Zetetiké

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Development of didactical knowledge of teachers in statistics: A teacher education experience

Desenvolvimento do conhecimento didático de professores em Estatística: uma experiência formativa

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Abstract

This article aims to analyze the development of didactical knowledge of teachers attending an in-service course focusing on statistical representations and investigations, two fundamental aspects of statistics education. This is a statistics course in a specialization program for middle school mathematics teachers, including 13 participants. The data collected was obtained from sessions by a logbook, audio recordings, and document collection and by interviews. The course was structured to articulate didactic knowledge from activities directed to teaching practice. The results show that the teachers deepened their knowledge of statistical representations and how to teach them. About statistical investigations, the teachers emphasize the importance of valuing these activities for the development of citizens who are critical and capable of dealing with information.

Keywords: Statistics; Statistical representations; Statistical investigations; Didactics knowledge.

Resumo

Este artigo tem como objetivo analisar o desenvolvimento do conhecimento didático de professores em formação continuada, com foco nas representações e investigações estatísticas, dois aspectos fundamentais da Educação Estatística. A formação é relativa à disciplina de Estatística de um Curso de Especialização para professores de Matemática do Ensino Básico, com 13 participantes. Os dados foram recolhidos no decorrer das sessões através de um diário de bordo, gravações de áudio, recolha documental e entrevistas. A disciplina foi estruturada de modo a articular conhecimentos didáticos a partir de atividades direcionadas à prática letiva. Os resultados mostram que os professores aprofundaram seus conhecimentos sobre as representações estatísticas e sobre o modo de as ensinar. Em relação às investigações estatísticas, os professores destacam a importância da valorização destas atividades para a formação de cidadãos críticos capazes de lidar com a informação.

Palavras-chave: Estatística; Representações estatísticas; Investigações estatísticas; Conhecimento didático.

Introduction

Nowadays, statistics is directly linked to daily activities and professional activities, so its teaching should be directed to develop in the student ability to make appropriate decisions in the light of available information. However, despite the importance of developing ability to deal with quantitative and qualitative data, the inclusion of statistics in the mathematics curriculum is relatively recent.

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In Brazil, only in 1997, with the publication of the National Curricular Parameters (NCP), statistics topics were introduced in the curriculum. In elementary school (1st and 2nd cycles), this was achieved through the "Treatment of Information" block, and in High School (3rd cycle) on the "Data Analysis" axis. According to Borba, Monteiro, Guimarães, Coutinho and Kataoka (2011), this inclusion was incipient when compared with other fields such as Algebra and Geometry. More recently, the National Common Curricular Base (NCCB) was published, whose main objective is to structure the mandatory national reference content in public and private schools. The document lists "Probability and Statistics" as one of its five thematic units. In this document, there is an incentive for students to be progressively involved in activities of investigation nature.

Considering that this is a recent subject in the curriculum, many teachers have a cursory knowledge of a variety of concepts, representations, and procedures used in statistics, which may result in hesitation about how to teach them. As Borba et al. (2011) affirm, the difficulties that teachers face in approaching statistical concepts derive from insufficient initial and continuing training regarding the analysis and practical use of teaching materials. Batanero, Godino and Roa (2004) consider that mathematics teachers who teach statistical concepts generally adapt their conceptions of mathematics teaching to statistics teaching, giving rise to a didactic aligned with calculations and procedures. Consequently, the meaning of statistical concepts is impoverished and the uncertainty notion inherent in the data interpretation proper to statistics is lost.

Thus, it is important to reflect on the knowledge teachers need to articulate to teach statistics. In addition, it is necessary to create opportunities for continuing and/or specialized training in which teachers can enhance this knowledge and change their conceptions about teaching statistics from their experiences. In this paper, we present a course inserted in a specialization program for mathematics teachers in order to constitute a teacher education process in statistics, trying to identify their learning. *Our aim is to understand how teachers, attending to a statistics training experience, develop their didactic knowledge about statistical representations and statistical investigations.*

Statistical representations and investigations

Statistics is a topic that has been included relatively recently in the curriculum, which has a different reasoning than what is done in mathematics. While mathematics uses deterministic procedures to solve its problems, statistics encompasses concepts of variability and randomness, resulting in activities of different nature (Ponte & Fonseca, 2001).

Although there are several ways to teach and learn statistics, it is important that the student develop the ability to handle available information critically and meaningfully, with statistical literacy playing a central role in this process (Franklin et al., 2005). Generally speaking, it is understood that literacy refers to a set of principles, ideas, interpretative and communication skills and abilities necessary to efficiently handle information involving quantitative data arising during life and professional situations (Martins & Ponte, 2011;

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Steen, 2001). In this teaching-learning process of statistics with valorization of literacy development, two fundamental aspects are evidenced: representations and statistical investigations.

Statistical representations

Considering they present a wide variety of information in a compact way, graphical representations, tables and diagrams emerge in various daily contexts and in statistics teaching. These representations allow the communication, classification and comparison of data, and assume a mediating character in the process of data interpretation (Curcio, 1989; Monteiro & Ainley, 2006). Martins (2018) affirms that components of a graphical representation (graphical specifiers, labels, title and background) interfere with the type of reading and interpretation of this representation. For the author, it is possible that the reading of a graphical representation is based on formal concepts of mathematics. However, often the visual and symbolic aspects inherent in representation receive greater importance. For Carvalho, Monteiro and Campos (2010), there is a dynamic process in graphical interpretation, thus requiring an interaction between conceptual and visual aspects, in which experiences and knowledge are mobilized and new meanings are constructed in the interpretation domain.

Curcio (1987, 1989) lists three cognitive levels for reading and interpreting graphical representations. Level 1 (reading the data) requires a simple reading of the data, taking into account only what is explicitly represented. Level 2 (read between data) involves the use of mathematical comparisons, concepts and techniques. Level 3 (reading beyond data) is associated with the use of predictions or inferences. For Ayoama (2006 and 2007), the development of statistical literacy is associated with a level at which students are able to read the values and trends evident in graphical representations, presenting the literal understanding of contextual meanings through what is explicit in the representation.

Statistical investigations

According to the NCTM (2007), statistical research develops in the student the ability to formulate questions that can be addressed through data, so that it is possible to collect, organize and represent data in a relevant way to solve the questions. According to this document, students should be able to select appropriate statistical methods for data analysis in way to make and evaluate inferences later, as well as data-based predictions. The GAISE Report (Franklin et al., 2005) clarifies that statistical investigations involve a cycle composed of four main steps: (i) formulation of questions; (ii) data collection; (iii) data analysis; and (iv) interpretation of results.

Again, according to the GAISE Report (2005, p. 11), “Formulating a statistical question requires an understanding of the difference between a question that anticipates a deterministic answer and a response based on varying data.” One may consider that variability grounds the basis of statistical research. In the data collection phase, planning is

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necessary to collect appropriate data in order to proceed with the organization and processing of this data through graphs, tables and statistical measures. This whole process allows the interpretation of the results, besides the formulation of conclusions. Therefore, investigations allow the student to be closer to the statistical process in order to see beyond the data and has significant learning, although naturally, in the context of a much less sophisticated statistical knowledge than a professional. It should be noted, however, that statistical investigations are hardly addressed in initial teacher education (Henriques & Oliveira, 2013).

Didactics knowledge and teacher education in statistics

Several authors have investigated the question of what knowledge should be taught by teachers. In this article, we will focus on knowledge directed, especially, to teacher's actions. In this context, Ponte (2012) considers four main domains: knowledge of mathematics for its teaching; knowledge of the curriculum; knowledge of the student and his/her learning; and knowledge of educational practice (Figure 1). According to the author, what makes different this model from the others is the fact that it clearly assumes the existence of a central core: the knowledge of educational practice, better known in English as instructional knowledge. This central core rests on the other three dimensions. Another divergence is that in this model there is no separation between the dimensions of knowledge, all being present at the same time in teaching activities. Although focused on teaching mathematics, the didactic knowledge domains described by Ponte (2012) can be adapted to different areas and themes, including studies aimed at Statistical knowledge (Quintas, 2017). Thus, we relate the dimensions of didactic knowledge proposed by Ponte (2012) to Batanero (2002) perspectives for the professional knowledge of the teacher who teaches statistics.

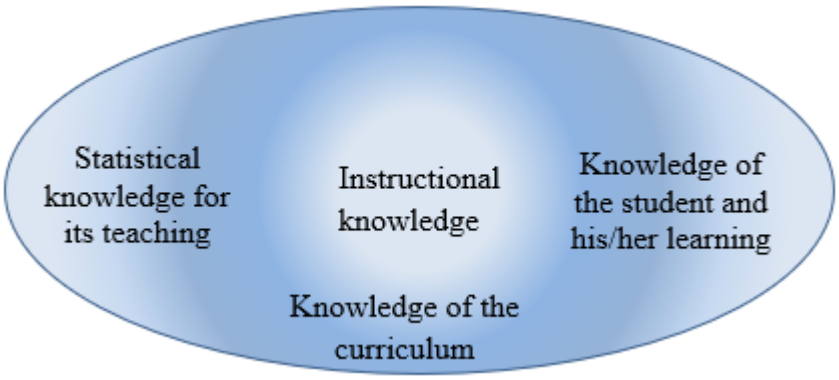

Figure 2 – Didactical knowledge dimensions of statistics teacher

Source: adapted from Ponte, 2012

Regarding to the aspect related to knowledge of mathematics, Ponte (2012) asserts that his model does not cover the knowledge of mathematics as a science, but the interpretation that the teacher makes of mathematics as a school subject, including notions of its varied representations and internal and external connections with different concepts. In the field of *Statistical knowledge for its teaching*, there is a need to develop the ability to reflect

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epistemologically on the meaning of concepts, in this case relating the nature of stochastic knowledge, its development and evolution (Batanero, 2002).

For Ponte (2012), in the *knowledge of the student and his/her learning processes*, students are seen as people with interests, tastes, reactions, values and cultural references. In addition to cognitive limitations of students, all these points are considered in the learning process and are decisive for the choice of activities proposed by the teacher. According to Batanero (2002), it is necessary to develop the perception of the difficulties and limitations related to learning of teachers, as well as problem solving strategies that guide more effectively the teaching and learning and the evaluation activity.

The domain related to *knowledge of the curriculum* includes the objectives, the organization of the contents, the ability to work with different materials and the knowledge of various ways to perform an assessment. Decisions about the contents and the time required to manage them, as well as what to prioritize and how, are also part of this knowledge. When considering that curriculum perspectives are constantly changing, it is important for the teacher to follow these changes. Thus, it is necessary to analyze the transformations of knowledge to adapt it to different levels of education, so there may be a reflection on the different possible levels of understanding related to the same knowledge, including its assessment and reflection on how a given concept may be taught to a particular person (Batanero, 2002).

In the field of didactic knowledge called instructional knowledge, Ponte and Oliveira (2002) say that this is a fundamental domain of didactic knowledge, as it covers lesson planning, the conception of tasks and all other issues that involve the classes, considering the time before, during and after each teaching-learning moment. In this way, Batanero (2002) highlights the observation of various methodological resources directed to the improvement of practice, as well as the analysis of the curriculum and learning situations.

Didactic knowledge of the teacher is elaborated from the first moment of his/her teacher education. However, the reality of each teacher, full of complexities and doubts about teaching, is what determines the direction it takes during of his/her teaching activity. In other words, his/her techniques, approaches and analysis depend on the context in which he/she operates.

A learning process centered on the transmission of concepts is not likely to engender, in the teacher, the ability to promote an active learning of students, especially with regard to statistics. Thus, we highlight the learning processes in which authentic classroom situations configure a material full of meanings capable of providing an environment of reflections and interactions among teachers (Day, 2001). An example of activity for this type of teacher education process is given by Smith (2001), who highlights the appropriate selection of tasks to use in the classroom as a strong reflection exercise on the goals to be achieved, as well as on previous knowledge and skills that are needed to the students. Specifically, in teacher education for teaching statistics, several undergraduate courses show that a training centered

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on active position of the participant is possible, in which content, pedagogy and assessment are the focus of discussion (Groth & Xu, 2011; Quintas, 2017).

Methodology

This study follows a qualitative and interpretative approach in order to know the meaning that participants attribute to their experiences in a context in which the process is observed and not only the results (Bogdan & Biklen, 1994). We seek to verify how teachers develop their knowledge and meanings in relation to their work, paying special attention to their interpretations and perspectives concerning the same specific situation (Cohen, Manion & Morrison, 2001). In this way, we examine the situations and the development of knowledge of teachers through their visions, interactions and reflections during a teacher education experience. Identification of these factors is supported by theoretical knowledge about statistics teaching, statistical knowledge to teach, as well as the teacher education of teachers who teach statistics.

This training experience took place in a postgraduate course (specialization) for mathematics teachers from basic education, in a statistics course. The program is offered by a college located in the West Zone of Rio de Janeiro, recognized for focusing on initial and continuing teacher education. The program structure consisted of 8 courses of 36 hours related to mathematics, in addition to 4 seminars related to didactics and the elaboration of a monograph. In courses related to mathematics, the curricular proposal was centered on a deepened approach of concepts concerning to the basic education curriculum. In this case, the course syllabus was structured in such a way as to enable the articulation between the subject and didactics of statistics.

The group of trainees was composed of 13 mathematics teachers who mostly worked in the second cycle of elementary school, with the exception of Giulia[3], who was not acting as a teacher, and Diana, who worked as a teacher in the 1st cycle of elementary school. The first author of this article was the teacher in this training experience and already knew the participants of curricular courses previously completed during the program.

In the initial phase of the statistics course, most participants reported that they had difficulty working on this subject. In this sense, several teachers reported that in their initial training, they had not analyzed the development of statistical literacy or the role of statistical investigations in the context of meaningful learning. They also reported that the way of teaching statistical concepts is generally not dealt in their professional contexts, being “a subject left aside”. Therefore, the statistics course was formulated based on the needs indicated by the participants, considering the difficulties which they pointed out, especially valuing aspects of didactic nature that teachers had not seen in their initial training or other teacher education processes.

[3] All the participants have fictitious names.

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Focusing on the development of Statistical literacy in school environment, the work realized with the teachers included moments of analysis of didactic materials and the use of student responses to tasks to promote collective discussions, as well as Statistical investigations that allowed the student to be brought closer to the statistical work in order to appreciate its importance and difficulties from topics of interest. All sessions were recorded in audio, and document collection and a logbook were also used at the end of each training session.

After collecting data, a content analysis was performed which consisted of three phases: (i) pre-analysis; (ii) exploration of the material; and (iii) treatment of results, inference and interpretation (Bardin, 1977). Initially, the categories of analysis were defined, which are based on dimensions of the didactic knowledge of the teachers presented by Ponte (2012). In a second moment, the units of analysis of each category were defined based on specifications of the knowledge related to statistics highlighted by Batanero (2002) and the studies on the representations and statistical investigations mentioned in the theoretical framework. Finally, in the last phase, a qualitative analysis was made, and an explanation was elaborated.

We examined the data in three distinct moments: analysis of teaching materials; task solving and student response analysis; and conducting a statistical mini-investigation by teachers and discussion of conducting classroom investigations.

Table 1 – Analysis categories and units

Categories	**Units**
Knowledge of the statistics curriculum	- Which aspects of the curriculum teachers arrange; - How they relate the didactic materials to the curricular proposal.
Statistical knowledge for for teaching	- Meanings they attribute to statistical representations; - How do they do the various stages of the investigation cycle.
Instructional knowledge	- Perceptions about tasks related to representations; - Knowledge about how to conduct classroom investigations.
Knowledge of the student and his/her learning	- How do they predict student responses. - How do they understand student responses and errors.

Source: Table elaborated by authors.

Results

The results of the research are presented below.

Analysis of teaching materials

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At this stage of the course, discussions were made about the potential of tasks in teaching materials regarding the development of statistical literacy as indicated in the curriculum guidelines, supported by the indications contained in National Curriculum Parameters for the approach of the Data Processing block[4]. The class was divided into groups, each group being responsible for analyzing the material directed to a given grade (from 6th to 9th grade). Participants chose to analyze the materials released by the Rio de Janeiro Municipal Department of Education[5] in the 1st Quarter of 2017, as they refer to the education network in which most of them work and have a great influence on the education of Rio de Janeiro citizens. The groups had a discussion moment and then presented their observations to the whole class. The interactions that the teachers established were analyzed, and then we discussed their considerations about the 7th grade task presented in Figure 3.

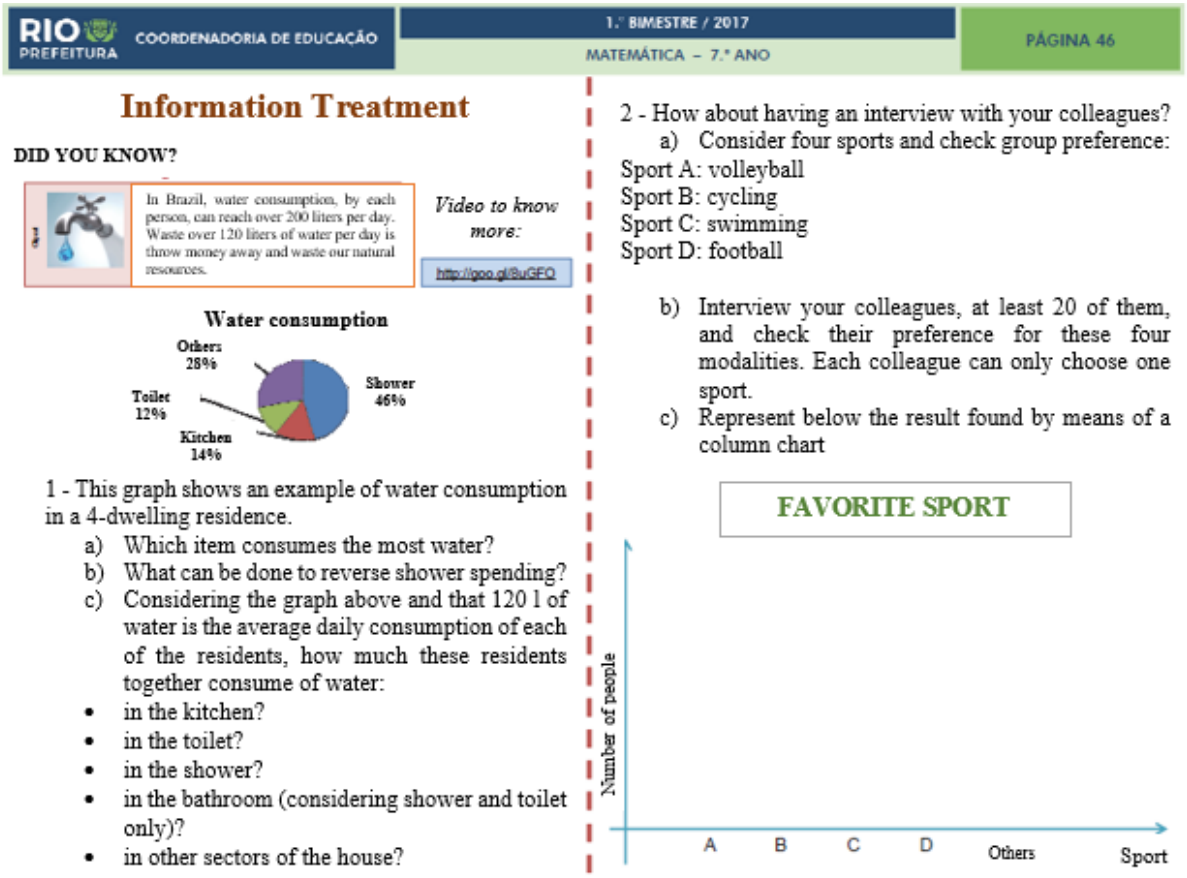

Figure 3 – 7th grade task analyzed by participants

Source: Rio de Janeiro Municipal Department of Education.

This material was formulated for the 1st Quarter in 7th grade classes, and the tasks above are the only ones directed to data processing. This fact caused inconvenience in the teachers:

[4] At the time of the training, the National Common Curricular Base (NCCB) had not yet been published.

[5] This public education network serves 461,002 elementary school students. The teaching materials used by this network are available at: http://www.rioeduca.net/.

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Norberto: It's a very synthetic approach...

Augusto: Summing up in two exercises, you can't! We need more.

Teacher educator: What things?

Norberto: It can be with exercises of the same line.

Augusto: I think you could explore the tables further. I think with tables the students understand well... They could also take the opportunity to include exercises on arithmetic average right away. Students at this level can already understand [...] They have already learned fractions in the other block.

Although Augusto points to the real need for the inclusion of other tasks, especially requiring the construction of tables, it is important to emphasize that there is also a concern to treat "soon" other concepts, such as the arithmetic mean. In this sense, the discourse of Augusto does not include many appreciations about the quality of the tasks, which shows that the teacher has no notion that varied tasks are needed to promote statistical literacy.

Giulia and Diana, however, seem to care about an education which can explore a critical sense of the students (as underlined in Franklin et al., 2005). Furthermore, they argue that an approach to statistics focused solely on mechanical calculations is poor and does not favor the development of statistical literacy:

Giulia: I think the first question is interesting and interdisciplinary, but you lose focus on statistics and critical thinking just by doing the mathematics. Accounts can be made at another time.

Diana: It seems that the purpose is just that and not to lead the students to interpret information.

In despite of the care with the promotion of learning associated with the development of critical sense in students, the teachers present superficial considerations about the tasks, since they do not initially discern important characteristics of the investigation cycle included in the second task, such as the formulation of questions, the collection of organization of these data and its further interpretation.

Giulia: The research proposed in the second exercise is cool for students.

Diana: Here the kids will just build the columns... It's a very simple activity for 7th graders.

Teacher educator: But is that all? Think for a moment about what they need to do before they build the chart.

Diana: They really need to interact, gather information, organize...

Teacher educator: And do you think this work can unfold to a bigger one?

Diana: Yes... They can include material from sociability of students and identify their collection task done while researching data shown in newspapers and magazines.

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The intervention of the teacher educator was enough for Diana to identify the task potential. Although at this stage the teacher still did not discern the proposal for statistical research in a global way (this topic had not yet been addressed), the teacher identified aspects of the investigation cycle, such as collection and organization of information, and was sensitive to a work capable to connect social context of students and statistical process, emphasizing the importance of working on interesting issues to students related to the world in which they live. The interaction and the shared experience could strengthen the *knowledge of the statistics curriculum*, so that they could associate the tasks with the curriculum proposals and respective objectives.

Task solving and analysis of student response

At this stage, the teachers were divided into four groups and performed exploratory tasks related to the reading and interpretation of graphs, collectively discussing their solutions. Then, each group analyzed the responses of some students to the same tasks. These tasks had previously been done in a 9th grade elementary school class of a municipal school in the West Zone of Rio de Janeiro, in which the first author was the teacher. After group discussions, the teachers presented their reflections to the whole class. We analyzed the statistical knowledge, as well as the interactions and reflections that the teachers made about the answers of the students regarding the "Favorite Pizza" task (Figure 3).

While discussing the solution of this task, some difficulties of the teachers were evident, namely, in understanding the meanings inherent in various graphical representations. Some teachers failed to realize initially that without the total number of boys and girls, it would not be possible to answer the questions posed. In other words, they had trouble distinguishing absolute and relative frequency, and did not understand the specifics of the information conveyed by the pie chart. In the collective discussion about the task, the teachers explain:

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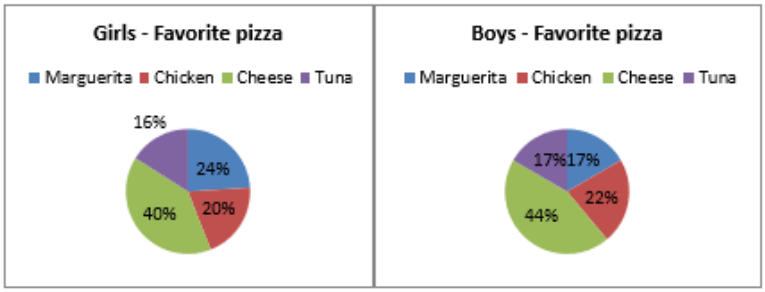
In one class, an interview was conducted to find out which pizza students liked the most. See the following graphs:



Answer T for true and F for false statements.

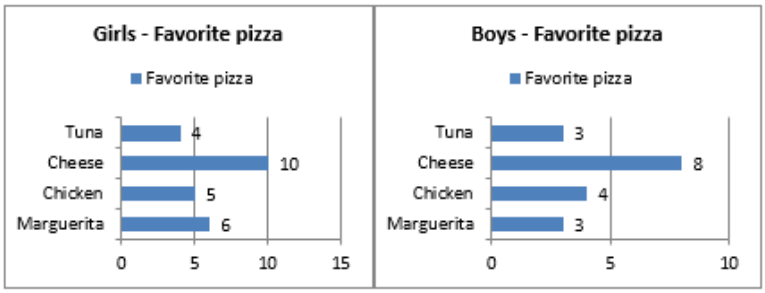
a) The number of boys preferring cheese pizza is higher than the number of girls who have a preference for this flavor.

b) The number of girls who like tuna pizza is more than the number of boys who has a preference for this taste.

c) The number of boys preferring chicken pizza is higher than the number of girls who prefer this flavor.

Explain the reasoning you used to find these answers.

Now look at another representation of the situation described earlier.



Do the conclusions remain the same? If not, please justify.

Figure 3 - Favorite Pizza (Source: task adapted from Martins and Ponte, 2011).

Hugo: *Percentages* are already showing that the statements are true.

Diana: Yeah... Just compare the *sectors*...

Vicente: It's true! The activity is kind of silly...

Teacher educator: Is that all? Let's see in another representation...

After presenting the second part of the task, the participants made deeper reflections about the meaning of the graphs:

Teacher educator: What now?

Márcia: Sometimes we don't see that each *graph serves to highlight different things*, right?

Augusto: Yeah... The pie chart shows the percentage and the other chart shows the exact amount...

Vicente: It's true... It would be necessary [to know] how many boys and girls.

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Hugo: We could only compare directly if the number of boys and girls were equal.

The graphics in the task are among the most popular in different media and illustrate what is evident in most data processing materials. Therefore, these are not new representations for teachers. The initial error was a proof that they paid attention to the visual aspects of the graph and not to its meaning. Overcoming this obstacle occurred through doing the task itself, which enabled a collective reflection on statistical concepts involved in the two graphical representations.

Having previously solved the task, teachers were able to identify possible misconceptions the students might make by analyzing their own responses:

Norberto: If we (teachers) were deceived, now you imagine the students...

Henrique: Yeah, these kind of activities are not usually in school materials [...]. This activity has a catch and then brings reflection. I think most will go wrong and then some may reflect with the other chart.

Following the discussion of difficulties and thoughts of students through their responses, we analyzed a student response (Figure 4) who demonstrated misconceptions in the first part of the assignment by not associating the pie chart with the idea of relative frequency. However, in the second part of the assignment, the student presented the following solution:

Do the conclusions remain the same? If not, please justify. (Task adapted from Martins and Ponte, 2011).

No, because it was not revealed the number of people interviewed in the first graph. Thus, it gave the impression that the number of people was the same in both graphs.

Figure 4 - Student response on “Favorite Pizza” assignment (Source: Survey Data).

The reasoning of the student yielded some comments from the teachers:

Márcia: What he meant is that he only did that because when he started reading, the interpretation was that the groups had the same number of people. If he didn’t think it was the same, he wouldn’t respond that way. I think in the next activity of this kind, the first thing he’s going to do is make sure the number is the same...

Norberto: I would say he is right, although he missed the first part.

Teacher educator: And for what reason, Norberto?

Norberto: For me, the student gave the answer that is the *summary of the class*. In the first part of the question *he answers what he sees*.

Diana: At the end, I think it is always necessary to have moments of discussion among the students... I think that way they learn from mistakes...

The comment made by Márcia denotes an attempt to put herself in the place of the student by inferring that she interpreted that the number of girls and boys was the same.

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Norberto refers to the "danger" of reading based on visual aspects of the graph, justifying that the student "responds what he sees". Diana, in turn, values the potential of collective thinking, so students can reconstruct their ideas through mistakes. It is important to emphasize that the teachers were motivated to consider the reasoning that the students showed and not just the fact of answering correctly or not.

This moment of the course showed significant traces of *statistical knowledge for its teaching*, namely, as regards to *statistical representations*. When solving the first part of the "Favorite Pizza" task (Figure 3), it was clear that they initially only *read the data* (Curcio, 1987, 1989) superficially. They did not associate reading with the concept of percentage, which would only allow comparison in absolute terms between the two groups (boys and girls) if there was information on the number of members in each group. Thus, we found that the teachers began by focusing on the visual aspects of graphs, not exploring their meanings, and did not realize the need to consider the notion of relative frequency (Martins, 2018). After the intervention included in the task itself, which proposes a new graph that shows the absolute frequency for each category, the teachers recognized their error and the respective reason. Therefore, the mediation proposed in the second part of the task was necessary for the teachers to read between the data, using comparisons and concepts of mathematics, in order to properly interpret the information included in the two representations.

Discussions about teaching materials and student responses highlight *knowledge of the statistics curriculum*, as well as *knowledge of the student and his/her learning*. Initially, some teachers show that they are unaware of the need to propose varied tasks for students to develop statistical literacy. Although some teachers still express concerns about quantity in relation to the quality of the proposed tasks, their discomfort is evident when they realize that data processing tasks are not properly valued in the materials. With the reflection and small interventions of the teacher educator, the teachers were able to identify important characteristics of the tasks, including valuing aspects related to the context of students. In the episode in which the student solution was discussed, the teachers highlighted valuing student errors as a starting point for learning (Cury, 2008).

Conducting a statistical mini-investigation by teachers and discussion of conducting classroom investigations

After studying statistical concepts[6] and reading research related articles[7], it was proposed that the groups conduct a mini statistical investigation. They were intended to do a very simple investigation that followed the usual structure of the steps in this process. The teachers were instructed to choose topics that interested them to do the research and the class was divided into three groups, each with three or four participants. At the end, we discussed the relevant aspects of conducting statistical investigations in the school context. In this way,

[6] Graphical representations, measures of central tendency and dispersion.

[7] We used the article "Investigações estatísticas no 6.º ano" (Sousa, 2002).

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we analyze the stages of the statistical investigation that the teachers made, besides their perceptions about the doing of activities of this nature with their students.

In the mini-investigation made by teachers in a computer lab, we sought to understand the profile of postgraduate students of the institution. The teachers decided to use their own data to delimit this profile for logistic reasons, thus configuring three groups for the research:

Diana: We decided to investigate how people do to come to the college location in terms of transportation...

Giulia: We also took the opportunity to see how long it takes colleagues to get to college location, whether they live near or far... And we also related to the people who work...

Diana: Yeah, teacher, we tried to research it all, but we couldn't organize this data on the computer. So, we made our own representation [...] I know it's not ideal, but as we have difficulties in the computer, we decided to do it anyway [Figure 5]. We have never done these activities before (undergraduate).

Giulia: We realize that even living near to work and without public transportation, it takes people a long time to get to work. Of course! With this traffic...

People

Answers	1	2	3	4	5	6	7	8
1	Far	Near	Far	Far	Near	-	-	Near
2	3h	15 min	5h	2h	30 min	-	-	30 min
3	Bus	On foot	Bus, train and subway	Car	Car	-	-	Car
4	Regular	Bad	Regular	Very bad	Very bad	-	-	Good
5	Yes	Yes	Yes	Yes	Yes	-	-	No
6	Yes	Reasonable	Yes	Yes	Yes	-	-	No

Figure 5 - Representation of Diana and Giulia

Source: Research Data.

The decisions of the groups revealed important aspects about the *knowledge of statistics*, namely, in conducting the investigation cycle. The inexperience with the technological support, besides the difficulty in associating the proper process to collect data with the question raised, led to an inconsistent data organization, which would not make data interpretation possible if the population was larger. The teachers found that even if individuals have their own transportation, their quality of movement remains unfavorable. However, as can be seen from the representation made, the answers to the questionnaire do

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not allow this conclusion to be reached, as they do not have a clear criterion of what it is 'near' or 'far'.

Another episode of the mini-investigation came from a group that investigated the academic background of the participants in the course. This group was able to formulate coherent questions, demonstrating to understand the complexities existing in the statistical research process, in order to do data collection giving priority to quantitative variables:

> **Luíz:** We decided to formulate questions that generate data for quantitative variables.
>
> **Norberto:** It's just that it's easier to handle data...
>
> **Fábio:** We aimed to relate the age of teachers with the time spent in studying as undergraduates and postgraduates and also with working time in the classroom. We also sought to know if teachers who have more time in service spent more time in teacher education.
>
> **Norberto:** We find it easier to organize the data in a table [...]. We did not think of another representation.
>
> **Manuel:** The representation we have built about the time of study throughout academic education and working time is interesting because it raises other questions. Another thing we saw is that the 60-year-old studied less, but what is the context? This takes into account the period you were born, the family, the financial conditions...

The teachers collected the information by organizing it into tables in Excel and concluded that the experiences are not proportional to age, as the younger teachers are more qualified and experienced in teaching. In addition, they found that the younger teachers were more involved in teacher education courses (specialization and further education). Although these teachers demonstrated greater skill in formulating the questions according to the variables that they preferred to work with, there was a limitation in the use of statistical tools. If the data were too large, they would have difficulty drawing conclusions, as evidenced by what Norberto says, who reported that they could not use a representation for bivariate data (such as a scatter plot) to relate the age of teachers to time in school education and work in the classroom. On the other hand, it is important to emphasize, the statement of Manuel, who understands that the process of data processing associated with a context allows new questions to be formulated, continuing the investigation process.

Regarding doing statistical investigations in the school context, Norberto and Henrique show limited perceptions about the potential of the investigation cycle, in its different stages:

> **Norberto:** In my point of view, research is very important because we can explore many graphs and measurements.
>
> **Teacher educator:** So that's all?

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Henrique: Oh, I think you just have to define the theme and the sample, make questions and put it all in one table.

Norberto evidenced the possibility of exploring different statistical representations such as graphs and measurements and Henrique highlighted part of the investigation cycle, including the choice of a sample for a given population. However, there are many more important aspects in a statistical investigation.

Other teachers went further and revealed, in their arguments, the concern to attend to the needs of students, besides providing attractive tasks connected to the social context in which they live:

Diana: I think what's most important is connecting the student to their reality, to the problems of their community...

Giulia: I think so too. This brings meaning to what they do in school.

Claudio: I think it's nice to work in a class, in a micro universe and try to do it in the series or at school... We won't do the research in the whole universe of the school... The television, which interview 2,000 people representing the whole of Brazil... Macro for them would be the whole school itself, right?... Creating a survey for them, they would take 1, 2 students from each class, asking some questions... And so, they were going to experience what happens in research like Datafolha, research at election time... I think it's something that everyone lives... Even children, right?... When we are with them, they are always asking "what does this percentage mean?", "how does it represent the whole of Brazil if only 1,000 or 2,000 and a few people were interviewed?" Getting them to do or create within the school is an opportunity for them to feel, collect information, organize... And see the result they have achieved and ask questions about the results.

Although Norberto and Henrique attributed the importance of investigation to the ability to organize information, they limited the scope of statistical literacy to various aspects to be worked on. Claudio, Diana and Giulia, in turn, demonstrated that they understand several elements present in investigations. Among them, the association with the real context is remarkable. Moreover, Claudio expressed, in his words, the importance of each process of the investigation cycle and the relationship between the choice of samples and the inferential process.

We highlight that there are different levels of understanding of the investigation cycle by teachers. In the work made by Diana and Giulia, the search for a diversity of variables was remarkable. However, the coherence between questions and alternatives was low, and teachers did not have adequate statistical tools to do the proposed task. On the other hand, the data presented in the investigation by Luiz, Norberto, Fábio and Manuel point to a partial understanding of the investigation process, showing coherence between the collected data and the formulated questions. In both cases, reduced use of statistical tools is noted, showing insecurity of them using a greater variety of representations. With the work done, it became clear to teachers the articulation between the various processes of the research cycle.

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In *instructional knowledge*, namely, in the investigation cycle, some reservations are notorious on the part of some teachers, whose main concern was "to make many graphs and tables". The environment of joint reflection and debate made teachers reframe their conceptions about the investigation cycle in the classroom, relating this process not only to the opportunity to carry out the construction of different representations, but also to develop several skills associated with statistical literacy.

Conclusion

This study reports how the teachers developed their knowledge for statistics teaching, namely, statistical representations and statistical investigations, through the proposed tasks and by the moments of collective discussion. Concerning to the knowledge of statistics, both in representations and in investigations, there were misunderstandings and limitations associated with the little work done in this field in the initial and continuous teacher education. Regarding to the didactics of statistics, the teachers initially did not realize the importance of working with different representations and involving students in investigations.

The activities carried in the course showed potential to complement knowledge of teachers, both in terms of statistics and its didactics. The analysis of didactic materials enabled the construction of a global view on the work to be done in statistics, as well as the potential and limitations of these materials. In addition, task-solving and student response analysis, based on authentic classroom situations, enabled a better understanding of the considered concepts, representations and statistical processes by the teachers, as well as the work of the students in their solution. Moreover, in the analysis of teaching and learning knowledge of statistics, the fact that the teachers made similar mistakes to those of the students gave more meaning to the discussion about the reasoning of students. Finally, conducting a mini-statistical investigation and discussion of statistical investigations allowed the teachers to gain awareness of the importance of this activity for students' learning, as well as the aspects to be considered in doing it. Therefore, this teacher education process helped to understand knowledge about statistics and its teaching in the teachers' point of view, as well as how the proposed activities may contribute to student development.

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