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About critical and behavioral competences in Statistics Education

Sobre as competências crítica e comportamental na Educação Estatística

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Abstract

Research in Statistics Education has advanced intensely with the planning of activities aimed at the development of three competences, literacy, reasoning, and statistical thinking, which allow more meaningful learning of the concepts of this science. The deepening of these researches allowed the identification of another competence also important for this universe, which is the critical competence. The aim of this paper is to deepen the reflection on critical competence, highlight its evolution in research developed in the field of Statistical Education, and to present a fourth competence, which we call behavioral competence. In our analysis, we have observed that critical competence develops itself on two streams, sociopolitical and epistemological. Finally, based on several practical examples observed inside and outside the classroom, we show how behavioral competence has been identified.

Keywords: Statistics education; critical competence, behavioral competence.

Resumo

As pesquisas em Educação Estatística têm avançado intensamente com o planejamento de atividades que visam ao desenvolvimento de três competências, a literacia, o raciocínio e o pensamento estatístico, as quais permitem uma aprendizagem mais significativa dos conceitos dessa ciência. O aprofundamento dessas pesquisas permitiu a identificação de outra competência também importante para esse universo, que é a competência crítica. O objetivo deste trabalho é aprofundar a reflexão sobre a competência crítica, evidenciando a sua evolução em pesquisas desenvolvidas no âmbito da Educação Estatística, e apresentar uma quarta competência, a qual chamamos de competência comportamental. Em nossas análises, pudemos observar que a competência crítica se desenvolve com base em duas vertentes, a sociopolítica e a epistemológica. Por fim, tomando como base diversos exemplos práticos observados dentro e fora da sala de aula, mostramos como a competência comportamental foi identificada.

Palavras-chave: educação estatística; competência crítica; competência comportamental.

Introduction

Statistics, as a data analysis science, is present in various contexts. For this reason, it is fundamental to comprehend it so that people can understand, evaluate and position themselves against the statistical data that circulate in the most diverse media.

Recognizing the frequency of Statistics in people's daily lives, Statistics Education researchers began to discuss the importance of this science being taught from basic education to the undergraduate level. This discussion was punctuated, initially, through two documents,

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one from the American Statistical Association (ASA) and one from the National Council of Teachers of Mathematics (NCTM), published in 1960 and 1967, respectively. Given the expressiveness acquired by these documents, from 1989 onwards, important academic magazines have published articles that highlighted two actions that would deserve attention in the following years: inserting Statistics in all school levels and giving greater attention to the ways of conducting teaching-learning processes of this science (Zieffler, Garfield & Fry; 2018).

In the Brazilian context, the concern with the Statistics' teaching-learning processes has also gained space in official documents. At the end of the 1990s, with the publication of the National Curriculum Parameters, Statistics and Probability were officially incorporated into the curriculum structure of basic education (Samá, 2018). More recently, the promulgation of the National Common Curricular Base (MEC, 2017) ratifies the inclusion of Statistics and Probability in basic education and suggests the approach of statistical concepts through situations from everyday life, science, and technology (Cazorla, Silva & Santana; 2018).

Both nationally and internationally, the dissemination of these ASA and NCTM documents has led many researchers to develop studies focused on Statistics Education. Thenceforward, research in this area has advanced worldwide, and one of the greatest achievements has been the identification of statistical competences that must be developed in students for effective learning of the concepts of this science.

In this line, the skills related to statistical thinking, statistical reasoning, and statistical literacy were identified. From the beginning, it was observed that there were some common aspects of these three competences that should be identified. A first assumption was that these competences would form an intertwining, which was represented by Delmas (2002), as shown in figure 1.

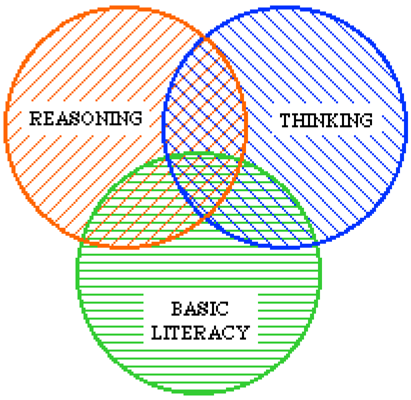

Figure 1- Independent domains with some intersection
Source: Delmas, 2002, p. 4

Another possibility observed by the same author suggests that literacy would have a more comprehensive domain, with the other competencies inserted in it, as suggested in figure 2.

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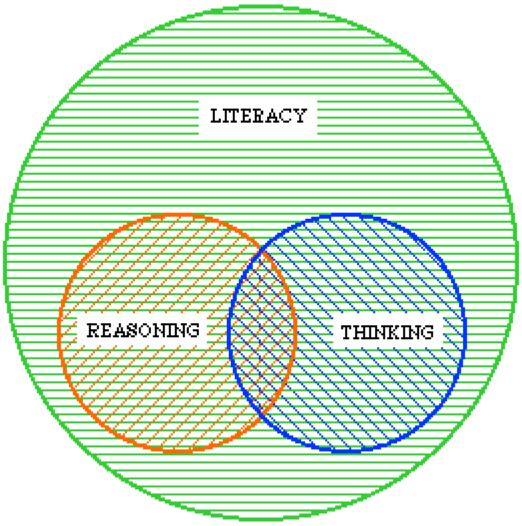

Figure 2 - Reasoning and thinking inside the literacy
Source: Delmas, 2002, p. 4

This second interpretation was more accepted by the academic community and researches began to work harder in the field of literacy, seeking to identify its determinants, levels of depth, how to evaluate its development, connections with Mathematical Modeling, with other literacies (Campos & Coutinho, 2019; Coutinho & Campos, 2018) and with other skills.

The deepening of literacy research enabled the identification of critical competence, which has served as a basis for what we now call Critical Statistical Education (Campos, 2007; Campos, Jacobini & Wodewotzki 2011a; Campos, Jacobini & Wodewotzki, 2011b; Campos, 2016; Perin, 2016). Other authors such as Sampaio (2010), Hollas & Bernardi (2018), Melo & Maçal (2012) also made valuable contributions to the development of studies in this area.

Research in Statistics Education is very dynamic, and researchers from all over the world contribute to its constant deepening, making this area to have a constant evolution in its most diverse aspects. Seeking to contribute to this evolution, we have identified a peculiar characteristic, which are emotional, social, psychological and/or cognitive factors that affect the decision-making of students who learn Statistics. Such factors are manifested in different ways in their behaviors and have influenced in a non-positive way the learning of this discipline.

In this context, the objective of this work is to deepen the reflection on critical competence, showing the evolution of this competence in research in Statistics Education. In addition, we aim to present a new competence that seems important to this universe, which we call behavioral competency, which can significantly influence the student's relationship with the concepts of Statistics, with consequences on several aspects related to data interpretation.

Our intention is to show that the three competences initially identified (literacy, thinking, and reasoning), need to be complemented with the other two (critical and behavioral)

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to provide a better understanding of the teaching-learning dynamics of Statistics at different levels of schooling.

The theoretical deepening of critical competence

The identification of the three original statistical skills was a milestone in the evolution of Statistics Education, as it allowed the construction of its theoretical and methodological foundations in a more objective way. It also guided the research to determine the characteristics, forms of development, evaluation, and levels of deepening of these competencies.

Campos (2007), in his study, found that the development of these skills requires an environment in which students are inserted in an investigative practice; work with topics from their concern; use data that is relevant to a particular context; manipulate a diversity of variables and experience the process of data generation and analysis; develop group activities; use technology; and are evaluated by the relationships and judgments they establish for a data set. To this end, the author advocates that didactic activities should always deal with relevant issues for students, linked to their daily lives and/or their professional training. Campos (op. cit.), when studying pedagogical strategies with a view to the development of these competences, has realized that the precepts of Mathematical Modeling from the perspective of Mathematics Education proved to be suitable for work in the classroom.

A basic condition for the elaboration of Mathematical Modeling works, in this perspective, is the data contextualization, which must come from real research, preferably collected by the students themselves. In this sense, Mathematical Modeling aims to provide students, in addition to the ability to deal with mathematical notions, to apply these notions in different contexts. Thus, it intends to develop the ability to reflect on these applications so that the student can exercise critical citizenship (Campos, Wodewotzki & Jacobini, 2011a). The basis for developing works with these characteristics is dialogue, which favors meaningful, political and democratic learning. It is hoped that the student will be able to recognize, reflect and ponder the socio-political application of knowledge.

In view of this understanding of Mathematical Modeling, Campos (2007), stressed that by bringing real and contextualized data to the classroom, the teacher would be thematizing and problematizing teaching, as well as stimulating debate and dialogue. Thereby, the author states that it encourages the de-hierarchization of the classroom environment between students and with the teacher and practices democracy in the pedagogical context. Thus, Campos (op. cit.) observed that the principles of Critical Education and Critical Mathematics Education, according to Freire (1965, 1979 and 2014) and Skovsmose (2008, 2014a and 2014b), respectively, complement the pedagogical strategies favorable to the development of statistical skills. More specifically, he conceived Statistics Education with a differentiated and integrating approach to these ideas and made concrete the convergence between the foundations of Statistics Education and Critical Education.

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Based on this understanding, Campos (2016) pointed out that statistical competences then lead to the development of a fourth competence, which is the critical one. This is because, when bringing to the classroom problem situations experienced by students, they would be challenged to think what the data indicate about their reality and, therefore, would make them more critical insofar as the treated themes deal with social, economic, policies, environmental issues, etc. For the author, problems based on real data are the key to develop creativity, criticality and encourage reflection on their own reality.

In this line, we understand that when solving problems arising from modeling and investigating situations that refer to data from a social, political, economic nature, etc., the tasks do not lead students only to their solution, but, above all, to dialogue and debate about the exposed reality, stimulating reflections and actions on the way in which we live and conduct the world around us.

Thus, the environment needed for the development of statistical skills, which are concerned with a multifocal look, attentive to everything that involves a statistical investigation process, is inseparable from the elements (dialogue, problematization, reflection, and awareness) cited by Freire (2014) as fundamental to Critical Education. Additionally, these elements will allow education to fulfill its political dimension as well, which is why Campos (2016) affirms that critical competence is intertwined with statistical competences. Talking about this fourth competency, Campos (op. cit.) argues that it is not a matter of creating a new dimension for Critical Education, but of highlighting its aspects within Statistics Education, so that pedagogical activities can be better planned and evaluated in that context.

In her turn, Perin (2019), while seeking the contributions of the Mathematical Modeling environment in the perspective of the development of the skills of Critical Statistics Education, observed, based on the students' statements, that critical competence can be built based on two distinct aspects: sociopolitical and epistemological.

a) Socio-political criticism is one that addresses issues related to the understanding of aspects of the world in which individuals live, as well as their actions in that world. In her research, Perin (op. cit.) identified that after carrying out a modeling job, students, in addition to questioning their relationships in society, also sought to identify different ways of acting in order to be more participatory and questioning with regard to a set of actions of social and/or community interest, in which the activity performed is reversed beyond itself, that is, in favor of the other. Next, we reproduce the students' dialogue that allowed this understanding.

> A3G1: […] Not to mention the difficulty of getting the data [...], we need to insist a lot!
> A1G1: It was sad to realize that people did it for fun!
> A3G1: But the cool thing is that I changed my view of taking surveys after this work. Before I didn't pay attention, I also didn't respond well.
> A1G3: [...] developing this research moved us […]. We thought about that day when the hospital staff was here to donate marrow, they had to almost beg here in the classroom for us to go down there […] and now that we were on the other side, we were much more sensitive and open to these things (Perin, 2019, pp. 188-189).

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These statements reveal the students' concern with their ethical-moral education in order to better consolidate social values compatible with the exercise of citizenship. When reporting the difficulties encountered at the time of data collection, students promoted reflections that broadened their social skills, becoming aware of important aspects of the world in which they lived and pointed out paths that lead to transformation.

b) Epistemological criticism refers to students' perceptions of statistical knowledge and is strongly related to elements that characterize the development of statistical skills. Based on the following excerpt, extracted from Perin (2019, p. 195), we discuss these elements.

> A2G3: […] I saw that Statistics does not just see numbers or percentages, but that it is a difficult job with many obstacles and surprises along the way [...].
>
> A2G5: Nowadays, I look at everything more carefully, checking the sources of information and analyzing the whole scenario to draw my conclusion about that data [...].
>
> A2G1: [...] before doing all this work and the discussions we had in class, I saw the world as it appears to be, but now I feel that I have the ability to look at things more deeply, analyze the situation better and make my decision better. I already think: Who did it? How did they do it? Did they respect all the care?
>
> A2G2: [...] before for me, a given number was exactly that, today I think there is a range of possible values […].

When recognizing the relevance of articulating conceptual knowledge with the sampling process to verify whether certain conclusions can be drawn based on available information, one sees characteristics of statistical literacy. Statistical thinking, on the other hand, can be perceived when students recognize that data analysis indicates a trend and not a certainty in relation to the phenomena. In these statements, we realized that students were able to understand that a statistical study cannot account for the totality of variables that involve a phenomenon, which has characteristics of statistical reasoning.

Facing this, we realize that students recognize that being well informed means analyzing what others say, evaluating and questioning what is shown in research, graphics, and numbers, that is, they understand the need to look critically at knowledge. Campos (2016) states that critical competence reaches its peak when students cry out for their rights, demand social justice, raising their voice against oppression systems imposed on them.

That said, we understand that although the three original competencies are somewhat related to the need for criticality for their development, critical competence must stand out and not be subject to them. As well as Campos, Jacobini & Wodewotzki (2011) and Perin (2019), who identified and deepened their knowledge of statistical literacy, reasoning and thinking, proposed pedagogical actions aimed at their development, critical competence must have the same treatment and, as we have shown, it finds fertile ground in the pedagogical strategy of Mathematical Modeling.

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Behavioral competence

Some Statistics' concepts are quite common to be applied in people's daily lives, as they are present in the media very often. However, it is perceived that ordinary people sometimes appropriate the statistical concept in a peculiar way, without concern for rigor as to its correct use or not. Our intention here is not to make value judgments, but to try to understand the behavior of people who use the concept inappropriately and reflect on the presence of behavioral competence in Statistical Education. To achieve this, we will cite some examples.

Example 1: Inhabitants of large cities, such as São Paulo, are used to listening to traffic information on the radio in the morning, such as: "Traffic now has X kilometers of congestion, which is within the average for the day and time." On September 6, 2019, there was a strike by bus drivers in São Paulo, and traffic was quite chaotic. During the long congestion that was formed on the main roads of the city, it was possible to hear on a radio station an interview of the mayor of the city saying: "We are operating below average[3] [...]". These examples make us reflect on whether what these people understand as an average is the same formal concept that we know in Statistics or not. In particular, it is up to us to reflect on what it means to be within the average. Let us present another example before proceeding with the analysis of this.

Example 2: Perin (2019), presented a modeling study carried out with students of an undergraduate course in the discipline of Statistics, in which a group researched some variables regarding the number of students in the college and presented a boxplot representation of the variable age, as shown in figure 3.

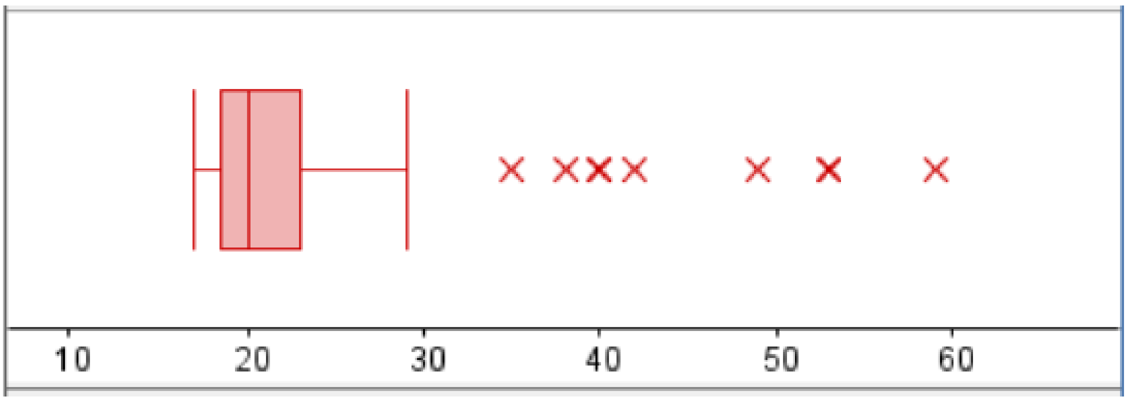

Figure 3 – Representation of the respondent's age variable.
Source: Perin (2019, p. 136)

The boxplot, box diagram, or mustache box is a very useful graph for analyzing the concentration and dispersion of data. The drawing box is limited on the left by the first quartile (Q1) and on the right by the third quartile (Q3) and it has inside it a line that represents the positioning of the median. The extension of the box (shown in figure 3 horizontally) represents the interquartile range, which contains 50% of the data of a distribution. The lines coming out

[3]http://cbn.globoradio.globo.com/media/audio/273670/sistema-nao-e-mais-ou-menos-eficiente-pela-quantid.htm

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of the box on the right and on the left (the mustaches), were built considering 1.5 times the interquartile range to cover about 99% of the data, whereas the line on the right should extend to the largest observation below the limit for upper outliers, while the line on the left should extend to the smallest observation above the limit for lower outliers (Bussab & Morettin, 2005). The extreme values were represented by a “×” positioned outside the mustache.

The representation that we see in figure 3 was made by the students, and it is possible to observe several outliers positioned to the right of the mustache limit. Following, we reproduce the students’ speech from the group that did this work, as described in Perin (2019).

> A1G1: Here is our first variable, age. You can see from the boxplot that it has a lot of disparate data. You can see that the youngest student is 17 years old and the oldest is 59, but it is more concentrated… it is more concentrated between 17 and 23 years old; most students are within this range that varies little. At other ages, you will notice that you are farther, wider. We will talk about this better when we present the average age.
> A2G1: We got to do the …
> A4G1: Histogram.
> A2G1: That!!! And we saw that most people are within that range here[4]. And if we were to exclud some of these students[5], the average would be well represented in this place (Perin, op.cit, p. 136).

From the students' speech, it is possible to notice that they had calculated the arithmetic mean of the ages considering all the results, but, excluding the outliers, they reached another result that made them more comfortable (“the average would be well represented”). Perin (op. cit., p. 137) states:

> We also think it is interesting to reflect on the speech of student 2 about excluding extreme values for calculating the average. In a data collection, we can find values that are out of the normal range and that are likely to cause irregularities in the results obtained through algorithms and analysis systems. However, in a data analysis process, we consider it important to look at these values in two ways: these values can negatively bias the entire result of an analysis; the behavior of outliers can be precisely what is being sought and understanding them can be fundamental to produce measures relevant to the study which is being developed. However, we understand that the student had a curious attitude when removing these values and recalculating the average. Nevertheless, these values can receive not only this treatment but other views that allow us to understand the reasons for their existence.

Example 3: The same author also reports the presentation of the work of another group that also researched the age of the college students, as well as the monthly income (in reais), and which showed the graphs that are in figure 4.

[4] The student points to a range between 18 and 24 years old.
[5] The student indicates the outliers.

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Figure 4 - Diagram elaborated by students
Source: Perin (2019, p. 142)

The students' speeches about the behavior of the variables differ substantially from what was observed previously.

> A2G5: Analyzing the age distribution graph, as in the presentations of the other groups, we noticed that the age does not vary much! I mean, it varies, but what I want to say is that most are here, in this region[6], it is a group of young students. […] But I'm also happy to see that people who didn't have a chance when they were younger are studying now. […] The income graph has a very different shape - excluding this last column, we could say that low income is for a small number of people, then increasing income, increasing the number of people, and finally, increasing income, the number of people falls.
> A1G5: I'll talk a little bit about what we thought when we built this income graph: in our reality, it was like this, as you can see. But in practice, thinking about a larger population, I think it would look like the age graph (Perin, 2019, pp. 142-143).

The author highlights this difference:

> Unlike students from Group 2[7], those in Group 5 took a different look at the values farthest from where most elements are concentrated. This can be seen from the comments they made about the age of students who are attending higher education. They did not ignore these values, as they were few compared to the frequency of other ages, but they tried to understand them, pointing out positive aspects of this fact (Perin, 2019, p. 143).

This idea of excluding outliers for averaging needs to be considered carefully. Outliers may indicate measurement error or response error, in which case they must be excluded in order not to bias the average (for example). However, these outliers can be real values, that is, a part of the whole, which, if excluded, can cause an inadequate change in the data, leading to a failure in the interpretation of the scenario under study.

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[6] The student shows the first two columns

[7] Despite mentioning students in group 2, the author here is referring to group 1, which we present in figure 1.

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Example 4: In a pedagogical workshop that we conducted with Mathematics teachers and students of the Masters and Doctorate courses in Mathematics Education, we presented a boxplot for the number of inhabitants of the Brazilian states. Without revealing the scale of values, we presented to the participants the graph shown in figure 5 and asked whether or not we should consider the outliers when calculating the average number of inhabitants of the states.

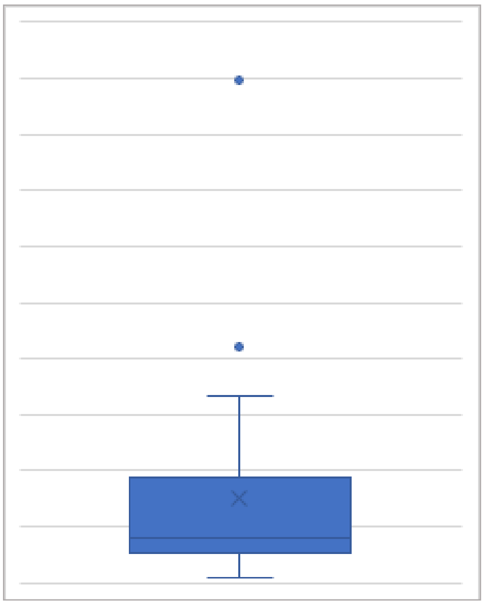

Figure 5 - *Boxplot*[8] showing the number of the inhabitant of Brazilian states
Source: our elaboration with data from IBGE (2016).

Some teachers answered yes that we should exclude extreme values. Asked about the reason (s) for exclusion, they replied that they were outliers, so they would bias the calculation of the average.

Following the activity, we showed the scale values and identified the states with different results (figure 6). We took the opportunity to debate the response of teachers who had excluded São Paulo and Rio de Janeiro from calculating the average number of inhabitants in Brazilian states.

[8] This chart was made using Excel, which positions the boxplot in vertically.

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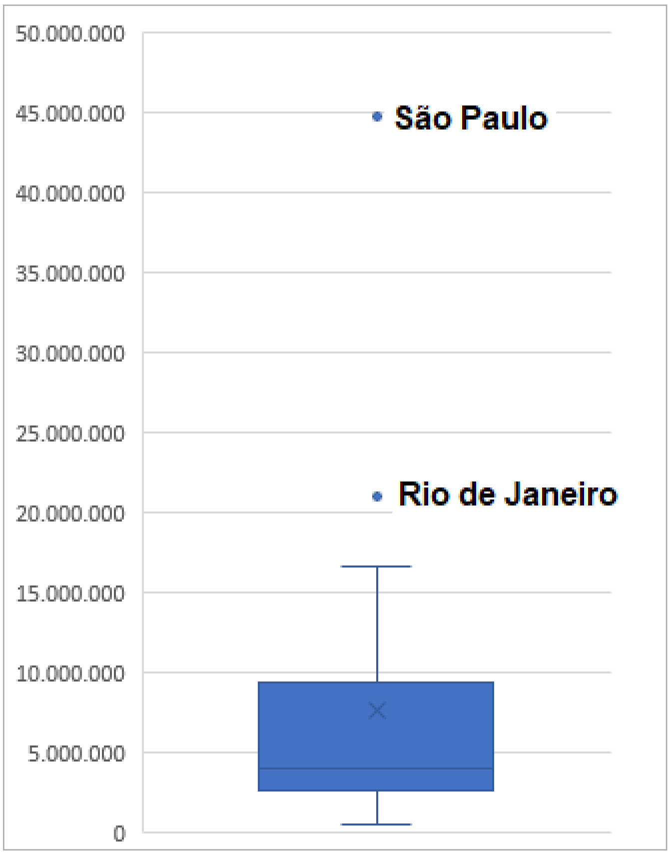

Figura 6 - Boxplot of the number of inhabitants of the states with identification of the scale and outliers.
Source: our own elaboration with data from IBGE (2016).

It may seem that teachers who felt that outliers should be excluded acted similarly to students who excluded the highest age observations. However, there is a significant difference in interpretation, as teachers did not have all the information about the problem. Before taking sides on the exclusion or not of extreme values, they should inquire about the origin of the data and reflect on the veracity of the information. Apparently, teachers opted for a more intuitive and less reflective reaction.

Now let's go back to the first example that we present, which refers to traffic in the city of São Paulo. The São Paulo Traffic Engineering Company (CET-SP) daily shows a graph of slow traffic in the city. With data obtained from historical records, CET-SP monitors the slowness index on approximately 863.6 km of city roads and calculates the average jam for each day of the week and for each hour, based on the records from the same days of the week of the previous twelve-months, excluding January, February, July, and December, as well as holidays and holiday amendments (atypical days). The graph in Figure 7 refers to a Friday, the same day of the week as the bus strike event mentioned. Unfortunately, we did not find the records of the strike day, which may have been deleted because it is an atypical day.

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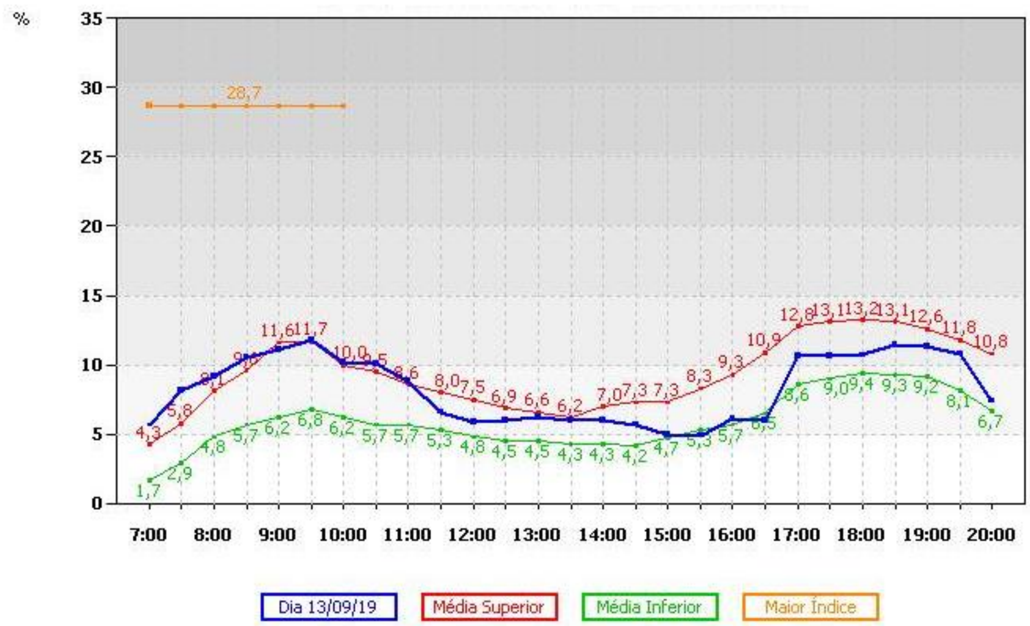

Figure 7 - Percentage of slowness per hour in São Paulo/SP, in 13/09/2019 (reference 863,6km = 100%). Source: CET-SP (http://cetsp1.cetsp.com.br/monitransmapa/agora/graficolimite.asp)

As you can see in the graph, there is a higher average and a lower average, whose calculation methodology is explained on the CET-SP website:

> The slowness graph shows the rate of slow traffic recorded every 30 minutes, from Monday to Friday, from 7 am to 8 pm, as well as the lines indicating the lower and upper limits, obtained through statistical calculations. These calculations are made taking into account the historical data of slowness on the same day of the week of the twelve months immediately preceding the current date. Values from January, February, July and December, holidays and holiday amendments and values whose difference in relation to the average value exceeds 1.5 standard deviations are discarded. The lower and upper limit line is the result of the application of the calculation of 1 standard deviation of the selected samples and represents the range of values considered acceptable when there is no occurrence that causes a major impact on traffic (CET-SP[9]).

As explained by the company, any event with congestion above or below 1.5 standard deviations taken from the arithmetic mean is discarded. Apparently, the CET wants to obtain a well-behaved average, and the number of days discarded ends up being large.

Regarding the speech of the mayor of São Paulo (audio from CBN radio), he quotes three media in the excerpt that we transcribe below:

> [...] traffic jam at 7h am was 27km when the average is 37km, at 7h30 am it was 37km when the average is 53km, and now, at 8h in the morning, we operate with 58km of jam when the average is 70km, that is, we are operating below average [...] (audio from CBN São Paulo radio, stretch between 1min35s and 1min57s)[10]

[9] cetsp1.cetsp.com.br/monitransmapa/agora/ajuda.htm

[10]http://cbn.globoradio.globo.com/media/audio/273670/sistema-nao-e-mais-ou-menos-eficiente-pela-quantid.htm

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To make the information more accurate, we observe that the data mentioned by the mayor referred to the upper average (figure 7). Otherwise, we will see:

7h: upper average = 4,3% of 863,6 = 37,1km

7h30min: upper average = 5,8% of 863,6 = 50,1km

8h: upper average = 8,1% of 863,6 = 70,0km

An error is perceived only at the average of 7h30 am, which the mayor says is 53km when in fact, it is 50.1km. In any case, what the mayor calls the average is, in fact, not the average since the value is added by 1 (one) standard deviation. In addition, the mayor's information could be questioned when we look at the graph of 9/13/2019 (figure 7), exactly one week after the bus drivers' strike, in which the level of traffic jams between 7h and 8h am was well above the upper average. However, our goal here is not to ask this type of questioning, but to try to understand what people's common-sense treats as being the average and how criteria are established to exclude outliers from their calculation.

When we see students who are not comfortable with the average result and, therefore, exclude extreme values for it to be better represented, when we see teachers also excluding important extreme values because they would bias the average result or even the mayor of the city forcing an approximation to reach a result within the average, we perceive a type of characteristic behavior that calls our attention.

Of course, we could argue that in any of these cases, the substitution of the mean by the median would solve the problem of representing the central trend of the distribution of values, without the need to exclude any number, and that it would demonstrate a certain development of statistical reasoning. However, what concerns us to observe more closely is the behavior, that is, the search for a value that is in accordance with the person's expectation, a *normal* result, in which the person feels comfortable.

When prioritizing a context in which the user of the statistical concept feels comfortable instead of looking for a plausible interpretation for the result of the calculations, it seems that the reality is modified only to please those who use it. When we start to notice the presence of this behavioral bias, we are able to observe it in many situations both inside the classroom and outside.

Example 5: In another workshop for elementary school teachers and students from undergraduate courses, we present the following probability problem, idealized as a test:

When tossing two honest coins, it is known that one of the results was head. How likely were the two results to be heads?

a) 1/4 b) 1/3 c) 1/2 d) 2/3 e) 3/4

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To solve this problem, it is useful to build the sample space of the possible results for tossing two coins, which are: S = {CC, CK, KC, KK}, where C is for heads e K is for tails. As we know that one of the results was head, we can eliminate KK and the sample space becomes reduced to S = {CC, CK, KC}. Now we can see that the event "two heads" represents one of three possible results, so the correct answer is the letter (b). However, several students responded (c). We observed that such students did not build the sample space. Asked about the reasons for not building it, they said that they responded intuitively. In this case, intuition prevailed over rationality and seemed to have made students comfortable to answer a problem that would require more refined reflection.

We know that probability is an object of study from Mathematics, but it should be noted here that we consider it as part of Statistics, given that it brings with it the concept of randomness and uncertainty, which are two of the pillars of statistical inference. Additionally, we point out that probability is present in the Statistics books (Bussab & Morettin, 2005; McClave, Benson & Sincich, 2009; Morettin, 2010; Vieira, 2012; Novaes & Coutinho, 2009; Braule, 2001; Clark & Downing, 2011).

We understand that this option for intuition, for the quick way to simplify the resolution of a problem, also constitutes a behavioral bias, perhaps of a slightly different nature than that described in the other examples, but that can be better understood with the help of figure 8, which represents the theory of the dual system, which states that we all have two distinct ways of thinking, which are defined as system 1 and system 2.

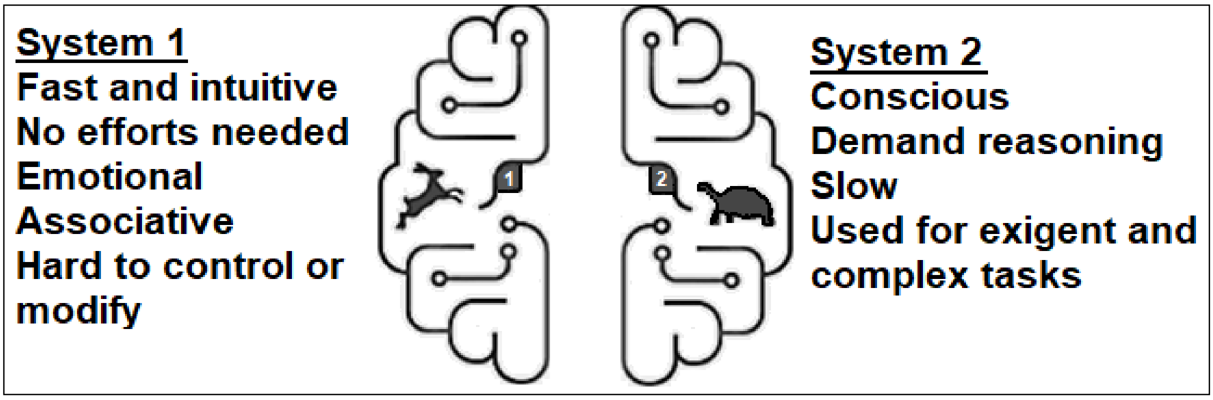

Figura 8 - Dual system theory scheme
Source: Adapted from Ávila e Inchausti (2017, p. 23)

System 1 is simple and involuntary, while system 2 is detailed, complex, and requires concentration. Ávila and Inchausti (2017) suggest that people's decision-making process follows the law of least effort: system 1, which cannot be turned off, proposes decisions to system 2, which is lazier, so it saves time and answers ok to system 1, and so we make our choices.

In these five examples, we saw different situations in which people's behavior seems to have been decisive in choosing the way in which they solve Statistics problems. The identification of some behavior patterns can be quite useful to promote more effective

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teaching/learning, as teachers may be able to predict and circumvent such bias that does not necessarily refer to the lack of knowledge of the learners, but to a care in relation to the attitude to be taken to resolve, interpret or validate the results.

Final Considerations

In this article, we seek to advance the theoretical assumptions of Statistics Education with regard to the development of some competencies.

Initially, we showed that the teaching and learning of Statistics must be structured based on the development of three competencies: literacy, reasoning, and statistical thinking. Additionally, we saw how some studies sought to understand the possible interrelationships between these competencies and the ways of representing them.

Subsequently, we present some studies that sought to identify the pedagogical guidelines necessary for the development of these statistical skills. In this search, Campos (2007, 2016) found significant convergences between these referrals and the precepts of Critical Education and Critical Mathematics Education, as this environment is structured based on dialogue, problematization, reflection and awareness, principles that constitute Critical Education. In addition, these ideas bring a critical and reflective look to your knowledge. For this reason, we corroborate the study, which understands that critical competence must be part of Statistical Education and should be considered as the fourth competence within this study field.

In addition, we showed studies by Perin (2016, 2019) on the relationship between Statistics Education, Critical Education, and Critical Mathematical Education that advanced in the understanding of critical competence, which covers two aspects, the socio-political and the epistemological. The socio-political stream refers to questioning and analyzing the individual's experiences and daily situations. The epistemological one represents criticism of knowledge itself and is linked to the recognition of some weaknesses of statistical tools, hence the importance of aggregating ideas in different ways and sources to draw any conclusions and to verify how and by whom the data were produced. We have shown examples of the practical application of these two aspects of critical competence.

Finally, we presented various examples that, in our understanding, constitute a new and important competence, which we call behavioral. Unlike the three original competencies, behavioral competence is not linked to the deepening of statistical content nor to its pure and simple learning. It is linked to the attitudes of users of statistical knowledge, which, in addition to the three primary competences and critical competence, require reflection on the acceptance or not of the results obtained, their validation in the adopted context, awareness of the possibility of causing a bias in the result and, still, not to adopt the intuition as controller of the procedural decisions.

With this, we seek to fulfill the objectives of this work, that is, to contribute to the deepening of knowledge about critical competence and to present the evidence of a new

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competence that seems to be equally important: behavioral competence. When thinking about this last competence, a saying came to us that says Statistics is the science by which you "torture data until they answer what you want" [11]. Obviously we cannot agree with this, but it possibly reveals a type of attitude that is related to the behavioral bias of people who prefer to work to obtain the results that make them comfortable instead of facing reality revealed by the numbers.

Behavioral competence needs more studies that allow us to deepen its investigation, in order to better understand and typify it, evolving to allow the identification of strategies for to face it, in order to highlight behavioral biases that lead to misinterpretation and misuse of the results obtained, among others. If the behavioral bias has an influence on the students' response to Statistics problems, then their knowledge must be of great importance for educators and researchers in the field of Statistics Education. Finally, we understand that research covering the articulations between the various competencies can also contribute to a better understanding of the topic's relevance.

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[11] Adaptation of the comment by British economist Ronald Harry Coase, professor emeritus at the University of Chicago and winner of the 1991 Nobel Prize in Economics, who said: "If you torture the data long enough, it will confess to anything", paraphrase Huff, D., *How to lie with Statistics*, New York: W. W. Norton & Company, 1954.

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