Zetetiké

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Tasks for learning teachers who teach mathematics in the elementary school

Tarefas para a aprendizagem de professores que ensinam matemática nos anos iniciais

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Abstract

Using tasks for teacher learning is a fundamentally topic for research in teacher education. Thus, this paper aims to *understand and explain how the construction of mathematical and didactical knowledge of teachers who teach mathematics at elementary school, in a formative process, on the different meanings of the equals sign, occurs.* The research is qualitative-interpretative and data, documentary and audio records, come from a teacher education process developed with 6 teachers from a public school in São Paulo. Analyzes showed mobilization of mathematical and didactical knowledge, which was reorganized, developed and constructed, by the teachers through getting involved in this collective formative process of planning, developing and reflecting a mathematics lesson on the different meanings of equal sign. It is understood that such movements occurred through the use of professional learning tasks and the mediations and the teacher educator.

Keywords: Continuous education; Algebraic thinking; Equals sign; Mathematical and didactical knowledge.

Resumo

O uso de tarefas para a aprendizagem de professores é um tema de fundamental importância para investigação na formação de professores. Assim, objetivou-se neste artigo, *compreender e explicar como ocorre a construção do conhecimento matemático e didático de professores que ensinam matemática nos anos iniciais em um processo formativo sobre os diferentes significados do sinal de igualdade.* A pesquisa é qualitativa-interpretativa e os dados, documentais e em áudio, são provenientes de um processo formativo desenvolvido com 6 professoras de uma escola municipal de São Paulo. As análises mostraram a mobilização, a reorganização

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e a construção de conhecimentos matemáticos e didáticos por parte das professoras que, coletivamente, planejaram e desenvolveram uma aula de matemática contemplando o sinal de igualdade e, posteriormente, refletiram sobre ela. Entende-se que tais movimentos se deram por intermédio do uso das tarefas de aprendizagem profissional e pelas mediações da formadora.

Palavras-chave: Formação continuada; Pensamento algébrico; Sinal de igualdade; Conhecimentos matemáticos e didáticos.

Introduction

The continuing education of teachers who teach mathematics in the first school years can be discussed based on connections and interlocutions between professional knowledge (Ball, Ben-Peretz & Cohen, 2014; Ball & Cohen, 1999; Silver *et al*., 2007; Smith, 2001) and the teaching practices employed by teachers (Ponte *et al*., 2008; Serrazina, 2013). There is the possibility of basing this educational context on the mobilization and construction of knowledge from teachers who continue to learn throughout their professional practices (Opfer & Pedder, 2011; Ponte & Oliveira, 2002; Ponte & Quaresma, 2016; Serrazina, 2013; Webster-Wright, 2009). Teaching practice scenarios can help in this context of mobilizing and thinking of mathematical and didactical knowledge.

In this sense, the knowledge from teachers assumes a fundamental role in their training, since "this is interrelated with the level of confidence teachers have, both in Mathematics and in teaching it, and in what they consider their students to be able to learn in Mathematics" (Serrazina, 2013, p. 77). Therefore, it seems that a positive relationship is established with teachers' confidence in their teaching practice, which increases as they build new knowledge specific to the content they teach. Consequently, this may also enable an improvement in knowledge about students and about teaching processes (Ball, Thames & Phelps, 2008), which, in turn, results in knowing Mathematics better (Serrazina, 2013).

In the field of Mathematics, the development of Algebraic Thinking (AT) and the importance of developing it since the early years stand out. There is great potential when students develop AT early and later engage in algebra studies in the subsequent years of elementary school (Blanton & Kaput, 2008; Britt & Irwin, 2011; Kieran *et al*., 2016). The present study, by focusing on the relevance of this approach, emphasizes working with the different meanings of the equal sign (Kieran, 1981; Ponte, Branco & Matos, 2009; Trivilin & Ribeiro, 2015).

Nevertheless, it is essential to intensify the continuing education of elementary school teachers, in order to expand and (re)structure their specific knowledge for teaching Mathematics and their knowledge about students and the curriculum. From this perspective, studies that investigate and point out new paths for continuing education (Silver *et al*., 2007; Barboza, 2019; Barboza, Ribeiro & Pazuch, 2019) are also important, in addition to other studies that may present possibilities for professional learning (Ball & Cohen, 1999; Smith, 2001). The use of Professional Learning Tasks (PLT) and the collective discussions that emerge from them should be considered as a means of enabling teacher learning (Ribeiro & Ponte, 2019; Ribeiro & Ponte, 2020).

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In this context, *the objective of this article is to understand and explain how, in a training course about the different meanings of the equal sign, the construction of mathematical and didactical knowledge by elementary school Mathematics teachers.*

This research[4] was based on a qualitative interpretative approach. The analysis will center around data gathered from 6 out of 14 meetings of a training course proposed by the first author of this paper, with 6 elementary school teachers from a public school in the city of São Paulo.

The next section presents the literature review. Subsequently, the research context and methodological aspects are explained. Afterwards, selected analysis episodes are discussed. Finally, the conclusions and final considerations are presented.

Teaching practice, professional knowledge and teacher learning

Planning lessons is an action inherent to the teaching practice of Mathematics teachers (Serrazina, 2017), and something that is essential to teaching (O'Donnell & Taylor, 2007). Therefore, it requires carefully thought out, structured and executed actions, enabling the expansion of the teacher's knowledge and, possibly, the improvement of teaching practices. The act of planning a Mathematics lesson with investigative tasks (Ponte, 2005) involves the teachers' knowledge about the Mathematics that will be developed, and the way they conceive student learning and classroom practices.

Serrazina (2017) ponders that the act of planning is not a simple task, as it is up to teachers to consider how students think and how they learn, when they are involved in a lesson plan. To plan is to trace a path to follow, in view of the work to be done with certain contents, establishing which goals need to be reached, which would be the best ways to mobilize thoughts, hypotheses and strategies for the content to be developed. In other words, teachers must establish where they want to go and what tasks can support this trajectory (Ponte & Oliveira, 2002; Serrazina, 2017). Therefore, the act of planning is a movement of learning and teaching for teachers, since for this action, which is inherent to their teaching practices, they need to think and rethink their teaching path and the proposals they make. Therefore, planning implies establishing possible relationships between what one thinks and how it relates to real challenges in the classroom (Ponte, 2005; Serrazina, 2017).

From this perspective, planning a lesson individually and, above all, collectively, can be considered a possibility for training courses (primary and continuing) for teachers, due to the action of sharing knowledge about teaching, about the way students can develop and register their mathematical thoughts, serving as a foundation against the (un)certainties that are part of teaching practice (Serrazina, 2017). Reflecting on how the planned task and the

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planned lesson were developed, from the initial plan, is also a form of professional learning (Ball & Cohen, 1999).

Thus, the act of planning lessons triggers some developments, whether in the choice of tasks for students (Ponte, 2005), or in the way these tasks will be developed (Stein *et al*., 2008), or in the mathematical goals that must be achieved. These developments can present themselves as necessary facets for teachers to mobilize mathematical knowledge in teaching (Ball, Thames & Phelps, 2008).

In our study, we assume that mathematical and didactic knowledge (Ball, Thames & Phelps, 2008) are important components of teachers' professional learning (Ball & Cohen, 1999). Thus, we will consider that professional knowledge feeds and connects to the act of planning, developing and reflecting on lessons, which makes it essential to mobilize this knowledge, resulting in the promotion of professional learning for teachers. Thus, one can ask: since reflection is an essential component for teaching, what types of knowledge are essential for training this professional? What relationships can be established between the professional knowledge of elementary school Mathematics teachers and their teaching practice?

According to Shulman (1986, 1987), one can only teach what one knows, and teaching others what one knows is the confirmation of having understood the subject and transformed knowledge itself into a possibility of teaching and learning. From the works of Shulman, later directed to the field of Mathematics, and the studies of Ball, Thames & Phelps (2008), the concept of Mathematical Knowledge for Teaching (MKT) was created.

The theoretical model proposed by Ball, Thames & Phelps (2008) is based on the assumption of the need to identify and understand what knowledge teachers require for their teaching practice, and how they can mobilize this knowledge. Therefore, the authors propose a mathematical and didactic knowledge foundation to support the development of tasks related to teaching.

Based on the aforementioned authors, Specific Content Knowledge refers to the mathematical content that will taught and is subdivided into: knowledge mobilized that goes beyond the teaching content, called Common Content Knowledge (CCK); mathematical knowledge that enables teachers to know how mathematical topics are constructed throughout the school curriculum, called Horizon Content Knowledge (HCK); and knowledge focused only on the teaching of Mathematics, that is, the type of knowledge that is considered indispensable to be a teacher, called Specialized Content Knowledge (SCK).

In addition, Pedagogical Content Knowledge, which refers to the way content can be taught, is also subdivided into three other types of knowledge: knowledge about what will be taught, in a way that allows teachers to anticipate the possible mistakes students can make, called Knowledge of Content and Students (KCS); knowledge that enables teachers to know about teaching and Mathematics, known as Knowledge of Content and Teaching (KCT); and

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knowledge of the specific contents that make up the curriculum to be developed, called Knowledge of Content and Curriculum (KCC).

The authors Russ, Sherin & Sherin (2016) carried out research on professional learning, aiming to answer the question: "How do teachers learn to teach?" These authors researched this topic from three perspectives: process-product, cognitive and sociocultural. The cognitive approach is underscored here, since it can bring evidence of how categories of professional knowledge are constituted and through which processes they are developed.

Studies like the one carried out by Ma (1999) highlight that one of the central processes of teacher learning is the formation of new knowledge structures, combining several of their spheres and potentially new knowledge, either individually or in collective spaces.

Since mathematical and didactic knowledge are essential to teach, considering the arguments given above, below we will discuss the importance of the development of AT since the first years of elementary school. Specifically, the importance of AT when working with the different meanings of the equal sign.

The development of Algebraic Thinking and the different meanings of the equality sign

Studies indicate the possibilities and the need for the development of AT since the first years of elementary school (Blanton & Kaput, 2005, 2008; Britt & Irwin, 2011; Kieran *et al*., 2016). This can contribute to the students' transition to a more formal study of algebra in the final years of elementary school.

Blanton & Kaput (2005) point out that AT in students of the first years of elementary school is

> [...] a process in which students generalize mathematical ideas from a particular set of examples, make generalizations through argumentative discourse, and express them, increasingly, in formal and age-appropriate ways. (Blanton & Kaput, 2005, p. 413)

Britt and Irwin (2011), when talking about some of the students' skills with regard to AT, refer to the mathematical knowledge of the teacher and the necessary training this professional must undergo to work with AT in the first years of elementary school. The authors also state that teachers need to develop their own AT skills in order to teach them. Corroborating this, Ponte and Branco (2013) argue that before promoting AT in the classroom, teachers need to develop their own understanding of what it means to think algebraically.

Considering the different meanings of the equal sign as one of the topics to be developed in the field of AT (Ponte, Branco & Matos, 2009), the research conducted by these authors takes into account the importance of the concept of equality in Mathematics, as the equal sign plays an important role in understanding the concept of equivalence. The authors point out: "mathematical equality or equivalence is always relative only with regard to a certain property." (Ponte, Branco & Matos, 2009, p. 19). It is important to remember that in

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Mathematics equality is an equivalence relation that respects three properties, namely: symmetric (4+2=6 or 6=2+4 or 3+3=1+5); reflexive (5=5); and transitive (2+5+3=7+2+1=8+2=10) (Ponte, Branco & Matos, 2009).

The study by Kieran (1981) gives three different meanings for the equal sign: operational, equivalence and relational. The operational meaning is the most developed in the first years of study, and, often, the only one. It is the operational meaning that gives students the idea that after this symbol – "=" – comes the result of an operation and that, generally, only one quantity is accepted as true (i.e. 5+13=18). Under these circumstances, students' understanding is limited to learning that the equal sign is "a sign that does something (Behr, Erlwanger & Nichols, 1980); a referred action that means: giving or doing (Stacey & Macgregor, 1997); an operator that transforms, for example, 3 + 4 into 7" (Trivilin & Ribeiro, 2015).

The second meaning of the equality sign, that of equivalence, is that which allows establishing many ways of representing 20 – for example, through true equations, such as 20=12+8; 17+3=20; 20=18+2 –, as well as by indicating the possibility of working on expressions such as 17+3=18+2, denoting a relationship of balance, of equivalence between the terms "before" and "after" the sign. Working on this meaning in the first years of elementary school is especially important to enable the understanding of algebraic concepts in subsequent years, such as the concept of equations, which is widely studied in the final years of elementary school (Ribeiro & Cury, 2015).

Finally, the last meaning of the equal sign is the relational one, by which relationships between expressions are established, and which implies the understanding and use of the properties of operations (addition and multiplication). In this case, the equal sign is fundamental, for example, for understanding the expression 10+12+15=10+10+17.

Next, PLTs are presented as one of the main data production instruments used in the present research.

Professional Learning Tasks and their possibilities

It should be noted that teachers need opportunities to learn: (1) the subject they teach (meanings and connections with everyday life, and not just procedures and information); (2) the knowledge of students: how they think, how they learn, why they make mistakes, how to listen carefully and how to help them move forward; (3) the need to develop the ability to overcome social and ethnic deviations and be sensitive to adjustments and adaptations necessary to reach each student, seeking strategies so everyone can learn (Ball & Cohen, 1999).

Thus, PLTs are composed of situations to be explored, enabling the formulation of mathematical conjectures, their validation, reformulation and the mobilization of knowledge necessary for teaching practice. They can be focused on the anticipation of students' thoughts or linked to the analysis of real and fictitious protocols that they can produce (Barboza, 2019).

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In line with previous discussions, Silver *et al*. (2007) further argue that PLTs have the potential to provide learning opportunities for teachers, when dealing with a situation – the construction of a mathematical concept and task resolution – as a problematic. These authors also assume that the interaction between teachers and lecturer when using the PLT can be an important factor in promoting learning opportunities based on practice.

Thus, PLTs seem to favor the emergence of professional learning opportunities (Ball & Cohen, 1999; Ribeiro & Ponte, 2019; Webster-Wright, 2009), since within them are usually contained "authentic examples of practice" (Ball & Cohen, 1999; Silver *et al*., 2007; Smith, 2001), that is, materials extracted from real classroom scenarios. With this, it is understood that spaces are open for criticism, questioning and investigation, enabling the (re)structuring of knowledge for teaching and about students, and the mobilization of mathematical knowledge. PLTs enable teachers to develop knowledge that is fundamental for teaching, since they engage in tasks and activities that are at the heart of their daily work (Smith, 2001).

Thus, one way to plan a PLT is to consider the work cycle of teachers and the nature of their activities. In other words, considering the act of planning what will be taught, what tasks could provide and elucidate the mathematical knowledge to be built. Subsequently, the plan is carried out in the classroom and, afterwards, there must be a reflection on what needs to be (re)formulated so that students can, in fact, learn the content (Ball, Ben-Peretz & Cohen, 2014; Smith, 2001).

The PLTs are potential tools for professional learning, as they are based on questions centered on practice; provide opportunities to establish comparative perspectives about the practice; contribute with personal and collective questions; and favor the (re)signification/transformation of practices (Ball & Cohen, 1999; Smith, 2001; Silver *et al*., 2007). Thus, it is understood that the PLTs are instruments or materials planned by the lecturer to enable discussions and reflections about the mathematical and didactic knowledge of teachers.

Having explored the literature related to our research question, as well as located the theoretical aspects of our study, the next section structures our research context and methodological procedures.

Research context and methodological procedures

This section presents the contexts in which the research was carried out, the methodological procedures adopted and the way in which the data was collected and analyzed.

The research took place in the context of a training course with 6 teachers from the early years of elementary school (Adionísia, Celeste, Kátia, Luciana, Márcia and Valdete), who teach in a public school in the city of São Paulo, with the participation of the first author of this article. The data were gathered in 14 in-class sessions, from August to October 2018.

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In broad terms, the first meetings of the training course focused on mapping the previous knowledge the teachers had about the different meanings of the equal sign and discuss theoretical and methodological elements for the instrumentalization of their lessons. In other meetings, PLTs were developed to mobilize mathematical and pedagogical knowledge. Considering the focus of this article, Table 01 presents a summary of meetings 9, 10, 11, 12, 13 and 14, which include the PLT that resulted in a collective lesson plan and its development, and another PLT that contemplated possibilities for reflection about said lesson.

Table 01 – Overview of some of the research meetings

Meetings 9 09/19/2018 and 10 09/24/2018	Developing PLT 3 (previously, this group worked with other PLTs, focusing on the different meanings of the equal sign), monitoring how teachers collectively build a lesson plan on the different meanings of the equal sign.
Meeting 11 09/27/2018	Administering the lesson plan discussed and developed collectively in the two previous meetings, in a class from the first years of elementary school.
Meeting 12 10/01/2018	Rethinking the five practices proposed by Stein *et al.* (2008) to orchestrate mathematical discussions in the classroom; analyze and reflect on the practical samples selected by the researcher-lecturer, in which she applied, in the classroom in which she teaches, a mathematical task on equivalence of values.
Meeting 13 10/08/2018	Develop PLT 4 to discuss and reflect on the practice samples referring to the previously administered lesson plan.
Meeting 14 10/16/2018	Developing PLT 4 to discuss and reflect on the practice samples referring to the previously administered lesson plan.

Source: (Barboza, 2019)

In the ninth meeting, the teachers were divided into two groups and the class was divided into two moments: in the first one, each group read, reflected on, discussed, recorded their conjectures and talked about the tasks that could be used in class to talk about the equivalence meaning of the equal sign. At the same time, there were particular interventions made by the researcher-lecturer (RL), who went around talking to both groups. In the second moment, the floor was given for both groups to share their issues, resolutions and the discussion they had developed. From the tenth meeting, after a suggestion from the teachers, the six of them started working jointly, in a single group.

From the methodological point of view, this is an interpretive qualitative research (D'Ambrosio, 2004; Esteban, 2010). Data were gathered through two *instruments and procedures*: (1) the PLTs, with focus on planning, developing and reflecting on a lesson, so as to enable different discussions and approaches related to specific knowledge, student knowledge and teaching processes, and to curriculum knowledge; (2) the audio and video recordings taken throughout the meetings, so as to obtain more details regarding the work done by the teachers over the development of the PLTs.

The PLTs are the main instrument for data collection and were structured by the researcher-lecturer based on: (1) what teachers need to know about Mathematics to be able to teach the different meanings of the equal sign; (2) which teaching practices will provide students with the possibility of interacting and building knowledge; (3) which types of tasks and approaches can be used to teach about the equal sign.

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These foundations are based on the MKT domains: recognizing the possible mistakes and errors made by students and understanding their nature (SCK); planning the lesson, considering student disposition, choosing and Building tasks, and anticipating answers from students (KCS); understanding the developments from teaching the different meanings of the equal sign since the first years of elementary school, and its impact on subsequent years (HCK and CCK); understanding the differences between the types of tasks and the possible interventions to deepen the understanding of the content (KCT).

It should be noted that, because the PLTs were research tasks – from what was proposed by Ponte, Brocardo & Oliveira (2003), due to presenting in their structure and development, an introduction, its development in small groups, them being shared with the whole group, and their systematization – with samples from teaching practices and mathematical tasks completes by students, their protocols for answers and strategies may create rich opportunities for teachers developing the ability to understand and make decisions in relation to their teaching practice. The PLTs are an instrumental source for teachers, so that while they plan and develop or, when in the classroom, administer a mathematical task, they can lend support to their students' learning, creating environments that promote communication among peers, the creation of hypotheses, questions and exchange of different solutions.

To build the **unit of analysis** of this article, *"*Mobilizing and building knowledge when planning collectively: from anticipating steps to reflecting afterwards." This unit of analysis considers the collective processes involved in planning, action and reflection about the teacher knowledge built within continued education courses. The transcriptions from meetings 9, 10, 11, 13 and 14 were considered, as well as the written materials produced by the group of teachers when developing PLT 3 and PLT 4. Subsequently, all this information was grouped, thus creating an inventory that was separated according to meeting and collection instrument. This procedure was carried out in a way so, when grouping the information in this way, it could be compared and analyzed.

From the inventory – a selection of all the information from the instruments used in this research –, an episode was selected for analysis, focused on the planning, development – and subsequent reflection – of a class about the equivalency meaning of the equal sign. The episode will be discussed on the next item, which will present 17 excerpts, with dialogues (11) and figures (6).

The episode selected came from the data generated with the work that was developed with PLT 3, whose objective was **to understand how teachers can make a collective lesson plan about the different meanings of the equal sign, selecting and anticipating the answers given to a task, and its goals**; and with PLT 4, whose objective was **to reflect about the practice samples** – which came from the narratives, audio transcriptions, written excerpts from the lesson administered and recorded in video, and the protocols from students – **previously selected by the RL, from the lesson administered by teacher Celeste in her**

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5th grade classroom, and how it related to the lesson plan developed collectively for PLT 3.

As a way of coding the information used in the analyses, we adopted the following procedures: after describing the information, the real name of participants is shown, followed by a letter and a number, which refer to the instrument (T3 or T4), for PLT 3 or PLT 4, respectively; (D) for transcriptions from the small groups; (Pn) for the transcriptions of the open discussions, where n represents the meeting number; and, lastly, the date of the meeting when the information was gathered. For example, to identify an information given by Luciana in the ninth meeting, on 09/19/2018, during the group discussion, this would be the code: (Luciana, D9, 09/19/2018). The same procedure was followed for students, entering the term "student," followed by the letter "E" (n), where "n" indicates if it was a pair or a trio; for the transcription of the general discussions, the letter "P" followed by the number of the meeting in which the teachers watched and analyzed the samples; finally, the date in which the meeting of the teachers occurred. For example, to identify an information given by a student from pair 1, another from pair 2, and another from pair 3, from the analysis of the teachers on the eleventh meeting, on 09/27/2018, the code would be: (Students (1), (2) e (3), P11, 09/27/2018).

On the next section we will discuss the results of our study, with the goal of linking our analyses and interpretations to data excerpts gathered from our field research.

"Collective planning - from anticipating steps to reflecting afterwards": Data analysis

For the analyses, we selected excerpts that point to an understanding of how the mobilization and construction of mathematical and pedagogical knowledge was developed for the six teachers who collaborated with this research and the discussions it proposes, considering their records and reflections after working on PLTs 3 and 4.

To begin working with PLT 3, the RL conducted a collective reading and each trio followed on their copy (Figure 1).

Both groups began by analyzing the mathematical tasks – what challenges and propositions they posed, in what way they could be developed in the classroom, what could be anticipated and how to develop them with students. Soon, it could be observed that the teachers were mobilizing and having the opportunity to build their knowledge about CK and their PCK, specifically regarding SCK, KCS and KCT. The group formed by teachers Celeste (C), Luciana (L) and Kátia (K) chose the task "The bowling game" (Figure 2).

During the discussions promoted by the trio Celeste (C), Kátia (K) and Luciana (L), on the first day of development of PLT 3 (Figures 1 and 2), When choosing the mathematical task, they initially anticipated possible resolutions for the chosen task, the difficulties students might have with it, and even suggested a few reasons for possible mistakes and developments:

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[1] K – *Look at this question* [referring to question 4 – **Figure 2**], *they will determine it. Each of them can have their possibilities, each one will create their own hypothesis.*

[2] C – *Yes, each one will distribute the way they think is best, as long as they reach 20 in total.*

[3] K – *Yes, there's many possibilities.*

[4] RL – *Do you think they are used to the possibility of having more than one right answer?*

[5] K – *No.*

[6] C – *No, because they know they can have different strategies to get to an answer, only one result. But many right answers...*

[7] RL – *And do you think this would be an extra challenge?*

[8] C – *I think it would be an extra challenge, yes.*

[9] K – *And they would be worried about their answer being correct, if it is the same as their classmate's.*

[10] L – *Yes. They would think: "Am I wrong?"*

[11] C – *Yes, because it has to be that amount, and the same as their friend's.*

[12] K – *And this is a good challenge.*

(Celeste, Kátia, Luciana, Researcher/Lecturer, D9, 09/19/2018).

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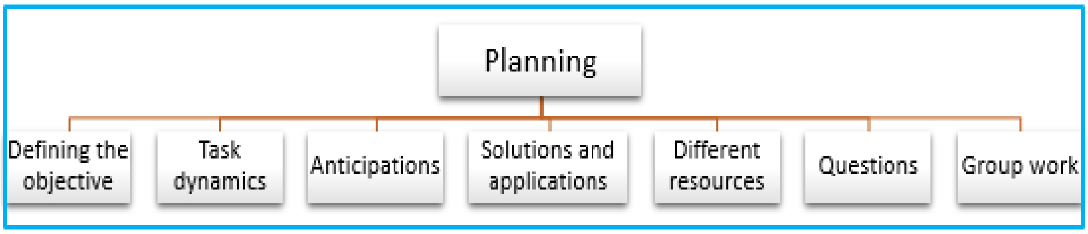
Following our discussions and readings from previous meetings, today I propose for us to develop a task to be administered to 5th grade A, the classroom taught by teacher Celeste. My proposal is for us to follow the lesson plan steps we talked about on our meeting on 08/28/2018, based on Serrazina (2017).

Let's first remember the steps proposed by the author. The establish we must define the objective(s) we want to work on; choose a task or create/adapt it, and establish what we can anticipate regarding possible resolution strategies proposed by students; anticipate possible difficulties that students may have when working on the task; anticipating alternative ways to solve the task and possible links between them; anticipating the possibility of using manipulative materials or resources to work on the task; anticipating some questions and key issues that connect to previously developed content; thinking about how to present the task in order to orchestrate mathematical discussions. As shown in the figure below, we have an idea of steps to follow, that do not necessarily need to be performed in the order presented.



Attached you will find some task suggestions, but you can develop new ones or adapt the ones I have suggested.

Some general questions that can guide your choices: (i) What previous knowledge will the class need to mobilize to answer the task?; (ii) What mathematical content is involved in the task?; (iii) How can you develop this task in the classroom?; (iv) If we wanted to use the operator meaning of the equal sign, what questions could we ask?; (v) If we wanted to use the equivalency meaning of the equal sign, what questions could we ask, or changes could we make?; (vi) What other questions could be asked?

Figure 1 – PLT 3: Planning the way
Source: (Barbosa, 2019)

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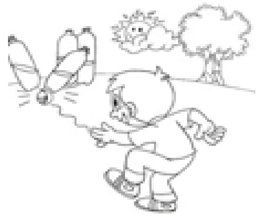
THE BOWLING GAME



Fifth grade A was divided into 4 teams to go bowling.

See the score that Professor Vagner managed to write down from the bowling rounds:

	1st ROUND	2nd ROUND	3rd ROUND	TOTAL
TEAM 1	12	13	15	
TEAM 2	15	7		35
TEAM 3		15	8	35
TEAM 4		10		

Some data was not filled in by the teacher. Answer the questions and help complete the picture:

a) How many points did Team 1 have at the end? Explain how you got to this result.

b) What score did Team 2 reach on the 3rd round? Explain how you got to this result.

c) What score did Team 3 reach on the 1st round? Explain how you got to this result.

d) Team 4 got, in total, only half of Team 1's score. Determine their total score in the 1st and 3rd rounds.

Figure 2 – Mathematical task chosen for the lesson plan
Source: (Barbosa, 2019).

Throughout the discussions, the RL introduced an issue as a way to propose a reflection about the equivalency meaning of the equal sign:

> [13] RL – *And do you think that with this or some other question they would be able to examine and establish equivalence relations?*
>
> [14] K- *Yes.*
>
> [15] RL – *And why? Please, elaborate.*
>
> [16] K – *Because they'll realize that...*
>
> [17] C – *You can add different numbers and get the same result.*
>
> [18] K – *Because it's not determined, so it can be different.*
>
> (Celeste, Kátia, Researcher/Lecturer, D9, 09/19/2018).

According to excerpts [13] and [14], for example, it is possible to conjecture that they understood the equivalency meaning of the equal sign and were developing their own

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algebraical thinking skills (Britt & Irwin, 2011; Ponte & Branco, 2013). Also, it is possible to conjecture that they were mobilizing and Building knowledge related to SCK [13] and [14], as mentioned before, regarding the broadening of their knowledge about the meanings of the equal sign. In addition, knowledge related to KCS [1], [2] and [3], and [7], [8] and [9], on the possibility of thinking about possible mistakes and errors made by students. Lastly, in [1], [4] and [6], it can be noted that teachers had opportunities to understand the differences in types of task and the possible interventions to deepen the understanding of the content (KCT) (Ball, Thames & Phelps, 2008).

Considering the discussions, the RL introduced another issue, as a way to propose new reflections about the equivalency meaning of the equal sign and an attentive look at the possible links that could be established between the data missing from the table (**Figure 2**).

> [19] RL – *Look at Team 2 and Team 3, what do they have in common?*
>
> [20] C – *The same result.*
>
> [21] RL – *And on their round, see if they are linked somehow.*
>
> [22] C – *Hm, both have 15.*
>
> [23] RL – *And on the other round, one has 7 and the other has 8. Do you think making this observation can be a way of looking at what is the relationship between these two gaps?*
>
> [24] L – *I don't think so, I think they'll just calculate it.*
>
> [25] C – *I don't think they would look at this. Because I think they'll add the parts, which is what they do, and then they'll take the smaller part and subtract from the bigger one to find the unknown number.*
>
> [26] L – *But maybe after calculating they could look at the table.*
>
> [27] C – *Ah, yes, and say: "Look what happened here."*
>
> (Celeste, Luciana, Researcher/Lecturer, D9, 09/19/2018).

This same trio also pointed out the reasons to justify choosing the mathematical task, and shared a few mathematical conjectures with the other teachers:

> [28] K – *We opted for the bowling one, because here they will have to find the results as well. They'll have to check between the teams. They'll have to do inverse operations to find the results. And with Team 4, we think it'll pose a big challenge for them as well, because here, each of them can give a result. And what will they see?! That in equivalency there are many possibilities to reach the same result. That what they found will not necessarily be what their classmate found. Because they Always think it has to be the same result. That different parts will get them the same result.*
>
> [29] L – *And then we found, with help from Lilian[5], that there is a relationship here between Teams 2 and 3, that one of the scores is repeated. And you can talk about equivalency here. Dismiss that idea of a linear calculation. Because in truth we have that idea of a linear calculation. Even us, we started from this idea, because then you go through the who dynamics of comparing, relating. Because we are not used to that.*(Kátia and Luciana, P10, 09/24/2018).

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[5] Lilian is the first author of this article, who acted as the researcher and lecturer for the training courses in this research.

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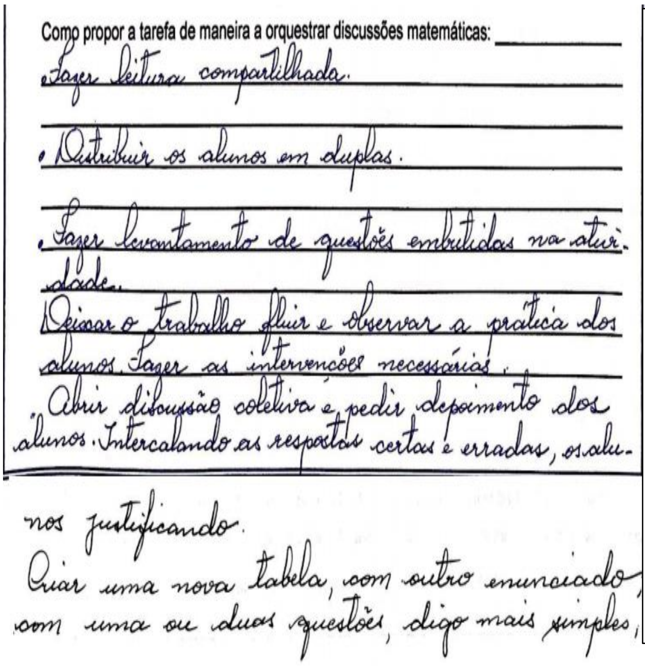
These discussions seem to reveal that, as the meetings and the development of the PLTs progressed, the teachers started to learn about the different meanings of the equal sign (Barboza, Ribeiro & Pazuch, 2019; Trivilin & Ribeiro, 2015) and expand their knowledge. They suggested different possibilities of resolution, using conventional mathematical terms, such as "equivalence," and also stated that students could make certain mistakes, because, in many cases, only one type of meaning of the equal sign is taught, or just a linear form of operation, which emphasizes only the operational meaning of the equal sign (Ponte, Branco & Matos, 2009).

After agreeing on which mathematical task (Figure 2) would be developed in the classroom (C), the six teachers established which goals could be developed in the class in which the task would be administered, based on the thematic unit "algebra," from the **National Common Core Curriculum** (Brasil, 2017), and on the "algebra" domain from the **Curriculum of the City of São Paulo** (São Paulo [município], 2017):


Como propor a tarefa de maneira a orquestrar discussões matemáticas:
Fazer leitura compartilhada.
• Distribuir os alunos em duplas.
• Fazer levantamento de questões embutidas na atividade.
Deixar o trabalho fluir e observar a prática dos alunos. Fazer as intervenções necessárias.
Abrir discussão coletiva e pedir depoimento dos alunos. Intercalando as respostas certas e erradas, os alunos justificando.
Criar uma nova tabela, com outro enunciado com uma ou duas questões, digo mais simples,

How can you propose the task in order to orchestrate mathematical discussions:

*Propose a collective reading;
*Divide students into pairs;
*Survey the questions embedded in the task;
*Let the task happen and observe how students work. Make the necessary interventions;
*Open collective discussion and ask students to share their results. Intercalate right and wrong answers, asking students to explain them;
*Create a new table with another statement, with one or two questions;
*I mean, a simpler one.

Figure 3: Protocol presented – how to propose the mathematical task PLT 3
Source: Adionísia, Celeste, Kátia, Luciana, Márcia and Valdete (PLT 3, 09/24/2018)

Afterwards, the teachers anticipated the possible difficulties the students might face when performing the chosen mathematical task (Figure 4):

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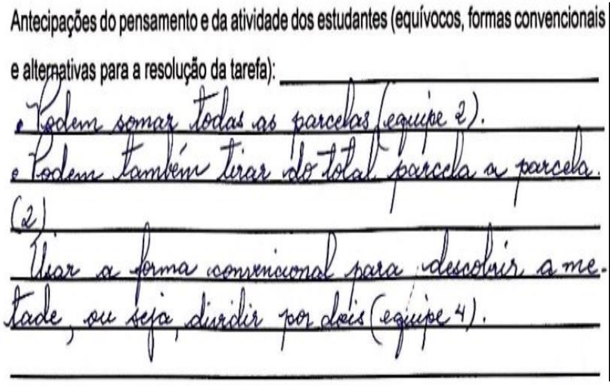
Antecipações do pensamento e da atividade dos estudantes (equívocos, formas convencionais e alternativas para a resolução da tarefa):
• Podem somar todas as parcelas (equipe 2).
• Podem também tirar do total parcela a parcela (2).
Usar a forma convencional para descobrir a metade, ou seja, dividir por dois (equipe 4).

Anticipating how students will reason and develop the task (mistakes, conventional and alternative ways to solve the task):

*They can add up all the parcels (Team 2);
*They can also subtract from the total, part by part (2);
*Use the conventional way to discover the half value, that is, dividing by two (Team 4).

Figure 4: Protocol presented – anticipation of student's reasoning - PLT 3
Source: Adionísia, Celeste, Kátia, Luciana, Márcia and Valdete (PLT 3, 09/24/2018)

The teachers were creating contexts for the classroom (Figure 3) so as to enable significant interactions around the content being developed, in a way in which students could advance in their knowledge (Russ, Sherin & Sherin, 2016). This can evidence how professional knowledge domains are constituted and through which processes they are developed. It can also be conjectured that at that moment, the teachers, from the PLT and the discussions, were mobilizing the KCS (Figures 3 and 4), [19], [20], [22], and, above all, the KCC [28, 29]. This can be inferred because the teachers demonstrated an understanding of the developments of teaching the different meanings of the equal sign since the first years of education, and the impact this could have in subsequent years, as part of the knowledge of content and curriculum.

During PLT 4, When the teachers watched two previously selected episodes, the students talked about the different possibilities for answering question D of the task (Figure 2). It can be observed that, in the transcriptions of dialogues 1, 2 and 4, in PLT 3, regarding the planning, the teachers stated that the fact that students would have different possibilities for reaching the result could be "an extra challenge." So, they were anticipating student reasoning and giving meaning to events that could take place in the classroom (Russ, Sherin & Sherin, 2016). It can be perceived that the students got involved with the task, as can be seen from the transcription of the video of part of meeting 11 (lesson administered to 5th grade class A), which corroborates what the teachers anticipated [8]:

> [30] E1 – *Team 4 was the most interesting to us [...] With Team 4 you could go 9 + 1 + 10, which is 20. You could also go 5 + 5 + 10, which is 20. And 6 + 4 + 10, which is 20. And 7 + 3 + 10, which is also 20.*
>
> [31] E2 – *So, each one did their own way, but it was all the same answer, but each one of us did it differently.*
>
> [32] E3 – *The equation and the numbers were different, but the result wasn't different, it was the same.* (Students (1), (2) e (3), P11, 09/27/2018).

The teachers were satisfied with what the students said and, especially, with the saying "it was all the same answer" [31], and stated that when these comments emerge it is

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possible to establish mathematical connections and systematize concepts [39]. They reflected on the episode chosen by the RL:

> [33] M – *Wow, isn't it at this moment that you can choose the concept you want to work with?*
>
> [34] C – *Ah, equivalency!*
>
> [35] M – *Yes [...]*
>
> [36] RL – *Linking it to the properties?*
>
> [37] M – *Yes, pointing to the properties. Closing the concepts. It has been worked before, go back to it. [...] This can be a strategy to do the opposite, right? First, the child explores, and then they close the concept. You draw a link between concepts. From this moment on, wouldn't it be ideal to make a collective record? This experiment is over, now let's systematize.*
>
> [38] C – *Yeah, I didn't do that.*
>
> *[...]*
>
> [39] A – *It's important to systematize using mathematical language.*
>
> (Márcia, Celeste, Researcher/Lecturer, Adionísia, P13, 10/08/2018).

Other than that, they proposed mathematical connections from the commentaries made by students; so, they mobilized their own mathematical knowledge. Also, we conjecture that when the teachers plan collectively, discuss their ideas and have the possibility of reflecting on their actions, they feel encouraged to go back to what they did or did not do, so as to implement new strategies from the interventions proposed by other teachers and from their personal reflection [33], [34], [37], [38], [39] and [46], [47], [48], [49]. So, if on the one hand they anticipated that students might not establish certain links, on the other hand, they felt encouraged to re-examine their vision and practice, because the students went beyond some of their expectations.

Thus, it can be understood that the logic established in the classroom was also revealed among the group of teachers, since they could exchange experiences, talk and share their difficulties, fears, uncertainties, shortcomings, their professional practice, their new knowledge, built based on theories (Ball & Cohen, 1999; Serrazina, 2013).

Another similar episode happened when the teachers were asked to develop PLT 4 (Figure 5), seeking to understand the narrative of the lesson. At the moment of the episode, students were expressing their hypotheses regarding the mathematical task about Teams 2 and 3 (Figure 5). During PLT 3, after the mathematical task was chosen (Figure 2), the teachers reflected on the equivalence relations that could be established by the students. It should be noted that, as can be seen from the third dialogue in this section, the teachers emphasized that the students would only calculate, without establishing relations between the numbers to be filled on the table (specifically referring to Teams 2 and 3), stating: *"I don't think they would look at this* [equivalence relation]. *Because I think they'll add the parts, which is what they do, and then they'll take the smaller part and subtract from the bigger one to find the unknown number"* (Celeste, D9, 09/19/2018).

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After all groups had shared, in response to a request by teacher Celeste, the researcher/lecturer wrote part of the scoreboard presented in the task on the board.

	1st ROUND	2nd ROUND	3rd ROUND	TOTAL
TEAM 2	15	7		35
TEAM 3		15	8	35

The researcher/lecturer asked the students if they noticed any link between the data of these two teams. Three pairs raised their hands and answered:

A4 – "Both teams had 35 as their total score."

A2 – "On both teams, one of the parts is 15."

A6 – "One team got 7 in a round, and in the other got 8 in another one."

The researcher/lecturer asked if any pair managed to find a link between the information pointed out by students A4, A2 and A6 and the values found. Quickly, pair A1 asked for the floor. Watch episode 8.

At the end, the researcher asked: "So, would it be necessary to do all the math you did to find Team 3's score on round 1?" The students answered all together: "No." Some pairs said that "if they had looked at these links more closely, they would have answered question C just by making these links, without the need for calculations and sanity checks."

Figure 5 – PLT 4: Practice excerpts: analysis and reflections
Source: **(**Barboza, 2019)

The teachers were invited to analyze what some students said when they shared their strategies and mathematical thoughts about the "equivalency" meaning of the equal sign (Figure 5). In this context they watched the episode, as revealed by the transcription of part of the video taken on Meeting 11 (lesson administered to 5th grade class A):

> [40] E1 - *V. and I had realized that on Team 2 we'd gotten 15 at first. And then on Team 3 we found 15 as well. [...] And the difference is 1.*
>
> The Other student from the group goes to the blackboard and says:
>
> [41] E1.2 – *Both have the same result, and since here it's saying that it's 35, they're equal. You could add 1 here, like, it could be 13 here, but it's 12, and here it could be 12, but it's 13. It's because this one [*pointing to 8*], has one more than this one [*pointing to 7*].*
>
> (Students (E. 1) and (E 1.2), P11, 09/27/2018).

One teacher said, as soon as she had watched the episode and listened to the students sharing their mathematical reasoning: "Wonderful, I liked it. They are showing they did understand [the equivalence relation of the equal sign]" (Márcia, P13, 10/16/2018).

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It can be considered that the teachers realized the mathematical strategies adopted by the children. On the other hand, during PLT 3 (Dialogue 3), they believed the students would not pay attention to the equivalence, or relation, between the equal numbers. However, in the previously discussed records (Figure 5 and Dialogue 7), students demonstrated these perceptions when they were invited to share what they had perceived in relation to Teams 2 and 3. These actions are suggested by studies that underline that if an unusual solution does not come up, it can be introduced to enable important mathematical discussions (Stein *et al*., 2008).

Another aspect raised by PLT 4 was the assessment of how children were disposed to perform the mathematical task and how it was administered. On the one hand, they stressed that, when planning, they suggested a collective reading, and this should have been done by the teacher, but it was done together by the teacher (C) and her students. On the other hand, in the episode the students pointed out and analyzed their opinions about working on mathematical tasks in pairs:

> [42] E 4 – *And especially because we were in pairs, we could count and it was way easier to do the math. [...] We could do many calculations and get the number 10. I was thinking about that, that when you get more people together, it gets more interesting. The answer becomes way more, like... smart.*
> (Student [5], P11, 09/27/2018).

The teachers agreed that mathematical tasks performed in pairs raise possibilities for exchanging different opinions, resolutions, verifications, and learning. Thus, they evidenced that they were (re)considering the way they work. And they shared their opinions:

> [43] C – *Yeah, I liked it. I'll do it like that more often, I already do that a lot in the humanities.*
>
> [44] K – *Working in pairs has this exchange, this possibility of seeing how other people did it.*
>
> [45] A – *But I usually work on Math in pairs, Only after I have developed the content individually. But the possibilities we have when we are working in pairs or trios, or in a group, and we let them share how they think, what they did, we are also building with them. Because it's usually focused on the teacher, who has mathematical autonomy, who holds all knowledge. And when we let students speak as well, we strengthen what students think and are capable of doing and sharing.*
>
> (Celeste, Kátia, Adionísia, P14, 10/16/2018).

The teachers recorded (Figure 6) their findings, after reflecting on working in pairs, as they had planned collectively:

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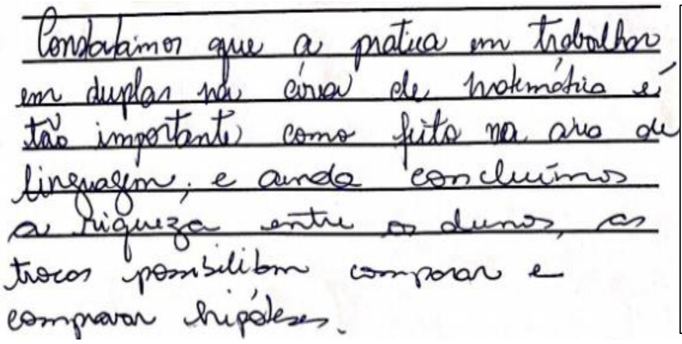
Constatamos que a prática em trabalhar em duplas na área de matemática é tão importante como feito na área de linguagem, e ainda concluímos a riqueza entre os alunos, as trocas possibilitam comparar e comprovar hipóteses.

We have found that the practice of working in pairs in Math class is as important as in language classes, and we also discovered the diversity among students, how exchanges make it possible to compare hypotheses.

Figure 6 – Protocol presented – analysis of the work in pairs – PLT 4
Source: Adionísia, Celeste, Kátia, Luciana, Márcia and Valdete (PLT 4, 10/16/2018)

When stating that they found that working in pairs is important for the study of mathematics, as it enables socialization among students and the comparison of hypotheses, the teachers are (re)structuring their knowledge about the content and students *(KCS)* [43], [44], [45] and (Figure 6), because they are talking about their disposition. In addition, they mobilize their mathematical knowledge, making it possible to know how mathematical topics are developed, that is, the horizon content knowledge (*HCK*).

When developing the lesson plan (PLT 3), they agreed that the teacher *should* select in advance which links could be made during while the students were sharing their responses, as shown in Figure 3. *However, teacher* Celeste waited for the entire class to finish solving the task and proposed they shared the strategies followed by the groups, following the order in which they were seated in rows, starting with the first row, from the end to the beginning (A4, A3, A2, A1), followed by the second row, from the beginning to the end (A5, A6, A7, A8, A9). She asked each group to say how and what they did to answer the questions. Then, in response to a student's suggestion, she asked that, following the row sequence, they would go to the front of the class to read how they arrived at the results. The teachers reflected on how they had suggested something different and suggested adjustments for future actions:

> [46] A – *But wouldn't it be to compare, who used addition, who used subtraction. And not just present it, and not everybody [*referring to all the groups/students*]?*
>
> [47] C – *Yeah, it could've been done that way. Because one pair did it one way, and the other pairs did differently [...]*
>
> [48] A – *Because the comparison could've been put on the blackboard, according to strategy, so the different ones could be discussed.*
>
> [49] C – *Yeah, I didn't do that.*
>
> Source: Adionísia and Celeste (P13, 10/08/2018)

After this proposal, the teachers jointly registered if they would make changes to the way teacher Celeste proposed the collective discussion (Figure 7) and how it could have been done.

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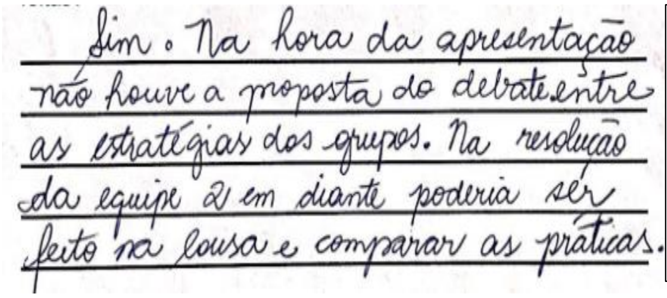
Sim. Na hora da apresentação não houve a proposta do debate entre as estratégias dos grupos. Na resolução da equipe 2 em diante poderia ser feito na lousa e comparar as práticas.

Yes. At the time of the presentation, there was no proposal for debating the different strategies used by the groups. The resolution of Team 2 onwards could have been put on the blackboard, comparing the practices.

Figure 7: Protocol presented – suggestions for changes in lesson development – PLT 4.
Source: Adionísia, Celeste, Kátia, Luciana, Márcia and Valdete (PLT 4, 10/08/2018)

Thus, it is possible to state, once more, that the teachers were (re)structuring their knowledge, based on the proposed reflections, from looking at professional practices centered in the classroom (Ball & Cohen, 1999; Silver *et al*., 2007), from the habit of reflecting on practices, and awareness that changes are needed to respond to the challenges of teaching Mathematics (Serrazina, 2013). It is also possible to conjecture that, during the development of PLTs 3 and 4, according to the excerpts presented here, the teachers were constantly building their mathematical and didactic knowledge (Ball, Thames & Phelps, 2008).

When concluding PLT 4, at the end of their reflections, they stated that learning and caring about the way children think is essential. One of them said:

> [50] L- *What will stay with me is [...] realizing their line of reasoning* [students]. *Because knowledge is a construction, not massification [...]. Because this* [pointing to the *slide* of the PLT] *is how we can get feedback on our work as teachers [...]*
> (Luciana, P14, 10/16/2018).

Another teacher also spoke up, stating that the opportunities provided by the researcher-lecturer could be something more constant for the professional training of the group: "what she brought us, we can do by ourselves, the talking, choosing a lesson and task, asking someone who is free in that period to film it. **Then, we can discuss it together, reflect on it, that thing about a researcher's way of looking at things.**" (Adionísia, P14, 10/16/2018). This statement allows us to conjecture about the importance of training processes like this, due to their possibilities for reflection and for professional learning, because research in/for practice was awakened.

Thus, below we will present the final considerations on this work.

Final considerations

The objective of this article was *to understand and explain how the construction of mathematical and didactic knowledge of Mathematics teachers who teach in the early years of elementary school takes place in a training course on the different meanings of the equal sign*. In line with the objective of the article, we believe that working with PLTs, the discussions that arise from them and their potential to build knowledge are possibilities to (re)think the training of teachers who teach Mathematics. Above all, in working with the different meanings of the equal sign.

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We inferred from the analyses the mobilization of specific knowledge for teaching and about students, which the teachers reorganized, developed and built, by being involved in this collective training course for planning, developing and reflecting on a Mathematics lesson plan about the different meanings of the equal sign, with the use of PLTs, followed by mediations, discussions and reflections on the class administered.

We understand that PLTs, by themselves, do not enable the (re)signification and mobilization of mathematical and didactic knowledge. We are not so naive to think that to build knowledge it is enough to propose good PLTs. However, it is possible to recognize the importance of teachers' reflections when working with them, based on questions from the lecturer and the discussions developed from them.

The teachers were asked to examine their own thinking and the way they teach. Thus, this professional learning through PLTs emerged from collective discussions. Exchanges with other professionals made it possible to understand, compare, (re)formulate their own (un)certainties, expanding their learning opportunities. The teachers looked at mathematical tasks and their potential, for collective discussions and their importance for learning (Ball & Cohen, 1999).

The six teachers who collaborated in this research reorganized mathematical aspects, as they properly talked about the equivalency meaning of the equal sign, and also reorganized their didactic knowledge. These conjectures are based on the fact that they verbalized/registered that they should propose mathematical tasks to be solved in pairs. They began to give more importance to the possibilities of solving and anticipating what students do/could do in the tasks and decide how/what to do to keep the mathematical discussions at a high level and the tasks challenging. They valued to the opportunity for students to explain how they thought and to seeing that they can exceed the expectations of their teachers.

Thus, the analyses were concluded and we propose that the excerpts presented here point to mobilization and construction of knowledge, since central questions were raised by the participants: directing the teacher's gaze to the students' reasoning as a resource for their own learning and recognition of their own work, as well as how to anticipate what students can do (Ball, Thames & Phelps, 2008; Russ, Sherin & Sherin, 2016). The excerpts also highlight the teachers' perception on the importance of making mathematical connections with the content developed in class, with what has already been done, and opening doors to what will be worked on later, as well as providing the possibility of working in pairs and trios to encourage mathematical discussions. In addition, they suggested the possibility of maintaining the habit of collectively planning some lessons, thinking together with their peers, discussing tasks and how they can develop a lesson and later reflect on it.

The excerpts presented here corroborate studies discussed in the literature review, as well as the possibility that teachers continue to develop new understandings of students' reasoning through interactions with students in their own teaching practice (Russ, Sherin & Sherin, 2016). The teachers examined mathematical tasks and their potential for collective discussions and their importance for professional learning itself, whether in planning, task

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selection or for reflecting on their practices (Ball & Cohen, 1999; Serrazina, 2013; Silver *et al*., 2007; Smith, 2001). Corroborating Ma (1999): a central process of teacher learning is the formation of new knowledge structures, combining several of its spheres and potentially some new knowledge, either individually or collectively.

Nevertheless, the teachers raised a point that can serve as a basis for other research, regarding the obstacles faced by early elementary school teachers of all disciplines (Portuguese, Mathematics, History, Geography and Sciences) and overcrowded classrooms. In addition, research is suggested on how these aspects can interfere in the professional learning of teachers who continue to learn while exercising their practices.

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