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The pedagogical game "playing with probability" for the early years of elementary school: the sample space

O jogo pedagógico "brincando com a probabilidade" para os anos iniciais do ensino fundamental: o espaço amostral

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Abstract

The present work has as objective to analyze the elaborated tasks for a pedagogical game related to the Anthropological Theory of Teaching (TAD). Taking as a reference the Common National Curriculum Base - BNCC, a Brazilian curriculum proposal regarding the contents and skills to be worked on in the thematic unit "Statistics and Probability" for the early years of Elementary School, we elaborate Questions (?) cards for the game, which are tasks based on problem situations whose objective is to favor the apprehension of probabilistic knowledge. It was shown the possibility of developing a pedagogical work based on games and problem solving involving probabilistic content, creating a resource that favors the rethinking of strategic methods, resizing them to minimize the gap between the daily playful activities performed. by the students, spontaneously, and the work triggered in the classroom.

Keywords: Educational game; Probability Teaching; Elementary School; Common National Curriculum Base.

Resumo

O presente trabalho tem como objetivo analisar as tarefas elaboradas para um jogo pedagógico relacionadas ao conceito de espaço amostral segundo a Teoria Antropológica do Didático (TAD). Tomando como referência a Base Nacional Comum Curricular – BNCC, proposta curricular brasileira no que diz respeito aos conteúdos e habilidades a serem trabalhados na unidade temática "Estatística e Probabilidade" para os anos iniciais do Ensino Fundamental, elaboramos cartas Perguntas (?) para o jogo, que são tarefas baseadas em situações problemas cujo objetivo é favorecer a apreensão do conhecimento probabilístico. Mostrou-se a possibilidade de desenvolver um trabalho pedagógico baseado em jogos e resolução de problemas que envolva conteúdos probabilísticos, criando um recurso que favoreça o repensar sobre os métodos estratégicos, redimensionando-os a fim de minimizar o hiato existente entre as atividades lúdicas cotidianas realizadas pelos alunos, espontaneamente, e o trabalho desencadeado em sala de aula.

Palavras-chave: Jogo pedagógico; Ensino de Probabilidade; Ensino Fundamental; Base Nacional Comum Curricular.

Introduction

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We believe that there is a real need for citizens to have mastery over basic knowledge of probability, believing that it has an important role in the formation of citizens. According to Bennett (2003) and Everitt (1999) the learning of probability contributes to the development of critical thinking, which allows citizens to understand and communicate different types of information present in countless situations of everyday life in which random phenomena, chance and uncertainty are present. And yet, because the teaching of probability is little evident in the early years of elementary school, requiring research in this area.

Thinking about these points of view and believing that the game is capable of enabling simulations of tasks that provoke and demand immediate solutions, being characterized as an effective resource for the exercise of active learning, we chose to create a board game whose movement of the pieces it will be done through the correct answers of the players in front of the cards, questions that will be composed by tasks that are part of the child's daily life.

Regarding games, according to the National Common Curricular Base - BNCC (MEC, 2017), a Brazilian curriculum proposal published in December 2017, indicates that it is important to distinguish between games as specific content and games as teaching aids. It is not uncommon that, in the educational field, games and games are invented with the aim of provoking specific social interactions among its participants or to fix certain knowledge. The game, in this sense, is understood as a means to learn something else.

Furthermore, introducing the game or other playful tasks in the classroom does not have to be a complex process in the teaching of mathematics, where various approaches and problems arise from solving problems that can be seen as a prize or an objective to be achieved . Some researchers have already analyzed the advantages of introducing games in class through the study of practical application cases (Torres, 2001; Chamoso, Durán, García, Martín & Rodríguez, 2004; Hernández, Kataoka & Silva, 2010; Bracho, Mas, Jiménez & García, 2011; Malaspina, 2012; Villarroel & Sgreccia, 2012).

Associating the game with problem solving, Grando (2004) explains that the core of problem solving is in the process of creating strategies and in the analysis, processed by the student, of the various possibilities of resolution, whereas in the game a similar fact occurs, because it represents a problem situation determined by rules, in which the individual seeks, at all times, developing strategies and restructuring them, to win the game, that is, to solve the problem. This dynamism characteristic of the game is what makes it possible to identify it in the context of problem solving.

And continuing to discuss this relationship, the teaching and learning processes of Mathematics through the methodology of problem solving and the use of games allows students to create strategies that favor the appropriation of mathematical concepts, so that “new understandings of mathematics embedded in the task ”(Van de Walle, 2009, p. 58) leads them to think, question and discuss their ideas and strategies in the activities carried out in individual work, in pairs or in small groups.

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Problem solving and pedagogical games in the teaching of probability

Considering how Probability was born, authors defend the idea that playing is the best way for children to learn probabilistic concepts such as Góngora (2011) by proposing that, in order to work Probability, games of chance should be used from a ludic and pedagogical approach. , so that, not only do students have a first contact with the field of Probability in a fun, but also meaningful way.

Considering the teaching of probability Vásquez and Alsina (2014) propose for the study of probabilistic concepts the use of concrete materials such as tokens, dice and games of chance, as they will be of great help in conducting random experiments that will reinforce the probabilistic concepts. Torra (2016) exposes several examples of activities with manipulative material for elementary school children and one can clearly see how experimentation and action favor the construction of knowledge.

Punctuating the use of games in education, Alsina (2011) explains that the game helps the child to escape reality to solve conflicts in a symbolic way and, thus, create a series of mental processes that help to internalize mathematical knowledge, but in a pleasant, playful manner and in which socialization is also encouraged. Playing motivates, excites and helps to overcome the fear of failure in the face of problems or operations.

Still focusing on games and indicating the importance of probability since the initial formation of Basic Education, Vásquez and Alsina (2014) recall that in the Chilean national curriculum, the teaching of probability begins with very simple activities in which chance is present, thus favoring the emergence of intuitions. And considering the importance of children learning the concept of chance, it is suggested to play random games, such as playing coins and dice

Highlighting some aspects of using games Londoño (2010) stresses that to learn intelligently and become aware of operations, there is nothing better than interacting with others and manipulating objects and, for example, it could be, designing a game to do it with colleagues where they have to roll a high number of times and check the probability of getting a specific number.

Completing this idea Florez and Vivás (2007) indicate that the game is a valuable recreational tool, as children can dedicate a lot of time to the same activity without getting bored. Playing arouses curiosity, the instinct for exploration, the taste for investigation, creating variants, moving things around, surprising them and surprising us with the results. The game favors mental development, promotes creativity and awakens joy.

Still associating the teaching of probability with games, Corbalán (2002) suggests the use of pre-instructional games, that is, those that are used before the acquisition of concepts or procedures assuming that in the process of introducing probabilistic concepts to students it is convenient to carry out a whole battery of activities before you can proceed to any kind of

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definition or formalization, although it doesn't seem very precise or rigorous. In this nucleus it is highly recommended to introduce games.

Highlighting probabilistic concepts, Edo, Deulofeu and Badillo (2007) point out that despite the existence of chance, when students are players in a pedagogical game, they must make decisions that can influence the game. During the game the following questions are generated: Which of my pieces have I advanced with? Is it better to save this piece? If I move, will I kill an opponent? etc. While games continue to depend largely on chance, the roll of the dice influences the outcome, but it also depends on how players position themselves in decision making.

Methodological procedures

The objective of this work was to show the process of creating a pedagogical game for the development of probabilistic concepts in the early years of elementary school, specifically the notion of sample space, proposed in the BNCC, Brazilian curriculum base published in December 2017, and supported by Anthropological Theory of the Didactic - TAD. We conceive that creating is the same as walking the path of the design of the educational game that we call “Playing with Probability”.

Starting from this objective, we list in this research some specific objectives. Are they:

- Create tasks supported by TAD that addresses the probabilistic content (notion of sample space) proposed by BNCC, based on Nunes and Bryant (2012) and converging on the proposals of Gal (2005), Coutinho (2001) and Batanero and Godino (2002) for the initial years of elementary school;
- Approach, through these tasks, the probability of everyday actions aimed at teaching sample space;
- Describe techniques that solve tasks;
- To substantiate theoretically and/or conceptually the use of applied techniques;
- Explore probabilistic concepts from the teaching methodology of problem solving.

We call tasks the various problem situations that will make up the “questions” cards of the game, situations based on the problem solving methodology. According to Van de Walle (2009) the game may not look like a problem, but it can, however, be based on a problem. If the game makes students reflect on ideas that they have not yet formulated very well, then it fits the definition of a problem-based task.

In order to achieve these objectives, this research was guided mainly by BNCC, which brings the contents and skills to be worked on in the early years of Elementary School and by TAD, which allowed a mathematical (probabilistic) and didactic praxeological analysis on the tasks.

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This praxeological notion, in its simplest form, can be described at two levels, that is, in the words of Chevallard, Bosch and Gascón (2001), in mathematical activity or in any other activity, there are two blocks that complement each other, tasks and techniques on the one hand, and technologies and theories on the other.

In the block considered practical-technical (praxis), the techniques associated with solving the task will be presented. According to Chevallard (1999), a praxeology related to task T needs (in principle) a way to perform, that is, a way to perform a certain task.

In the knowledge block (logos), the first component is a rational discourse, called technology (θ) and theory (Θ), which represents a higher level of justification, explanation and production that plays the same role in relation to technology (θ). that it has in relation to the technique (τ) (Chevallard, 1999).

Thinking about this praxeology, the problem situations that make up the game cards are composed of tasks, consisting of a sequence of subtasks that can be performed using various techniques, justified by technology, which uses theories related to probability as an object of study.

We believe that through TAD it will be possible to broaden the view on each proposed task, from the strategies to the theoretical discourse on those strategies.

The pedagogical game in the light of TAD

Starting from our object of study and what research carried out in the area reveals to us about the contribution of games associated with problem solving, we seek to create a game based on problem solving that addresses probabilistic content for the early years of Elementary Education, especially the sample space.

The pedagogical game as a tool for teaching probability in the early years of elementary school

The game was created considering the contents proposed by the BNCC in order to assist both in the apprehension of the contents by the students, and to assist the teacher in identifying possible difficulties of the students in relation to such contents.

In this respect, the proposed game offers the teacher some possibilities for working in the classroom, which can be used both to assess the children's previous knowledge, and to introduce a subject that can later be developed in the classroom, or, introduce a subject in the classroom and then reinforce with the game, as well as to deepen a specific content.

In addition, some experiments that are proposed in the game cards can be carried out in the classroom, which would bring the possibility to promote intervention in favor of building the notions of probabilistic concepts.

By involving probability in a playful environment of a board game, we intend to provide the feeling of being in opposition to a formal learning situation. The game consists of

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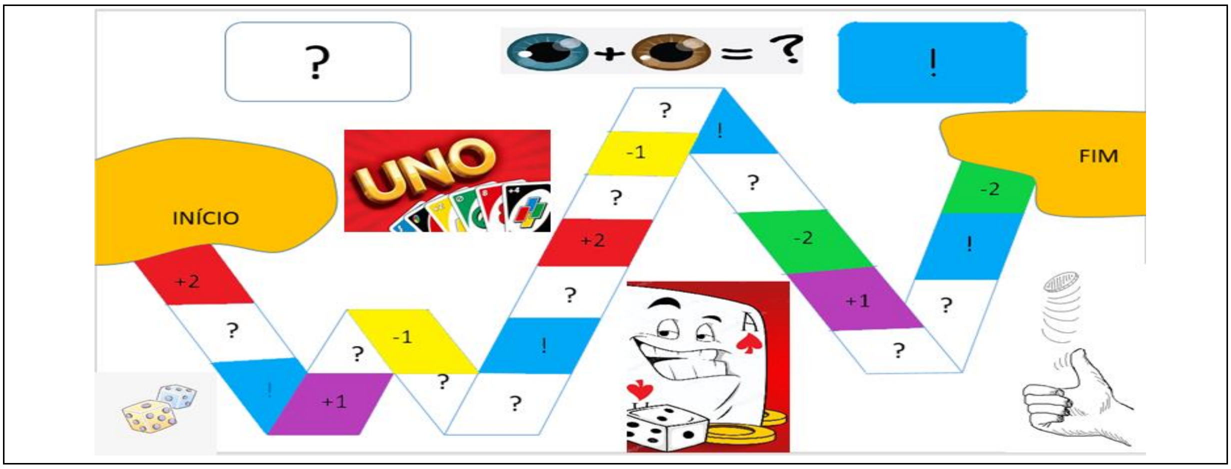
a board (Figure 1), in the shape of a course, whose intention is to bring playful elements of provocation during the game.



Source: Prepared by the authors.

Figure 1 - Game Board “Playing with Probability” for the early years of elementary school.

Important characters from the history of Probability and Statistics, such as Karl Pearson and Ronald Fisher, were made in biscuit to represent each group (Figure 2).

Source: Prepared by the authors.

Figure 2 - Photos by Karl Pearson and Ronald Fisher and their biscuit versions to represent each group in the game.

Bringing information about these characters, Castro (2007) says that Karl Pearson redefined statistics itself, as an abstract science in its own right, related to all sciences, in addition to the social and actuarial studies to which it was restricted and Rosário (2009) considers that Ronald Aylmer Fisher was the greatest statistician of the 20th century, making it practically impossible to carry out science without the ideas he developed about maximum likelihood, small sample theory, neo-Darwinism, selection theory, among others.

In order to run the board with the famous statistical/probabilistic characters in biscuit, a common dice must be used (Figure 3). We highlight historical aspects of the origin of probability considering that games with dice (or similar objects) have existed since Ancient Egypt (Lopes & Meirelles, 2005) and have been part of the history of Probability since long before this area was consolidated within Mathematics.

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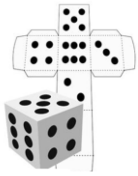

Source: Prepared by the authors.

Figure 3 - Representation of common dice to proceed with the game.

In addition, it should be considered that the game cards involve the probabilistic content (sample space) that we identified as Questions (?) being the proposed tasks.

As for the organization, we suggest dividing students into groups composed of at least two and at most four members and we list the rules of the game which are as follows:

1) To start the game, the characters must be placed in the “Start” box and immediately afterwards the groups will roll the dice to determine who will start the game.
2) The group that gets the highest number on the dice roll starts the game by taking a letter from the pile of “questions” (?), which one of the components will read aloud for the group to answer.
3) The group answering the question correctly, must advance the number of squares indicated on the value obtained in the dice roll on the board.
4) If the group misses the question, it will not be able to move the piece from place and the other group will have the right to answer the question.
5) If the two groups do not get the solution to the problem right, if there is a possibility, the teacher may intervene in the game with questions that help students in the search for the solution, so that together they perceive and comment on the “mistakes” made;
6) Whenever the character falls in the box "Asks" (?), a card from that pile must be removed and this process is repeated.
7) After walking a house, it will be the other group's turn to play.
8) When the character representing the group falls in the box “Advance houses” (+1), (+2), the number of houses must correspond, in the same way, if it falls in the house “Return houses” (-1), (-2), it should return to the number of corresponding boxes;
9) The group wins the game that first reaches the end of the board, that is, in the “End” box.

The teaching of probability and problem solving in the pedagogical game for the early years of elementary school in the light of TAD

Using TAD as methodological support, in the elaboration of the tasks that make up the game cards, we sought to connect to BNCC (MEC, 2017) and the Teaching Program developed by Nunes and Bryant (2012) developed in England to be applied in the early years

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Basic Education, in order to also enable a dialogue with research carried out in the area of probability, as proposed by Gal (2005), Coutinho (2001) and Batanero and Godino (2002).

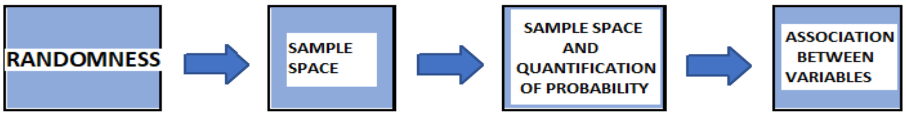
According to BNCC (MEC, 2017), the study of probabilities in the early years of elementary school should focus on the development of the notion of randomness, providing situations in which the child begins to realize that not all events are deterministic and from there , gradually expanding the idea of a random event.

Starting from teaching proposals indicated in research developed by Gal (2005), Coutinho (2001) and Batanero and Godino (2002), it is indicated that the study of probability should develop the understanding of three basic notions: perception of chance, idea of random experience and notion of probability.

Reinforcing teaching proposals, the Teaching Program developed by Nunes and Bryant (2012), focuses on two aspects in relation to the teaching of probability: (1) Promote children's understanding of the concepts of randomness, assessing the improvement in their ability to solve mathematical problems in situations involving uncertainty; (2) Promote children's understanding of numerical operations in a context in which results can be assured and, from there, assess whether understanding of mathematical ideas involving certainty can contribute and also improve their knowledge of Probability.

It is worth noting that this Teaching Program includes some of the elements described in the probabilistic literacy model proposed by Gal (2005) and related to the treatment of randomness, probabilistic calculations and critical issues.

In the figure below brought from Nunes and Bryant (2012), figure 4, we present the scheme of this teaching program that starts with the simplest ideas about randomness, goes through the quantification of probabilities and reaches the understanding of the risk (association between variables).



Source: Nunes and Bryant (2012)

Figure 4 - Stages of the teaching program on probability and risk.

In the second unit of the proposal by Nunes, Bryant, Evans, Gottardis and Terlektsi (2015), the concept of Sample Space is approached, which, according to these authors, is the set of all possible results of a random experiment that have a role that cannot be underestimated in the processes of teaching and learning probability. Nunes et al. (2015) indicate the need to identify the sample space in any task to understand and calculate the probabilities of specific events.

Reinforcing the definition of sample space, we bring Magalhães and Lima (2005) that defines it as the set of all possible results of a certain experiment or random phenomenon.

Tasks addressing the concepts of sample space

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The curricular activities developed by the problem proposition have their creation process considering the contents of the BNCC curriculum proposal (MEC, 2017), in order to enable students to initially understand basic concepts of probability, such as the analysis of the idea of chance in everyday situations, chances of random events and sample space (table 1).

Table 1 - Objectives and Skills of the probabilistic contents proposed in the National Common Curricular Base - BNCC of the 3rd and 5th years of Elementary School.

Year	3rd year of Elementary School	5th year of Elementary School
Objectives	Analysis of the idea of chance in everyday situations: sample space.	Sample space: analysis of chances of random events.
Skills	Identify, in random family events, all possible results, estimating those that are more or less likely to occur.	Present all possible results of a random experiment, estimating whether these results are equally likely or not.

Source: MEC (2017, p. 284-285; 292-293).

We consider that the sample space is fundamental content, which serves as a starting point for the study of probabilities.

Indicating definitions of sample space, Magalhães and Lima (2005) say that a phenomenon or random experiment is a situation or event whose results cannot be predicted with certainty and present the following examples: (1) next Sunday's climatic conditions that do not they can have them established with complete accuracy; (2) similarly, next month's inflation rate.

Coutinho (1994) adds, stating that the random experiment is one that is possible to repeat as many times as one wants under exactly the same conditions, being possible to identify all possible results without being able to identify a priori the one that will occur.

Starting from the construction of probabilistic concepts, for Bryant and Nunes (2012) we must recognize that the first and essential step in solving any probability problem is to discover all possible events and sequences of events that could happen. The set of all possible events is called "sample space" and the elaboration of the sample space is not only a necessary part of calculating the probabilities of a given event, but also an essential element to understand the nature of the probability.

Also for Keren (1984) and Chernoff (2009) working with the sample space is the first and essential step to solve any probability problem. They add that, in many, it is the most important, since the solution is usually quite obvious to someone who knows and lists all the possibilities of a given random experiment. However, this aspect of probability has been overlooked in research on children's ideas about chance, which largely focus on children's understanding of randomness and the ability to quantify and compare probabilities.

Highlighting important aspects in the teaching-learning process of this concept, Campos and Carvalho (2016) say that the sample space involves reasoning against intuitive and combinatorial where the set of all possible events has a role that cannot be underestimated in the teaching and learning processes of probability and Nunes et al. (2015)

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argue that it is necessary to be able to work with any sample space in any task to understand and calculate the probabilities of specific events.

Also according to SEB (2014), document of the National Pact for Literacy at the Right Age, to find the probable results and the chances of occurrence of each event, it is necessary to first identify all possible results - define the sample space and Borba (2017) says that the formation and categorization of the sample space plays an important role in understanding probabilistic situations, and the survey of the possibilities that compose it is fundamental to understanding the probability, since the probability calculation is based on your analysis.

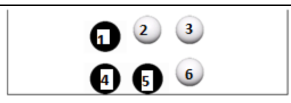
Finally, we consider Bryant and Nunes (2012) when they highlight the importance of sample space when raising a general cognitive issue, which is quite obvious, but has never been discussed. To represent the sample space, the child must imagine the future in a particular way and must think of all possible events that could occur in a particular context. Studies of this aspect of probability thinking are sorely needed.

Based on these considerations, we present tasks related to “Sample Space” aimed at the objectives to be achieved in the 3rd and 5th year of Elementary School, according BNCC.

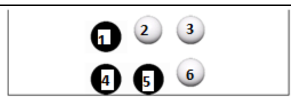
These tasks and their subtasks (problem situations), figures 5 to 10, make up the pedagogical game focused on the principles of TAD in the didactic and mathematical (probabilistic) praxeological organization.

We remind you that each of the subtasks presented is a game card. The aim is for students to recognize how to represent all the possibilities that can be listed from the proposal of a problem aimed at situations that may even be experienced.

Approaching the didactic praxeology for teaching probability, the objective is related to the skill (EF05MA22) of the BNCC (MEC, 2017), which indicates presenting all possible results of a random experiment, that is, determining the sample space. Task 1 (T1), figures 5 to 10, is configured to expand the idea of sample space through different contexts that involve different daily situations presented in the game cards.

1. Professor Nilceia Datori presents Box A in the classroom, consisting of white and black balls, each of which contains a number. Looking at Box A shown in the figure to the side, indicate all the balls you can select.			 BOX A
A	**B**	**C**	**D**
{1, 4}	{2, 3, 1, 5}	{5, 4, 3, 6}	{2, 3, 6, 1, 4, 5}

Source: Prepared by the authors.
Figure 5 - Subtask 1: Determine the sample space.

2. The teacher Nilceia Datori presents Box B in the classroom, consisting of white and black balls, each of which contains a number. Looking at Box B shown in the figure to the side, indicate all the balls you can select.			 BOX B
A	**B**	**C**	**D**

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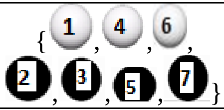
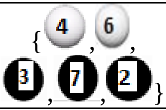
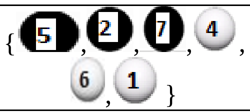
			

Source: Prepared by the authors.

Figure 6 - Subtask 2: Determine the sample space.

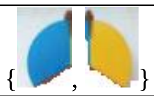
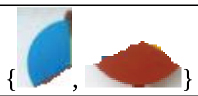
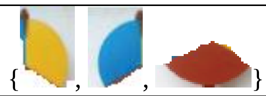
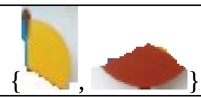
3. Observe the roulette shown in the figure beside. Among the numbers that appear which are all possible results that can come out after spinning the roulette wheel once?

A	B	C	D
{2, 1}	{2, 1, 4, 3}	{3, 2, 1}	{3, 4}

Source: Prepared by the authors.

Figure 7 - Subtask 3: Determine the sample space.

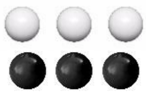
4. Observe the roulette shown in the figure on the side. Among the colors that appear which are all possible results that can come out after spinning the roulette wheel once ?

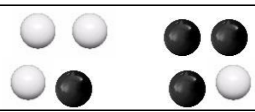
A	B	C	D
			

Source: Prepared by the authors.

Figure 8 - Subtask 4: Determine the sample space.

5. The teacher Nilceia Datori brings 6 balls to the class, three white and three black and asks a student to remove two balls with his eyes closed. Indicate all pairs of balls that you can select.

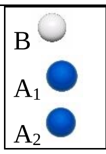


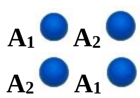
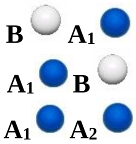
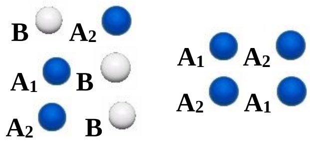
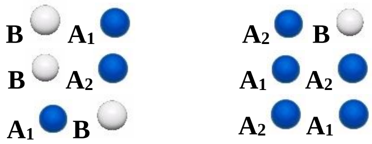
A	B	C	D
			

Source: Prepared by the authors.

Figure 9 - Subtask 5: Determine the sample space.

6. A box contains a white ball (B) and two blue balls (A1 and A2) and you can remove two balls at random without replacement. Indicate all pairs of balls that you can select.



A	B	C	D
			

Source: Prepared by the authors.

Figure 10 - Subtask 6: Determine the sample space.

We still support in Cobb, Comfrey, diSessa, Lehrer and Schauble (2003) that in proposing a sequence of activities, after initially having explored consistently the notion of randomness, then introduced the notion of sample space.

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This sequence of activities is still supported by training for learning notions regarding the probability of elementary school conceived by Nunes and Bryant (2012).

Considering the mathematical (probabilistic) praxeology of the subtasks (t1 to t6), figures 5 to 10, task T1 seeks, from daily situations, to appropriate the concept of sample space in the identification of the possible results of a random experiment, table 2.

Table 2 - Description of the practical-technical block or know-how regarding the TAD of the activities in figures 5 to 10 that present tasks related to the concept of sample space.

Task 1	Subtasks		Technique	
Determine, from problem situations, the sample space from different random experiments.	t_1	It consists of determining which balls can be selected from Box A which contains three white balls and three black balls.	τ_1	Identify among the listed options the one that determines the list of all possible results of the proposed random experiment, that is, the sample space.
	t_2	It consists in determining which balls can be selected from Box B which contains three white balls and four black balls.		
	t_3	It consists in determining what are the possible numbers that can occur after turning the roulette with the numbers 1, 2, 3 and 4 once.		
	t_4	It consists in determining what possible colors can occur after turning once the roulette wheel that has three different colors: yellow, blue and red.		
	t_5	It consists of determining all pairs of balls possible to remove two balls, without replacement, from a box containing six balls, three white and three black.		
	t_6	It consists in determining all pairs of balls possible to remove two balls, without replacement, from a box containing a white ball (B) and two blue balls (A1 and A2).		

Source: Prepared by the authors.

Resuming the subtasks in table 2 and detailing the technique τ1, we have in table 3 this detailed description.

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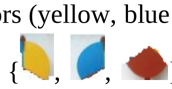
Table 3 - Detailed description of technique 1 (τ_1).

Technique	Subtasks
τ_1	In the case of subtask t1, the answer to the proposed task is option "D", as it expresses all the possible results of the proposed random experiment which are the balls that you can select from Box A, since it contains three balls of color bank and three black colored balls. The option also indicates the set notation, for example: D = {}. It should be noted that any ordering of the three white balls and the three black balls is the answer to the problem. We also indicate that options "A", "B" and "C" are not possible results, as they do not present all the balls that are present in Box A.
	In the case of subtask t2, the answer to the proposed task is option "A", as it expresses all the possible results of the proposed random experiment which are the balls that you can select from Box A, since it contains three balls of color bank and four black colored balls. The option also indicates the set notation, for example: A = {}. It should be noted that any ordering of the three white balls and the four black balls is the answer to the problem. We also indicate that options "B", "C" and "D" are not possible results, as they do not present all the balls that are present in Box B.
	In the case of subtask t3, the answer to the proposed task is option "B", as it expresses all possible results of the proposed random experiment, which is to spin a roulette that contains four different numbers (1,2,3,4) a single turn. The option also indicates the set notation, for example: B = {2, 1, 4, 3}. It should be noted that any ordering of the four numbers is an answer to the problem. We try not to present the answer to the problem corresponding to the numerical sequence to assess whether there is an understanding of what the sample space is. We also indicate that options "A", "C" and "D" are not possible results, as they do not show all the numbers on the roulette wheel.
	In the case of subtask t4, the answer to the proposed task is option "C", as it expresses all possible results of the proposed random experiment, which is to spin a roulette wheel that contains three different colors (yellow, blue and red) only once. The option also indicates the set notation, for example: C = {}. It should be noted that any ordering of the three colors is an answer to the problem. We also indicate that options "A", "B" and "D" are not possible results, as they do not have all the colors shown on the roulette wheel.
	In the case of subtask t5, the answer to the proposed task is option "C", as it expresses all the possible results of the proposed random experiment, which is to designate all possible ways of leaving two balls, between six balls (three white and three black) removed at random and without replacement. It should be noted that the ordering of the two balls makes a difference in the response of the problem. We also indicate that options "A", "B" and "D" are not possible results, as they do not present all possible orderings.
	In the case of subtask t6, the answer to the proposed task is option "D", as it expresses in an orderly manner all the possible results of the proposed random experiment, which is to designate all possible ways of leaving two balls, between three balls (one white and two blue ones), removed at random and without replacement. It should be noted that the ordering of the balls makes a difference in the response of the problem. We also indicate that options "A" and "B" and "C" are not possible results, as they do not present all possible orderings.

Source: Prepared by the authors.

The theoretical-technological discourse ($\theta 1$, $\Theta 1$), which allows to justify and explain

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the technique τ1 is based on the definition of Sample Space according to Meyer (1982) indicating that for each experiment ε of the type we are considering, we will define the sample space as the results possible de ε. We will generally represent this set by S. In this context, S represents the fundamental set, that is, the set of all possible results of an experiment. And we complement with the definition of Magalhães and Lima (2005), which calls the sample space the set of all possible results of a certain random phenomenon, being represented by the Greek letter Ω (omega).

We also indicate the definition by Bussab and Morettin (2004) who say that the sample space, Ω, consists of the discrete case, of the enumeration (finite or infinite) of all possible results of the experiment in question, that is, $\Omega = \{\omega_1, \omega_2, \omega_3, ..., \omega_n, ...\}$, where the elements of Ω are all sample points or elementary events.

Also addressing the didactic praxeology for teaching probability, the objective of the following subtasks, figures 11 to 14, is related to the skill (EF05MA22) of the BNCC (MEC, 2017), which indicates presenting all possible results of a random experiment, estimating whether these results are equally likely or not.

Thus, task 2 (T2), figures 11 to 14, is configured in which the players expand the idea of sample space focusing on the estimation of results that are configured as equally likely, or not.

7. The teacher Nilceia Datori presents the following game to her students: They must remove a ball from one of the following boxes (A or B) with their eyes closed. They win if they get a white ball. Which box do you prefer to extract?

Box A　　　　Box B

A	B	C	D
Box A	Box B	Either box	None of the boxes

Source: Prepared by the authors.

Figure 11 - Subtask 7: Determine which of the boxes is more likely to hit a white ball.

8. The teacher Nilceia Datori presents the following game to her students: They must remove a ball from one of the following boxes (A or B) with their eyes closed. They win if they get a black ball. Which box do you prefer to extract?

Box A　　　　Box B

A	B	C	D
Box A	Box B	Either box	None of the boxes

Source: Prepared by the authors.

Figure 12 - Subtask 8: Determine which of the boxes is more likely to hit a black ball.

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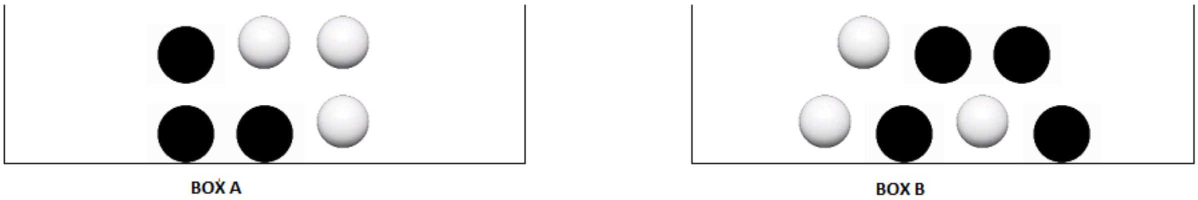
9. The teacher Nilceia Datori presents the following game to her students: They must remove a ball from one of the following boxes (A or B) with their eyes closed. They win if they get a black ball. Which box do you prefer to extract?

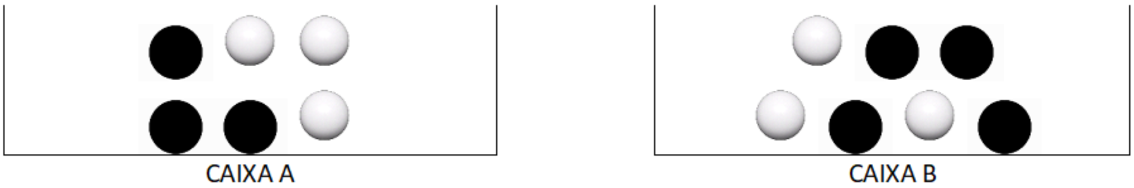


A	B	C	D
Box A	Box B	Either box	None of the boxes

Source: Prepared by the authors.

Figure 13 - Subtask 9: Determine which of the boxes is more likely to hit a black ball.

10. The teacher Nilceia Datori presents the following game to her students: They must remove a ball from one of the following boxes (A or B) with their eyes closed. They win if they get a white ball. Which box do you prefer to extract?



A	B	C	D
Box A	Box B	Either box	None of the boxes

Source: Prepared by the authors.

Figure 14 - Subtask 10: Determine which of the boxes is more likely to hit a white ball.

Considering the mathematical (probabilistic) praxeology of the subtasks (t7 to t10), figures 11 to 14, task T2 seeks, from daily situations, to appropriate the concept of sample space focusing on the estimation of results that are configured as equally likely or not, table 4.

Table 4 - Description of the practical-technical block or know-how regarding the TAD of the activities in figures 11 to 14 that present tasks related to the estimation of results that are configured as equally likely, or not.

Task 2	Subtasks		Technique	
Identify the solution based on problem situations, focusing on estimating results that are equally likely or not.	t_7	It consists in determining which box (A and/or B) is preferable to extract a white ball, with the eyes closed.	τ_2	Identify among four possible options the one that determines the solution to the proposed question, starting from the idea that all possible results of a random experiment are equally likely.
	t_8	It consists in determining which box (A and/or B) it is preferable to extract a black ball, with the eyes closed.		
	t_9	It consists in determining which box (A and/or B) is preferable to extract a black ball, with the eyes closed.		
	t_{10}	It consists in determining which box (A and/or B) is preferable to extract a white ball, with the		

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		eyes closed.		

Source: Prepared by the authors.

Resuming the subtasks in table 4 and detailing the technique $\tau2$, we have in table 5 this detailed description.

Table 5 - Detailed description of technique 2 (τ_2).

Technique	Subtasks
τ_2	In the case of subtask t7, the answer to the task is option “C”, because when comparing the two boxes (A and B), it can be identified that both boxes have the same number of white (four) and black balls (four). Taking into account that each ball has the same chance of being selected, either of the two boxes is the solution to the proposed problem.
	In the case of subtask t8, the answer to the task is option “C”, because when comparing the two boxes (A and B), it can be identified that both boxes have the same number of white (four) and black balls (four). Taking into account that each ball has the same chance of being selected, either of the two boxes is the solution to the proposed problem.
	In the case of subtask t9, the answer to the task is option “B”, because when comparing the two boxes (A and B), one can identify that box B has four black balls and box A three black balls. Thus, taking into account that each ball has the same chance of being selected, as in box B there is one more black ball than in box A, there is a greater chance that a ball of this color will be selected.
	In the case of subtask t10, the answer to the task is option “A”, because when comparing the two boxes (A and B), it is possible to identify that both boxes have the same number of white balls (three), however the Box A has the least number of balls in total. Taking into account that each ball has the same chance of being selected, in Box A we will have a greater chance of selecting a white ball, so it is a solution to the proposed problem.

Source: Prepared by the authors.

The theoretical-technological discourse ($\theta2$, $\Theta2$), which allows justifying and explaining the $\tau2$ technique, can be described according to Pinheiro, Cunha, Carvajal and Gomes (2015), by saying that sample space is the set of all possible results of the random experiment , being denoted by S, considering that the sample space is finite uniform when it has a finite number of elements, all of which are equally probable.

Final considerations

Probability concepts are complex with a high degree of abstraction, so it is necessary to progress gradually towards an adequate understanding of the specific language of probability.

We believe that the study of the sample space should begin with children in the early years of their schooling. For this, we must conceive as strategies for approaching the concept of probability in the early years a series of activities, games and didactic sequences, among other methodological procedures to help children in understanding the situations in which randomness is present.

Our proposal, guided by the BNCC (MEC, 2017), sought to explore probabilistic concepts based on the teaching methodology of problem solving inserted in a pedagogical

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game because we consider that this brings important achievements and developments to students.

For us, thinking about the basic concepts of Probability is to develop research that meets the needs of the primary school, so that we can contribute to the growth and development of an autonomous, critical, active and capable of making decisions regarding information which you encounter.

We also believe that the game is a resource that can help students and teachers in the process of teaching and learning the probability for the early years of elementary school.

Our aim was to contribute to the teaching and learning processes and we believe that the creation of this game will provide the student with moments of pleasure, exchanges and learning, and the teacher, a theoretical resource of the contents that are being worked on.

We also believe that through TAD, through the mathematical and didactic organization, it is possible to broaden the look in relation to the several existing possibilities that surround each activity that is being developed, from the mathematical "knowing" to the "doing".

And, in terms of TAD, its use allowed to identify a set of praxeologies that makes it possible to characterize both the mathematical object (probabilistic) and the didactic approach for such an object. The praxeological organization was composed of four elements:

1. Task (T) and its subtasks (t), which characterized the action demanded by the proposed problem situation for the game question cards. For example, to identify and list all possible results of a random experiment (sample space) as well as its events (subsets).
2. Technique (τ), identifies the way of accomplishing the task and its subtasks. Each task has at least one technique associated with it. For example, to determine the different events resulting from the flipping of a coin, a technique that can be associated is the enumeration of the elements of the subsets of the sample space associated with this random experiment, that is, "$\varnothing$", "face", "crown" and "face, crown". Considering other experiments, like those proposed in this work, we will have other possibilities combined.
3. Technology (θ)/Theory (Θ), was specified by the set of definitions, properties, axioms and theorems that justify the technique. For example, the technology that justifies the technique is the definition of sample space, that is, for each random experiment E, sample space S is defined as the set of all possible results of this experiment.

We are aware that the apprehension of these concepts does not occur in a simple way, even because they are not simple concepts, but we believe that the didactic transposition adopted on a daily basis makes all the difference.

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The creation of this game is also justified by the fact that we consider that the playful environment attracts the attention of the child, who spontaneously participates and shares his knowledge. Successes and mistakes go hand in hand and the child learns in a pleasant and meaningful way.

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