higher scores in the episodic buffer component, achieved better accuracy in the NP number line estimation task. Considering this same component of working memory, EB, there was a significant difference between high and moderate scores (H(2)=22.28, p<0.05) and between high and low scores (H(2)=24.97, p<0.05) in the PN task, which suggests the higher the capacity of the episodic buffer, the better the performance in PN number line estimation task.

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Discussion

The aim of the present study was to look into the relations between the components of working memory and performance in number line estimation, as well as to relate performance in mathematics, using quantitative reasoning tasks, with performance in the same number line estimation tasks.

One of our hypotheses was that there would be a correlation between working memory and the two proposed number line estimation tasks both in the 3rd and 4th school grades. This hypothesis was confirmed and, more specifically, we found the score in central executive and episodic buffer were significant for performance in the two number line estimation tasks. Furthermore, we found the most significant difference to be between pupils that have high and low scores in the tasks that evaluate these components of working memory, indicating the greater the capacity of the central executive and the episodic buffer, the better the performance will be in number line estimation tasks.

It is worth mentioning that, although it was not the focus of this study, we found an association between working memory and performance in mathematics. The phonological loop and the episodic buffer components only correlated with additive reasoning, whereas the central executive component was associated with the two types of quantitative reasoning: additive and multiplicative. The involvement of these components seems quite reasonable, given that it implies the retention of auditory information, which is processed by the phonological loop, while the episodic buffer integrates currently provided information with that from the long-term memory. Simultaneously, the central executive manipulates and modifies the information to arrive at the result. All of which are important and necessary processes for the resolution of mathematical problems. These findings confirm previous studies demonstrating the existence of an important relation between working memory and mathematics, specifically in solving arithmetic problems, which were assessment measures in this study (Corso & Dorneles, 2012; Corso, 2008; Gathercole & Alloway, 2004; Passolunghi, Vercelloni, & Schadee, 2007).

According to the results found with quantitative reasoning task, the 4th graders performed better overall than 3rd graders, as expected. When the types of reasoning were analyzed separately, the 4th graders obtained better results in both the additive and multiplicative reasoning tasks, but this difference was only significant for additive reasoning. This suggests that, according to the content proposed in the Brazilian school curriculum for these school grades (MEC, 1997), students are about to start learning multiplicative reasoning and the relations between multiplication and division, which may explain the fact there was no significant difference in terms of performance between the 3rd and 4th graders in these types of problems.

Of the additive reasoning problems, the one with the highest percentage of correct answers was that involving simple transformation, in which an initial quantity is given, a second quantity is subtracted, and the result is requested. In this problem situation, students

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use the subtraction scheme together with counting (Nunes, Campos, Magina, & Bryant, 2005). In this problem, the students obtained over 95% of correct answers in the 3rd and 4th grades, the same result obtained by children from São Paulo in the research by Nunes and colleagues (2005).

The students' performance in the quantitative reasoning task also correlated with the two of number line estimation tasks, with additive reasoning presenting the highest level of correlation in both the NP and PN tasks. Regarding the correlation rates, one can see that quantitative reasoning was the measure that presented the highest rate of correlation with the ability to estimate numbers, in comparison to the rates of the working memory tasks.

Similar results were found in a study by Xenidou-Dervou and colleagues (2014), which linked working memory with additive problems and number-to-position tasks. In their study, the authors found a correlation between the phonological loop, visuospatial sketchpad and central executive components of the working memory in different types of additive problems with number line estimation tasks, while also showing an association between working memory and addition problems, which confirms and adds consistency to the findings of the present study.

However, some limitations must be considered when interpreting the present findings. Firstly, the restriction of the tasks selected for the assessment. Consideration should be given to the diversity of the available tasks' types and their non-standardization for use in assessing number line estimation, working memory and quantitative reasoning, none of which are standardized for the Brazilian population. The collective application of the number line estimation and quantitative reasoning tasks also limited the analysis of the data in this study. However, the study's findings are applicable, mainly, in the first grades of elementary school, as they point to the need to perform tasks, in the classroom, that stimulate aspects that currently are rarely explored, such as number line estimation and working memory, and that seem to have a direct influence on performance in mathematical tasks. These findings may provide new insights for the discussion of the role of number line estimation in mathematics achievement and, to some extent, the influence of working memory in the ability to perform number line estimations.

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