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Computational Simulation: Aspects of Frequentist Probability Teaching

Simulação Computacional: Aspectos do Ensino da Probabilidade Frequentista

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Abstract

This article discusses didactic aspects of the frequentist approach to probability through computer simulation, with the aid of an applet that simulated the *franc-carreau* game. Data for discussion were collected from a workshop for teachers. Brousseau's theory of didactic situations and Gal's probabilistic literacy model constituted the theoretical framework, along with premises based on second-generation didactic engineering as a research method, given that our objective addressed teachers' continuing education. Participants' reports on activities dealing with the frequentist approach revealed that using an Excel spreadsheet to handle cumulative relative frequencies proved relatively useful to overcome difficulties in the use of technologies, expanding the possibilities for addressing aspects of probabilistic literacy, with reflections and discussions on its application to basic and higher education.

Keywords: Frequentist probability; Simulation; Teacher training.

Resumo

Neste artigo discutem-se aspectos didáticos da abordagem frequentista da probabilidade por meio de simulação computacional. Para tanto, utilizamos um *applet* que simula o jogo *franc-carreau*. Os dados para discussão foram colhidos em oficina para professores. O referencial teórico abrangeu a teoria das situações didáticas, de Brousseau e o modelo de letramento probabilístico, de Gal, adotando-se pressupostos baseados na engenharia didática de segunda geração como metodologia de pesquisa, uma vez que o objetivo era relacionado à formação continuada de professores. Nos relatos dos professores sobre atividades que trabalhavam com o enfoque frequentista, observamos que a manipulação das frequências relativas acumuladas em uma planilha Excel mostrou-se relativamente propícia para superar dificuldades no uso de tecnologias, abrindo possibilidades de ampliação de aspectos do letramento probabilístico, com reflexões e discussões sobre sua aplicação na educação básica e superior.

Palavras-chave: Probabilidade frequentista; Simulação; Formação de professores.

Introduction

The 21st century is immersed in a complex data-driven technological world. Literacy for citizenship and preparing for work require to make students able to make decisions based on data, to analyze, infer and predict, skills that require probabilistic knowledge. On the other hand, technology and games are areas in which students show to be interested today.

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Probability has a wide range of applications in many areas of knowledge (such as physics, economics, meteorology, genetics, insurance). Joining technology and games to approach probability as part of the structure of the learning objectives of different disciplines - including mathematics - can be valuable for teachers and students.

We will discuss here aspects of the computer simulation of the *franc-carreau* game (Badizé, Jacques, Petitpas & Pichard, 1996), proposed by the French mathematician and naturalist George-Louis Leclerc (1707-1788), Count of Buffon, which consists of tossing a coin on a square tile floor. The activity was used in a workshop for teachers who teach mathematics in elementary school and high school, using a free access applet (Erdrich, n.d.), included in the GeoGebra software (n.d.). The applet allows both the manipulation of the "coin" within the "tile" and the tossing of that "coin" as many times as desired. Whoever obtains the *franc-carreau* position wins the game, that is, when the coin stops within a single tile.

Objective

We will discuss the development of probabilistic literacy in Gal's perspective (2005) and the role of computational simulation to make such insertion of probability possible in the teaching of mathematics. Throughout his work, Gal points out the need for the real world to be part of thinking, which includes things thought, evaluated and valued at school, and for this to occur, it is necessary that literacy is developed both in adults and in school-age youth. Gal (2005) provides examples of how perceptions of probability can be affected by context, which includes knowledge of the world, personal dispositions, the interpretation of phrases related to probability or the ability to understand, manipulate and analyze critically quantitative content of information given or implied.

We agree with Gal (2005) when he points out that contexts include the understanding of variation and familiarity with graphics and some descriptive statistics, among others, and when he points out that:

> [...] the knowledge of the probability is of relevance, mainly for the functioning in personal, community and society situations, where situations demand *interpretation* of the probability, statements, *generation* of probability judgments or *decision making*. (p. 49)

Silva (2018, p. 51) highlights that:

> It is necessary for adults to know the notion of randomness, to understand that events vary in their degree of predictability or independence, but also to understand that some events are unpredictable (and therefore that competition from certain events does not mean that they are necessarily related or cause each other).

Later, we will deal in more detail with the components of probabilistic literacy in the terms of Gal (2005), when discussing our theoretical and methodological references.

Some research works

In Brazil, the importance of including content related to probability is expressed in official documents such as the National Curriculum Parameters for Mathematics in Elementary

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Education (MEC, 1998) and the National Common Curricular Base (BNCC) (MEC, 2018). Both explain that one of the purposes of the study of probability is that the student understands that many of the everyday events are of a random nature and that it is possible to identify their possible results. BNCC (MEC, 2018, p. 274) prescribes that students participate in activities in which “they do random experiments and simulations to compare the results obtained with the theoretical probability - frequentist probability” and stresses that theoe activities must compose the “calculation of probability through many repetitions of an experiment (frequency of occurrences and frequentist probability)” (MEC, 2018, p. 304).

Coutinho (2001) already pointed out the importance of the articulation between the classical and the frequentist approaches, with the first focusing on the ratio between the number of successes and the total number of cases when the sample space is equiprovable, while the second, by Bernoulli (1713), emphasizes the value around which the series of accumulated relative frequencies of the event stabilizes.

In this scenario, researchers such as Lopes (2007), Silva and Cazorla (2008), Fernandes, Correia and Contreras (2013), Carvalho (2015) and Batanero, Contreras, Díaz and Cañadas (2013) have dedicated themselves to works involving concepts of probability, indicating teaching and learning possibilities. However, probability is a topic that students find difficult to learn (Ben-Zvi & Garfield, 2004; Koparan & Kaleli Yılmaz, 2015). According to Koparan and Kaleli Yılmaz (2015), the use of engaging learning activities in the teaching of probability has had a positive effect on the mathematical knowledge of students in several countries, besides improving thinking skills. Gürbüz (2008) advocates the use of computers in activities and games as a support for teaching probability, together with the different teaching techniques that this technology environment can provide.

Theoretical and methodological framework

The research on which this article is based was developed in the light of the theory of didactic situations (Brousseau, 1986), using as methodology the assumptions of the second generation didactic engineering - didactic engineering for training, according to Perrin-Glorian (2009). Already discussing such references in the context of the workshop developed on geometric probability, we will present the specifics that were worked on and their foundation.

In the adidactic situation (Brousseau, 1986) built for the research, the return phase occurred when the problem of calculating a probability in a geometric context involving the *franc-carreau* game was presented to the 20 teachers participating in the workshop.

As for the dialectics of action, formulation and validation, when contacting the game, the participants resorted to their knowledge to build their own strategies to solve the problem of estimating or calculating the probability targeted: that the coin would stop in the *franc-carreau* position, that is, freeze within a single tile. The consequent natural behavior is verbalization to the partner, in an attempt to explain the strategy formulated and to convince. Such verbalization configures the second component of the dialectic that is established between the action phase and the formulation phase, also suffering the effects of the validation phase acting in this dialectic - that is, the participants seek to validate their results through their own

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knowledge. For example, they compare results against each other and check if their results are less than 1, among other validation tools available as stabilized knowledge.

The last phase of the situation is that of institutionalization, in which the concept of probability and its classical and frequentist approaches and the role of computer simulation in the estimation of frequentist probabilities were discussed. The didactic aspects to approach these concepts in the high school classroom could also be discussed and systematized with the teachers in this institutionalization phase (Details of what is actually done are presented below).

The aim of the workshop was also to provide teachers with didactic-pedagogical resources for their classes, which characterizes the use of the assumptions of second generation didactic engineering, in the terms of Perrin-Glorian (2009), who exposes that in the second generation didactic engineering (which she designates as training didactic engineering - FDE) the theoretical control is carried out in the following dimensions:

- Analysis of knowledge: In this article, we present the frequentist approach as we wish to approach it, as well as the simulation tool used.
- Definition of the situation, of the *milieu*: In this item, Perrin-Glorian (2009) admits concessions to FDE, given its specificities - and here we will work with the assumptions of an FDE. We emphasize, however, that the *milieu* is mainly constituted by the specific knowledge of the participating teachers, by their didactic knowledge of this content, by the knowledge of the partner teachers (as they worked in pairs) and by the knowledge available in the computational tools (simulator and spreadsheet with pre-stipulated commands).
- Knowledge about participating teachers: From the registration forms, we only knew the educational levels at which they taught mathematics.
- The role of the teacher (in this case, the researchers-authors of this article): In this item, Perrin-Glorian (2009) highlights the need to characterize situations in a clearer and lighter way. In our workshop, we chose to act as mediators, intervening in each pair only to elucidate specific doubts and making local institutionalizations whenever necessary.
- Institutional restrictions: In discussions with teachers, we also focused on situations in which they did not have computer labs for their classes, cases in which they could use other simulators available on smartphones. We even discussed the possibility of resorting to the use of mechanical simulations, in which they would use the sum of the results of the flips made by each of their students, to obtain a total of repetitions acceptable from the point of view of the frequentist approach of probability.

Probabilistic literacy

Both Caberlim (2015) and Silva (2018) discuss probabilistic literacy in computer simulation situations, noting the effective contributions of this resource to the development of the competences and skills that constitute this literacy. In their work, they consider the model proposed by Gal (2005).

For Gal, probabilistic literacy is fundamental and we agree with his view when he affirms the importance of the attention that we must pay to the demands of the real world,

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although these should not "be the only factor influencing curriculum planning or teaching practices", but rather, "be part of the considerations that guide what should be planned, taught, evaluated and valued in the classroom" (Gal, 2005, p. 39).

Probabilistic literacy is composed of knowledge and dispositions that are mobilized by the subject in an articulated way, and to enunciate these elements Gal (2005) is based on the same logic that he had used in the construction of his statistical literacy model (Gal, 2002): five knowledge and three dispositional elements.

The knowledge elements are:

- big ideas (randomness, independence, predictability/uncertainty);
- calculating probabilities (ways to find or estimate the probability of events);
- language (the terms and methods used to communicate chances);
- context (understanding of the role and implications of probabilistic results and messages in several contexts and in personal and public discourses);
- critical issues (questions to reflect on when dealing with probability) (Gal, 2005, p. 46).

The dispositional elements are:

- critical posture;
- beliefs and attitudes;
- personal feelings about uncertainty and risk (for example, risk aversion). (Gal, 2005, p. 46)

Observing these eight points in relation to the simulation of the *franc-carreau* game, we can identify contributions to the development of the subjects' probabilistic literacy. Coutinho (2001) and Caberlim (2015) observed this fact with students from the first and second grades of high school, respectively. We are currently in the process of developing work with mathematics teachers in initial and continuing education, both in face-to-face and distance learning.

Particularly for the *franc-carreau* game, in our *a priori* analysis, we have the following points:

- Great ideas: With each coin toss, the system returns to its initial state, which makes the tosses independent and pseudo-random (an algorithm randomly generates a position for the center of the coin). There is no possibility to calculate or determine the exact position that will result from the coin tossing.
- Calculation of probabilities: The probability of the *franc-carreau* event can be estimated by observing the stabilization of the relative frequencies accumulated (*a posteriori* or frequentist probability) or by considering the measures provided by the system (*a priori* probability). In the system we are presenting, the *a priori* calculation results in 0.36, while the frequentist estimation can be estimated by 0.35 or higher.
- Language: Use of accessible language, for example designating the activity as a game of coins, tiles, grouting, freezing, probabilities, frequency stabilization.
- Context: The use of game contexts is quite common both at school and in the daily lives of students and teachers, as pointed out by several studies, including Rodrigues's work

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(2018), who reports that teachers who participated in his research recognize the randomness more easily in games than in other contexts.

- Critical questions: The *franc-carreau* game originated in the 18th century, a period of social problems due to a pronounced addiction to gambling. The activity involving this game can provide a moment of debate about criticality, citizenship, the role of social relations and the social and economic role of games in today's life.
- Critical posture: The debate mentioned before can promote critical posture in the face of game situations.
- Beliefs and attitudes: The computer simulation of a game favors the manifestation of beliefs and attitudes related to them to a lesser extent than the manipulation of physical objects for the realization of the game (in this case, the manipulation of coins).
- Risk aversion can be seen in simulations that allow manipulating game parameters, such as the radius of the circle that represents the coin and the side of the square that represents the tile. In these cases, actions that seek to minimize the radius and maximize the side can be observed, in order to guarantee the occurrence of the *franc-carreau* position.

The simulation we present therefore allows us to work with each of the elements pointed out by Gal (2005) for the constitution of probabilistic literacy. Such work will certainly depend on the practices of the teacher responsible for the content in the classroom.

The role of simulation for school probability in gaming environments

Several researchers propose the use of computers in teaching and learning concepts that mobilize probability to facilitate the understanding of abstract aspects or those that are more difficult to understand (Mills, 2002; Gürbüz, 2008). Batanero and Díaz (2007) advocate that students do simulations that help them solve simple probability problems. Borovcnik and Kapadia (2009) consider that combining simulation and technology is the most appropriate strategy to focus on concepts and reduce technical calculations.

Citing Heitele's 1975 study, Fernandes, Batanero, Contreras and Díaz (2009, p. 162) highlight that:

> Through simulation we match two different random experiences, with the proviso that each elementary event of the first experience corresponds to one and only one elementary event of the second, so that the events matched in both experiences are equiprovable.

Besides the positive points of using computer simulation, these authors also indicate negative points, for example, that the frequentist perspective addressed by the simulation does not consider other views of the concept of probability and only estimates the theoretical value of probability, which can eventually deviate from the true value if the number of experiments is small. In fact, that is a problem we observe quite frequently in Brazilian textbooks, which, when adopting a frequentist approach, take as bases a poor number of repetitions of the random experiment, as Coutinho (2013) found out. When analyzing three didactic collections, Carvalho, Silva and Paraíba (2016) found that about 87% of the activities that deal with

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probability involve its classical meaning. They also highlight that, among the 156 activities found, only three are in a geometric context. For these researchers:

> [...] students are not presented with explicit experimentation activities that allow them to move from experimental to theoretical, that is, a coherent articulation between the frequentist and classical meanings, such as the data entry, registration in tables and following for graphic construction. (Carvalho, Silva & Paraíba, 2016, p. 8)

But what would be a sufficiently large number? Just remember the definition given by Ventsel (1962) in *Théorie des Probabilités*, that the probability can be none other than the number around which the values of the frequency series stabilize. In current terms: $P(A) = \lim_{n\to\infty} Fr_n(A)$.

As the probability value is unique (see axiomatic definition), the frequentist approach gives us the same value for $P(A)$, namely: $P(A) = \frac{n.sucessos}{n.total\ de\ casos}$, being the sample space equiprovable, which is the definition proposed by Pascal and Fermat, which, axiomatized by Laplace (1814) in his *Essai Philosophique sur les Probabilités*, it is still used in schools today.

The game

The *franc-carreau* game consists of establishing the type of tile and effectively tossing a previously chosen coin (the value of the coin is fixed, since the radius of the circle representing it will influence the probability sought). The manipulation of the coin must also enable the construction of a hypothesis for the value of the probability sought, even if we take generic measures: a square tile of side *a* and a coin with radius *r*. The manipulation of the coin within this tile allows the visualization of possible strategies for solving the proposed problem (Figure 1).

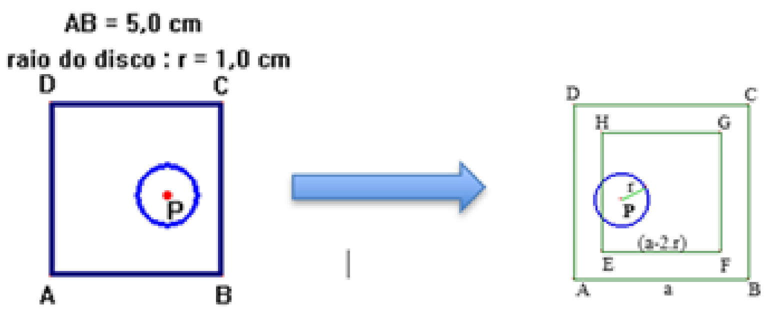

Figure 1. Solution scheme of the *franc-carreau* game.
Source: the authors.

When visualizing the relationship between the radius *r* of the coin and the side *a* of the square (tile), the student realizes whether the center of the coin is located inside the EFGH square after immobilization, obtaining the *franc-carreau* position, which allows the student to build the relation derived from the second principle enunciated by Laplace (ratio between the number of successes and the total number of cases), adapted to the geometric probability: being FC the *franc-carreau* position, then $P(FC) = \frac{área\ de\ EFGH}{área\ de\ ABCD}$. Caberlim (2015) found that this

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procedure was not problematic for students attending the second grade of high school, confirming what was observed by Coutinho (2001) in first graders of this level of education. Such a strategy is a type of validation of the results observed in the simulation, presented after this section, as stated by Coutinho (2001) - that is, if we consider the action-formulation-validation dialectic discussed when we present the adidactic situation, the comparison between the observation of the stabilization of the series of accumulated frequencies (frequentist approach) and the value obtained in the *a priori* calculation (classical approach) is configured as an effective validation tool.

To estimate the probability of obtaining the *franc-carreau* position, we need to observe the value around which the accumulated relative frequencies stabilize, in the terms proposed by Ventsel (1962). For this purpose, 25 repetitions were performed with 40 flips each, in order to total 1,000 flips, which is a statistically acceptable value from a teaching perspective, since the number of repetitions tends to infinite, according to a frequentist definition of probability. It is worth mentioning that in pragmatic terms, stabilization is observed from 1,000 repetitions.

Table 1. Simulation results of 1,000 flips in the *franc-carreau* game.

Flips	Accumulated flips	Successes	Accumulated successes	Accumulated relative frequency (successes)
40	40	4	9	0.225
40	80	5	23	0.2875
40	120	4	33	0.275
40	160	4	47	0.29375
40	200	3	55	0.275
40	240	10	70	0.291666667
40	280	5	84	0.3
40	320	9	100	0.3125
40	360	4	109	0.302777778
40	400	7	123	0.3075
40	440	4	133	0.302272727
40	480	11	151	0.314583333
40	520	7	161	0.309615385
40	560	8	180	0.321428571
40	600	2	185	0.308333333
40	640	6	199	0.3109375
40	680	8	218	0.320588235
40	720	7	231	0.320833333
40	760	6	245	0.322368421
40	800	7	258	0.3225
40	840	8	279	0.332142857
40	880	5	294	0.334090909
40	920	7	309	0.335869565
40	960	7	328	0.341666667
40	1000	12	352	0.352

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Source: the authors.

Building a line graph (Figure 2) makes it easier to see the stabilization of relative frequencies.

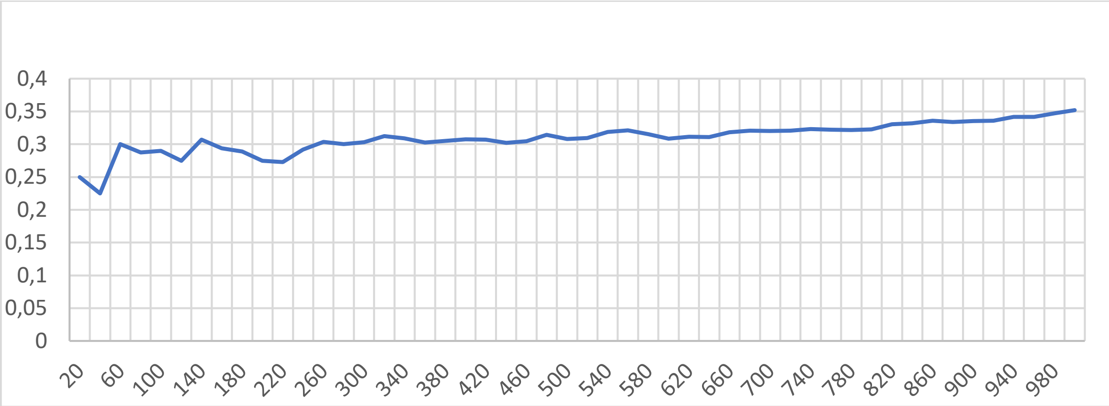

Figure 2. Accumulated relative frequencies.
Source: the authors.

We can see that the graph tends to stabilize around 0.34, but for a better result it would be necessary to have more flips, which in the classroom would not always be feasible. In the case of a sufficiently powerful computer system, we could expect to have 3,000 or more repetitions of the random experiment that we want to simulate. Note that simply assuming the value observed for the frequency relative to the end of the 1,000 repetitions of the experiment does not necessarily reflect the value of the probability sought, as the line graph (Figure 2) indicates that there is not yet sufficient stabilization for an effective estimate.

Making 2,000 flips makes the stabilization of the relative frequencies accumulated around 0.35 (Figure 3) clearer, without the need for analytical resolution, as pointed out by Biehler (1991). We also highlight that stability is observed after 1,000 repetitions, as shown in Figure 3.

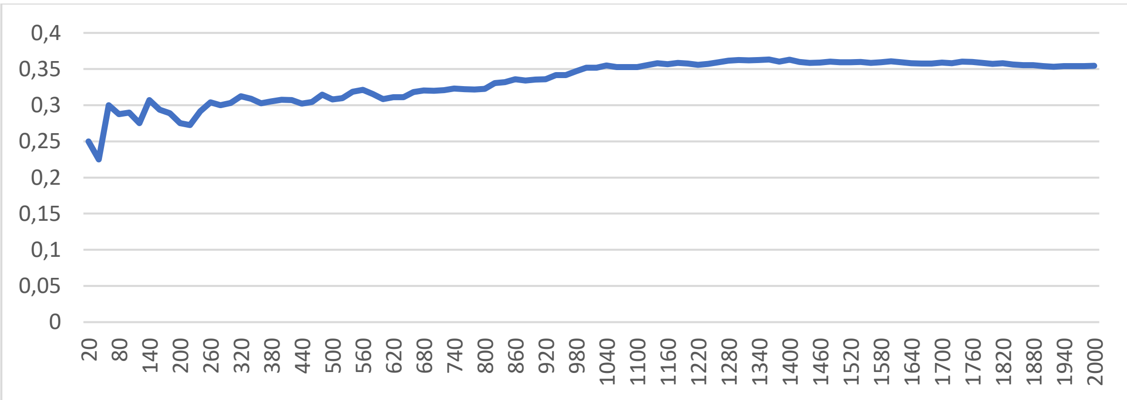

Figure 3. Frequency stabilization with 2,000 flips (simulation).
Source: the authors.

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The most usual is that each student performs a certain number of repetitions and the teacher organizes the table adding the results of all students, in order to achieve the largest possible number of repetitions of the random experiment as a whole - in this case, the coin toss with verification of its position on the tile after its immobilization.

Note also that the simulation using this applet does not require mastery of the *GeoGebra* software. Such mastery, however, would allow the teacher to build other simulations to with his students.

Discussion of the results

Workshop participants began making flips on the applet, whose interface was translated into French. At first, they did not find it difficult to handle it and identified all the information that the interface offers (Figure 4).

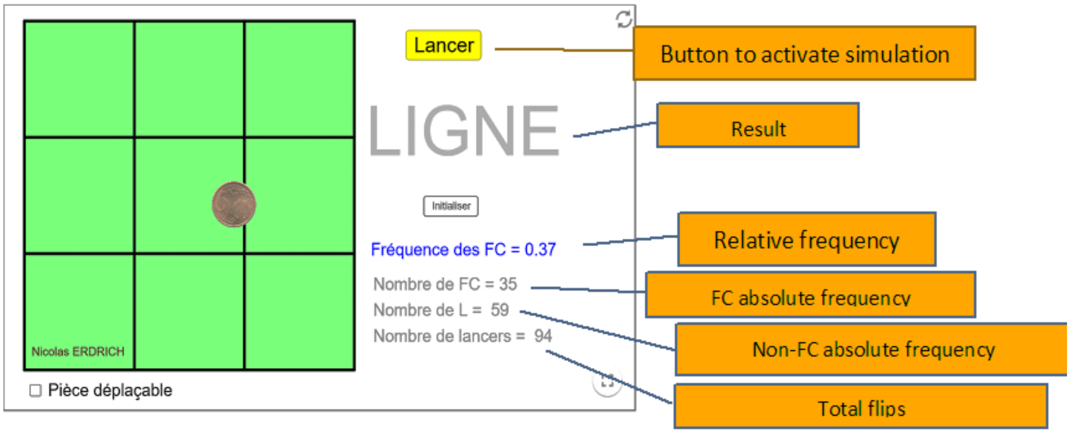

Figure 4. *Franc-carreau* game interface information provided to participants in printed material.
Source: the authors.

The participants started the simulations with 10 series of 20 flips and realized that these would not be enough to stabilize the series of relative frequencies, as we expected (Table 1) in our *a priori* analysis, as the relative frequencies varied greatly between flips, in addition to the fact that the stabilization of the series of relative frequencies is more noticeable for smaller, even unitary intervals, according to Ventsel (1962). Then they made flips by 40s and identified the need to increase both the number of observations and the number of series, concluding that flips by 100s would be enough for stabilization. Again, the perception of the need for a greater number of repetitions of the random experiment agrees with our *a priori* analysis, whereas the interval between observations is contrary to what we expected.

Most participants found the applet easy to understand, and there seems to be a consensus on what one of the pairs expressed in their final report:

> *Pair A: The applet is easy to apply. The justifications for the results indicated are more evident when exposed in Excel, which interprets the calculations of the relative and accumulated frequencies and exhibits the results comparatively.*

These participants stated that the game is not enough to show the calculation of the probability by the frequentist approach and that it was the association between calculations,

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tables and graphs that allowed them to achieve understanding. Such a conclusion was only reached at the end of the workshop, because, during its course, we realized that these participants were in doubt whether stabilization would actually occur, which leads us to suppose that, although they were teachers, they usually did not carry out a sufficiently large number of repetitions of the random experiment so they could observe stabilizations. This delay in perception was also due to the very wide range of amplitude for the observations of the repetitions of the random experiment. This refers to what Carvalho et al. (2016) point out: there are no such activities in the textbooks. We can thus infer that these teachers do not use such experiments in their practices; that they may not have experienced them in their initial training; that the number of repetitions present in the few activities with which they had contact was not enough; or that they did not address the frequentist approach properly.

In this stage of simulating in the applet and recording the results in an Excel spreadsheet, we observed that the greatest difficulty encountered by teachers was to use the Excel tool to record the relative and accumulated frequencies as a procedure that facilitates the calculations. Inserting the columns with the formulas for those frequencies took time. Even with the help we gave them in PowerPoint showing the steps to be followed, some pairs chose to do manual calculations. Therefore, the study revealed that most of these teachers, although they report working with this resource, do not mobilize such knowledge in a stable way. They are not used to (oral testimony, not registered) using Excel, often due to institutional limitations, as we infer in the presentation of our methodology**.**

The idea of the workshop was to discuss the teaching and learning of probability when the activities are placed in the form of problem solving that involve computer simulation, in the context of the articulation of two approaches (classical and frequentist). The applet proved to be motivating for the participants, but we observed that at no time did they resorted to the calculation of the classical probability to know which number the simulation stabilization should approach. When asked if any of the pairs had calculated the probability of *franc-carreau* occurrence, all responded negatively and, when invited to calculate, they showed difficulty in organizing a strategy that would provide such probability, contrary to what was observed by Coutinho (2001) and Caberlim (2015). At that time, we had to interfere to raise concepts of geometric probability, for whose calculation the data are found in the applet itself (Figure 5).

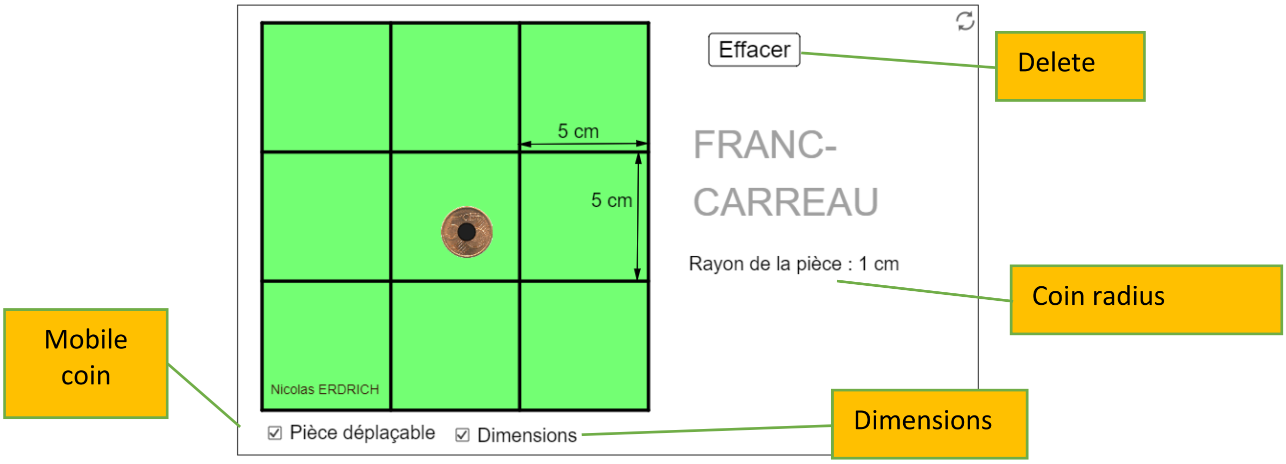

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Figure 5. Information about the *franc-carreau* game translated orally for the participants.
Source: the authors.

We raised some questions so that the participants could identify what elements they would take to calculate the probability. Each of them collaborated with the group until it was accepted that for such a calculation we would need, in principle, the area of only one of the nine squares (Figure 1) and, thus, analyze the "path" that this coin could take "not to touch" the sides of the square. After that, the understanding that this strategy could be extended to all nine squares without interfering with the probability of the ocurrence of the *franc-carreau* position in each of them became more natural, and we built together the calculation on the blackboard, arriving to the following conclusion: $P(FC) = \frac{9.3^2}{9.5^2} = \frac{9}{25}$, that is, $P(FC) = 0.36$. This observation result led us to suppose that those teachers do not associate the geometric context with the classical probability, due to the already mentioned fact that the textbooks work little with geometric probability.

During the workshop, there were many questions from the participants, of which we highlight those that dealt with probability under a frequentist approach, possibilities of using technology in teaching probability, other calculations of geometric probability and indications of work for the various grades of basic education for the probabilists concepts involved in this workshop. A general doubt was about the use of the electronic spreadsheet and from which level of education they could use it, as they had experienced some difficulties in dealing with it.

The simulation seems to have awakened those teachers to the possibilities of practice with their students, but they recognize that the applet imposes a certain initial difficulty for the visualization of the *franc-carreau*, which one of the pairs pointed out in the final report:

> *Pair B: The geometric space is not shown (useful space considered for the probability), as simulation, it is a valid resource, but for a better understanding of the results it is necessary to complement the Excel graphically. There is no view of the coin in the area of each square that demonstrates the maximum (useful) area of the probability.*

The participants discussed the possibility of applying this activity with simulation of the *franc-carreau* game, use of the applet and use of Excel to build the spreadsheet, and all agreed that the activity could be applied to basic education students, but left in their reports some observations on how to apply it, distinguishing between elementary and high school students. According to such reports:

- Elementary school students should test the coin flip in concrete situations before using the applet. They suggest the construction of a physical model with squares and the use of a coin so that they first understand what the franc-carreau game means , the concept of geometric probability and randomness; so that they also understand the importance of the reproducibility of the experiment, in order to understand the frequentist view to, afterwards, have contact with the simulation with the construction of Excel spreadsheets.

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- In the case of manual simulation, they suggest that students in a class do several simulations and then, together with the teacher, manage to join the totals observed by each student in a single table, so that they perceive the stabilization. However, they believe that without computational resources the activity would require more work and a lot of time to be carried out, since during the workshop, stabilizations occurred in approximately 2,000 flips.
- Before being introduced to the *franc-carreau* game, students should know geometric probability, as they themselves consider that almost no one works with this perspective of combining classical probability with area calculations.

In the suggestions that were discussed in this workshop with teachers, we observed the possibility of using the game with concrete situations, aiming to work on the geometric probability, and build a board and use 10 cents, 50 cents and a real coin, keeping the same square of 15 cm × 15 cm, formed by nine squares of 5 cm × 5 cm. They believe this would be a great activity for students in the early years of elementary school.

All reports pointed to the activity as conducive to discussing the classical probability combined with the geometric probability treated by the frequentist view, which allows us to perceive the random character of the probability, with the possibility of associating the construction of statistical tables and graphs. However, the participants state in their reports that the use of technology to develop these concepts involved in the activity would require them to prepare it so that they can associate it with the reality of their students and to train their own skill with the tools before taking it to the classroom.

In general, those teachers consider that activities that involve games are motivating for both basic education and higher education students. They realize that with the use of technology, there is greater student engagement in the activity. Such consideration also brings us to Borovcnik and Kapadia (2009), who not only consider it a challenge to teach probability in order to enable students to understand and apply it, but believe that there is a need to focus on creating approaches to the probability that are more accessible and motivating, besides the fact that the use of technology helps to reduce technical calculations and to focus the learner on the concepts.

Some considerations

The discussion about the development of probabilistic literacy and the role of computational simulation to enable such insertion of probability in the teaching of mathematics led to reflections among participants in an activity carried out in a workshop given to teachers of basic and higher education with computational resources. The activity proved to be conducive to reflections on classical, frequentist and geometric probability using the *franc-carreau* game and using an applet and an Excel spreadsheet to record and visualize the establishment of the frequency of occurrences obtained with simulations. Carrying out the activity in this environment offered teachers the opportunity to organize and configure their own knowledge and evaluate the concepts of probability from different perspectives, enabling

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the development of elements of the probabilistic literacy model, in the terms proposed by Gal (2005).

Teachers managed to advance in such a construction, despite difficulties in handling the Excel spreadsheet. They claimed to know and work with accumulated relative frequencies when addressing frequentist probability with their students. The use of the applet was not problematic, in agreement with the findings of Biehler (1991) and Batanero and Díaz (2007), but the difficulty in perceiving the stabilization of the frequencies proved to be problematic, contrary to Ventsel's (1962) findings, which it was a consequence of the difficulty in dealing with Excel. The *a priori* calculation of the probability was not done spontaneously, although the measures of the squares were provided by the researchers, which indicates they were not familiar with the articulated use of the classical and frequentist approaches, in contrast to what was observed by Coutinho (2001). The perspectives that are opened include the development of new, longer training courses, addressing the points identified here as fragile for the development of the probabilistic literacy of the participating teachers.

Like Batanero, Contreras, Fernandes and Ojeda (2010), we believe that teachers need support and training to have a balance between intuition and rigor when teaching probability, although, unfortunately, due to time constraints, teachers do not always receive a good preparation to teach probability in their initial training. However, situations like the one experienced in the workshop can serve to simultaneously increase the teacher's knowledge about probability and professional knowledge, that is, to increase the possibility that he will take such practices to his classes.

It should be noted, as a positive aspect, that the computational environment provided participants with the opportunity to observe situations for larger samples. This was because the high number of attempts in the simulation could take place in a short time. Such simulations involve the visualization that is necessary for the study of aspects of randomness and frequency stabilization after a sufficiently large number of repetitions of the random experiment in experimental probability problems, which is not always possible to achieve in traditional classroom environments. Furthermore, the simulations contributed to the creation of an environment for class and group discussions.

We suggest that teachers recognise the worth of educational materials that are appropriate to the different teaching methods and use them, aiming to promote greater effectiveness in teaching and learning probability, which can occur in a pleasant way. Furthermore, teachers can disseminate their findings and achievements. We emphasize the need for more research to clarify the essential components of teachers' preparation to teach probability, as well as studies that make it possible to explain appropriate methods for teaching each component.

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