Zetetiké

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Probability in Mathematics Middle School textbooks: different ideas

Probabilidade em livros didáticos de Matemática dos Anos Finais: diferentes concepções

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Abstract

In the present paper from the analysis of the collections of mathematics textbooks approved by PNLD 2017 we look at the distribution of probability activities in the different collections and their volumes, as well as the different conceptions of probability presented in this material. 875 activities were identified, a quantity that is not evenly distributed among the collections, nor among their volumes. With regard to the probability conceptions approached, as expected, it was found that an absolute majority of problems focus the classical probability (81%). The founded results point to the need for major changes in the next editions of these materials, considering the prescriptions presented at the BNCC, which highlight the work with Probability in the Elementary School, and, specifically, gives attention to the frequentist probability in Middle School.

Keywords: Probability; Textbooks; Middle School.

Resumo

No presente artigo, a partir da análise das coleções de livros didáticos de Matemática aprovadas pelo PNLD 2017, volta-se o olhar para a distribuição das atividades que trabalham com a Probabilidade nas diferentes coleções e em seus volumes, bem como para as diferentes concepções de Probabilidade presentes neste material didático. Ao todo, foram identificadas 875 atividades, quantitativo que não está homogeneamente distribuído entre as coleções, nem, tampouco, em seus volumes. No que se refere às concepções de Probabilidade abordadas, como esperado, foi constatado que uma maioria absoluta de problemas trabalha com a probabilidade clássica (81%). Os resultados encontrados apontam para a necessidade de grandes mudanças nas próximas edições destes materiais didáticos, tendo-se em vista as prescrições apresentadas pela BNCC, que trazem grande destaque ao trabalho com a Probabilidade no Ensino Fundamental, e, em especial, à probabilidade frequentista nos Anos Finais.

Palavras-chave: Probabilidade; Livro Didático; Anos Finais.

A look at textbooks

The great influence exerted by the textbook in the classroom is undeniable. This didactic material is in direct contact with the teacher (who often uses it as a basis to select the content taught and to guide the teaching process) and with the student (who has it in his

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possession inside and outside of the school). The textbook thus acts as a link between teachers, students and the content it carries.

The National Textbook Guide[2] (Mathematics), published by the National Textbook Program[3] - PNLD 2017 highlights that:

> textbooks bring to teaching and learning processes another element, its author, who starts to dialogue with the teacher and with the student. In this dialogue, the textbook presents choices about: the knowledge to be studied (Mathematics); the methods adopted in order to students being able to learn it more effectively; the curricular organization over the years of schooling (MEC, 2016, p. 13, free translation).

The content presented by the textbook, which largely guides the teaching practice in the classroom, must therefore be guided by official curriculum guidelines. As a result, the construction of such teaching material aims to first reflect the prescriptions regarding the respective stages of schooling to which they refer. Sacristán (2000) points out that materials such as the textbook tend to be closer to the teacher, which reinforces the importance of the textbook being in dialogue with the official curricular documents.

> There are a number of means, [...], which usually translate for teachers the meaning and contents of the prescribed curriculum, making an interpretation of it. Prescriptions are usually very generic and, to the same extent, are not sufficient to guide educational activity in class (Sacristán, 2000, p. 104-105, free translation).

From this perspective, the importance of textbook analysis is pointed out when the objective is to identify how a specific area of knowledge or specific content has reached the classroom. In the present article, which presents an excerpt from an ongoing thesis study (Lima, 2018), we look at Probability, seeking to understand how this field of Mathematics, which has gained even more prominence recently, with the homologation of the National Common Curricular Base[4] - BNCC (MEC, 2018), had been presented in textbooks for the Middle School approved by the last PNLD (MEC, 2016). The analysis conducted sought to find out which conceptions of Probability are present in the activities proposed throughout the volumes of each collection of textbooks, since the BNCC (MEC, 2018) brings the importance of working with the frequentist conception of probability and its comparison with the classic conception (most commonly present in formal classroom work).

Given the presented above and understanding that a function that has been exercised by the textbook is “to take to the classroom the didactic and pedagogical changes proposed in official documents, as well as results of research on the learning of Mathematics” (MEC, 2016 , p. 14, free translation), there was an interest in analyzing the collections approved before the approval of the BNCC (MEC, 2018) and raising reflections on the expected changes for the collections that will arrive in classrooms in the upcoming years.

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[2] Guia Nacional do Livro Didático
[3] Programa Nacional do Livro Didático
[4] Base Nacional Comum Curricular

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Probability

Probability in Middle School

Probability is a field of study that "creates, develops and in general researches models that can be used to study experiments or random phenomena" (Morgado et al., 1991, p. 119, free translation). Godino, Batanero and Cañizares (1991), argue that the knowledge of this area of Mathematics

> provides a way of measurement of uncertainty, as a result, probabilistic models are the foundation of most Statistics. This implies that knowledge of probability theory is necessary for an adequate understanding of statistical methods, which today are indispensable tools in the scientific, professional and social fields (p. 11-12, free translation).

These authors also highlight the potential of the articulation between mathematical situations of a probabilistic nature and everyday situations, arguing that the study of Probability allows the student to have contact with uncertainty, giving a new focus to school instruction in Mathematics, which tends to present a strong deterministic idea. Thus, it is the understanding of Probability that will allow the student to explore random situations, including estimating the probabilities of the occurrence of different events, classifying them in certain, probable, improbable or impossible events.

Several authors have highlighted the importance of studying Probability throughout schooling so that students can develop their probabilistic reasoning and build a broad understanding of essential concepts such as randomness and sample space, which precede the calculation of probabilities (Fischbein, 1975, Bryant & Nunes, 2012, Campos & Carvalho, 2016). In this sense, the PNLD 2017 (MEC, 2016) points out that:

> the study of probability at the fundamental level of basic education offers students the opportunity to recognize and quantify the uncertainty associated with random events by establishing pillars for further studies in other stages of schooling. [...] The notion of probability is adopted as a measure that quantifies the uncertainty of an event in a random experiment (p. 49, free translation).

This statement agrees with the National Curricular Parameters of Mathematics[5] - PCN (MEC, 1998) for Middle School, which highlights that:

> with the basic notions of probability students will learn to determine the chances of occurrence of some events (currencies, dice, cards). Thus, they will be able to familiarize themselves with the way Mathematics is used to make predictions and understand the importance of probability in everyday life (MEC, 1998, p. 70, free translation).

Another idea of Probability, associated with the realization of experiments and simulations (the frequentist Probability) appears timidly in this document, which states that

[5] Parâmetros Curriculares Nacionais – Matemática (PCN)

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the study of Probability "aims to make students realize that through experiments and simulations they can indicate the possibility occurrence of a given event and compare it with the expected probability using a mathematical model" (MEC, 1998, p. 86, free translation).

Table 1 shows, specifically, what is expected from the work with Probability in Middle School, as prescribed by the BNCC (MEC, 2018). It is possible to note the greater emphasis given to the work with Probability in all years and, in particular, the great focus on the frequentist conception. Such prescriptions are the basis for discussions about expectations for Mathematics textbooks published after the PNLD 2017 (MEC, 2016), analyzed here, which will already be based on this new official curriculum document.

Table 1 – Probability in Middle School (prescriptions)

Year	**Knowledge Objects**	**Abilities**
6º	Calculation of probability as the ratio between the number of favorable results and the total possible results in an equiprovable sample space; Calculation of probability by means of many repetitions of an experiment (frequency of occurrences and frequentist probability).	(EF06MA30) Calculate the probability of a random event, expressing it by a rational number (fractional, decimal and percentage form) and compare that number with the probability obtained through successive experiments.
7º	Random experiments: sample space and probability estimation by frequency of occurrences.	(EF07MA34) Plan and carry out random experiments or simulations that involve calculating probabilities or estimates using frequency of occurrences.
8º	Fundamental counting principle; Sum of the probabilities of all elements of a sample space.	(EF08MA22) Calculate the probability of events, based on the construction of the sample space, using the fundamental counting principle, and recognize that the sum of the probabilities of all elements of the sample space is equal to 1.
9º	Analysis of the probability in random events: dependent and independent events.	(EF09MA20) Recognize, in random experiments, independent and dependent events and calculate the probability of their occurrence, in both cases.

Source: National Common Curricular Base, BNCC (MEC, 2018).

Succeeding the work with elementary probabilistic concepts in primary school, the prescriptions in question are mainly related to the calculation of probabilities. The visibility given to the frequentist probability, the realization of random experiments and the relation of such conception of Probability with Statistics (calculation of probabilities from research data) stands out. The decentralization of school work with classical (Laplacian) probability calls attention to the existence and importance of different conceptions of probability. According to Godino, Batanero and Cañizares (1991), the classic, frequentist, subjective, logical and

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formal conceptions of probability are considered in this text, they are discussed in the following section.

Different conceptions of Probability

The classic concept of Probability supports calculation of the probability of the occurrence of an event that occurs a priori, taking the ratio between the number of favorable cases and the number of possible cases in an equiprovable sample space. This conception is the most commonly used on school problems, however it can come up against conceptions arising from previous experiences of the students, inside or outside the school, in which equiprobability is not present.

On the other hand, the frequentist conception of probability allows the calculation of probabilities to be carried out posteriori, based on the results of experiments and/or simulations. It is, therefore, objective, separate from any consideration resulting from personal experiences. In applicable situations, when the number of experiments is large enough, the value obtained from this conception of probability is close to that calculated by the ratio derived from the classical conception, it means that “the greater the number of events, the greater the proximity between the a posteriori probability and a priori probability, calculated without experimental manipulation, based on theoretical data and the classic concept” (Santos, 2015, p. 47, free translation). In a school context, the comparison between these two conceptions of probability (discussions about their similarities and differences) gains strength with the publication of the BNCC, which not only explicitly prescribes the teach of both conceptions in the classroom, but also highlights the importance of the confrontation between the results obtained from either conception being part of the time dedicated to the study of Probability in the Middle School.

According to the subjective conception, Probability is “an expression of personal belief or perception” (Godino, Batanero & Cañizares, 1991, p. 25, free translation). Thus, such a conception is strongly based on particular experiences of those who estimate the probability of a certain event. In this sense, the estimation and calculation of probabilities are centered on the subject and, therefore, different people can predict different probabilities for the same situation. Such conception “is not based on the repetition of any process, since it is possible to assess the probability of an event that can occur only once” (Godino, Batanero & Cañizares, 1991, p. 25, free translation). This conception of probability is often present in the judgment of everyday situations, such as games and bets (for example, when betting on a lucky number). Although it is not an explicit objective to work with this conception of probability in the classroom, since it is not part of curricular prescriptions, it is important to note its influence coming from the students' previous knowledge and experiences (including extra-school) for the development of their probabilistic reasoning. That is, it cannot be ignored that the understanding of randomness and other probabilistic concepts can face obstacles that are the result of erroneous, subjective conceptions by students.

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According to the logical conception, Probability is based on induction, that is, it defines a logical relationship between a statement and a hypothesis derived from it: “it translates a degree of rational belief, that is, the confidence rate granted to a proposition *p* to light of information from another proposition *q*. Probability is treated as a special type of relationship between the two statements” (Godino, Batanero & Cañizares, 1991, p. 23, free translation). In this sense, the confidence rate is measured in two extreme ways: certainty and impossibility. In the first case, *p* is a consequence of *q* and the proposition *q* gives *p* a probability equal to 1. In the case where the propositions *p* and *q* are contradictory, the probability given by *q* to *p* is equal to 0. The work with this conception of Probability is also not present in the prescriptions for Basic Education.

Finally, in the formal conception, which opposes the classical one, given that it does not impose the equiprobability of events, the probability of occurrence of an event is measured when a sample space (E) and a subset (A) of it are chosen. The probability is then calculated from the quotient between the measure of A and the measure of E, the result of which is between 0 and 1. Godino, Batanero and Cañizares (1991) also highlight that this conception of probability as a measure allows problems of geometric probability to be solved. Geometric probability is also discussed by Bittar and Abe (2013), who point out that it involves concepts of geometry such as length, area and volume, with the sample space constituted by continuous sets that refer to measures of the same nature.

It is important to say here, that such conceptions of probability coexist and, depending on the random situation to be treated, one may prove to be more adequate than the other, since “when we compare the different conceptions exposed, we see that each one can be applied with advantage in some circumstance” (Godino, Batanero & Cañizares, 1991, p. 28, free translation). Reinforcing this point, Santos (2010) says that “situations related to uncertainty can be interpreted in different ways, by different probabilistic concepts, leading or not people to the appropriate responses” (*apud* Santos, 2015, p. 50, free translation).

The results presented in this article had as main contribution this discussion about the diversity of possibilities to the approach of Probability (based on the work with different conceptions). The methodological processes developed are explained in the section that follows.

Methodological path

The quantitative and qualitative analyzes conducted were based on a documentary perspective. The 44 volumes that make part of the 11 collections of Mathematics textbooks for the Middle School approved by PNLD 2017 (MEC, 2016) were analyzed. The collections in question are listed in Table 2.

Table 2 – Collections approved by PNLD 2017: Mathematics

	Name	**Authors**
Collection A	Praticando Matemática	Álvaro Andrini e Maria

DOI: 10.20396/zet.v28i0.8656908

		José Vasconcelos
Collection B	Descobrindo e Aplicando a Matemática	Alceu Mazzieiro e Paulo Machado
Collection C	Matemática do Cotidiano	Antonio José Bigode
Collection D	Matemática – Compreensão e Prática	Ênio Silveira
Collection E	Projeto Teláris – Matemática	Luiz Roberto Dante
Collection F	Projeto Araribá – Matemática	Maria Regina Gay
Collection G	Matemática – Ideias e Desafios	Dulce Onaga e Iracema Mori
Collection H	Matemática – Bianchini	Edwaldo Bianchini
Collection I	Matemática nos Dias de Hoje – Na Medida Certa	José Jakubovic e Marília Centurión
Collection J	Convergências – Matemática	Eduardo Chavante
Collection K	Vontade de Saber – Matemática	Joamir Souza e Patricia Pataro

Source: National Textbook Guide – Mathematics, PNLD 2017 (MEC, 2016).

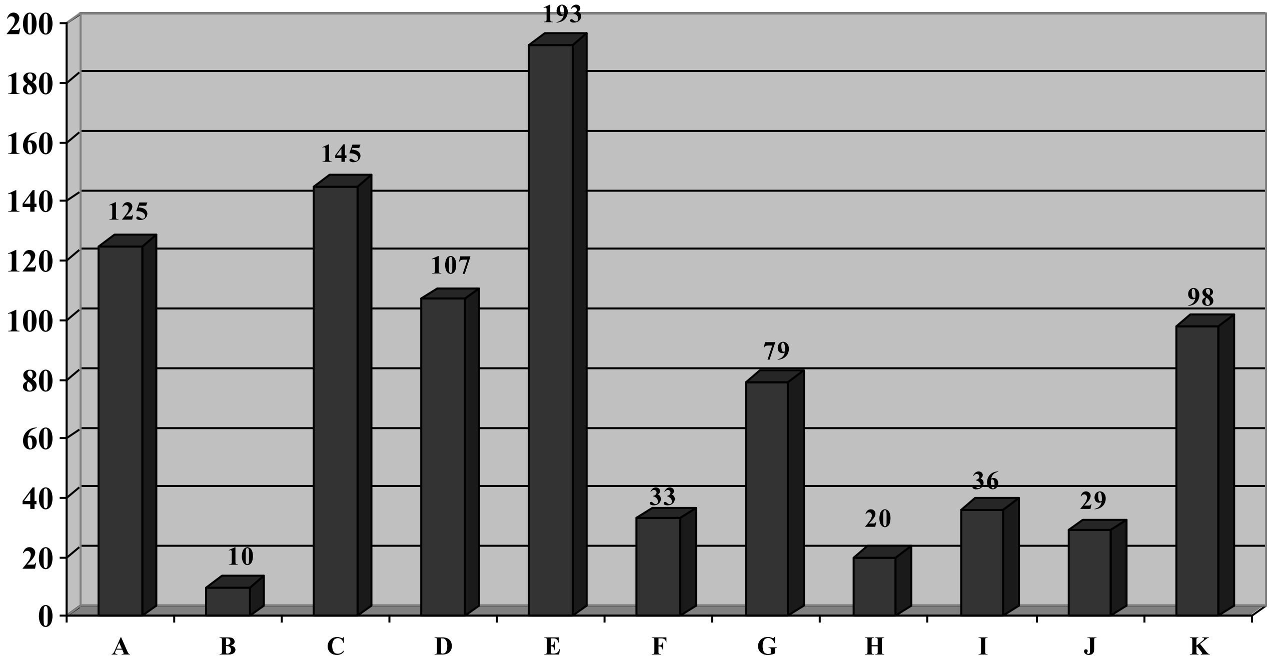
Initially, the chapters that would be analyzed were identified from the summary of each textbook. Those chapters that pointed to the work with Probability, possibilities, treatment of information and related topics were elected. From this, the survey of activities that work with probabilistic concepts was made. It is also worth noting that, given that the same question can address different probabilistic ideas, each item in the analyzed textbooks was considered as different activities (for example, items a, b, c were considered and classified as three distinct proposed activities).

The discussions presented below consist of a section of analyzes carried out in the thesis study, of which the analysis of textbooks constitutes one of the stages. Here, three variables are considered, referring to the distribution of the probabilistic activities surveyed, and which therefore indicate evidence of how the work with Probability takes place in the classroom from the use of this didactic material. Such variables are: 1. *collection*; 2. s*chooling year* and 3. *Probability conceptions addressed.*

Presentation and discussion of results

In total, 875 activities that explore probabilistic ideas in the analyzed textbooks were identified. Graph 1 shows the distribution of such activities in the analyzed collections (see Table 2).

DOI: 10.20396/zet.v28i0.8656908



Graph 1 - Quantitative of probabilistic problems (collections)
Source: The author.

A very important result that is evidenced from Graph 1 refers to the lack of homogeneity in the strength given to the work with Probability in Middle School by the authors of the different collections: while some collections bring only some problems (such as Collection B, with only 10 problems over the four volumes), others bring more than 10 times this amount.

It is needed to reinforce the influence that this didactic material has in the classroom. Thus, the choice of a collection that does not propose a more robust work with problems of a probabilistic nature may prevent this work from being a focus in the classroom, which would impair students' contact with varied concepts and problems, which could provide the development of their probabilistic reasoning (Vergnaud, 1986; 1996, Bryant & Nunes, 2012).

Such inconsistency in the distribution of work with Probability was also observed when observing the different volumes corresponding to each of the years that make up the Middle School (Table 3).

Table 3 – Quantitative of probabilistic problems (volumes)

	6º grade	7º grade	8º grade	9º grade
Collection A	-	-	12	113
Collection B	2	8	-	-
Collection C	9	136	-	-

DOI: 10.20396/zet.v28i0.8656908

Collection D	4	53	30	20
Collection E	-	76	68	49
Collection F	6	10	7	10
Collection G	12	21	15	31
Collection H	4	3	-	13
Collection I	6	-	30	-
Collection J	-	29	-	-
Collection K	2	38	58	-
Total	**45**	**374**	**220**	**236**

Source: The author.

There is a great concentration of work with Probability in the 7th grade to the detriment, especially, of a work with more basic concepts since the 6th grade. In percentage, there are, in the 6th, 7th, 8th and 9th grades, respectively: 5%, 43%, 25% and 27% of the activities identified.

In Table 3, it is possible to observe, still, that some collections bring the Probability only in specific volumes, with a certain agreement of a greater focus in the 7th grade (in these materials the quantification of probabilities from the classic ratio between the number of favorable cases and the number of possible cases appears strongly from this year). This leads to an expectation that this distribution will be rethought in the upcoming years, considering the guidelines present in the BNCC (MEC, 2018), which prescribe different learning objects to be worked on progressively since the 6th grade, guaranteeing a space for Probability in all volumes of textbook collections for the Middle School.

As previously mentioned, analyzes presented here were based, in particular, on the different conceptions of Probability present in Mathematics textbooks. Among the different conceptions considered (Godino, Batanero & Cañizares, 1991), were identified activities that address three of them: classical or Laplacian conception, frequentist conception and formal conception (under a geometric approach). Table 4 presents the quantitative panorama of the data obtained.

Table 4 – Quantitative of probabilistic problems (conception of probability)

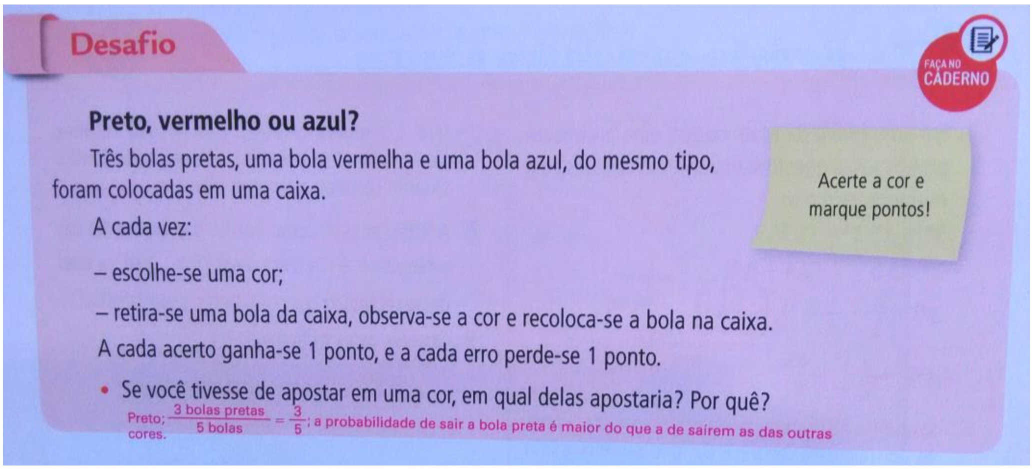
	Classic	Frequentist	Geometric
Collection A	100	14	11
Collection B	10	-	-
Collection C	104	3	38
Collection D	94	2	11
Collection E	138	43	12
Collection F	26	7	-
Collection G	72	7	-
Collection H	17	2	1
Collection I	33	2	1
Collection J	29	-	-
Collection K	84	7	7
Total	**707**	**87**	**81**

Source: The author.

DOI: 10.20396/zet.v28i0.8656908

As expected, the classic conception gains much more focus in the analyzed collections. In percentage, there are: 81% of the activities addressing the classic conception; 10% addressing the frequentist conception and 9% addressing the geometric conception. Following are some examples that illustrate how the approach to each of these concepts is conducted in the analyzed textbooks.

Regarding to the classical conception, the activities analyzed explore different probabilistic concepts from contexts such as raffles and games (Figure 1), mostly, with situations with data, coins and sweepstakes in ballot boxes. A smaller part of the problems explores contexts that have an articulation with Combinatorics (Figure 2).


Desafio

FAÇA NO CADERNO

Preto, vermelho ou azul?

Três bolas pretas, uma bola vermelha e uma bola azul, do mesmo tipo, foram colocadas em uma caixa.

Acerte a cor e marque pontos!

A cada vez:

– escolhe-se uma cor;

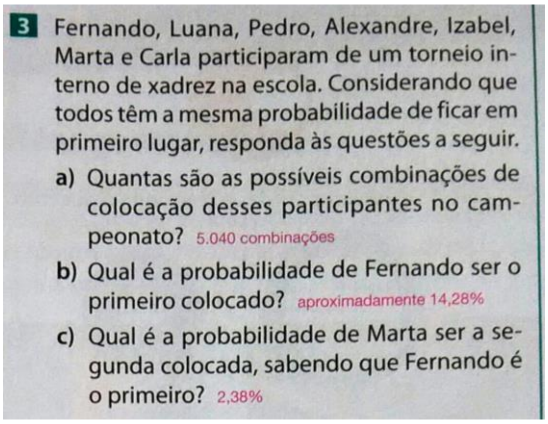
– retira-se uma bola da caixa, observa-se a cor e recoloca-se a bola na caixa.

A cada acerto ganha-se 1 ponto, e a cada erro perde-se 1 ponto.

- Se você tivesse de apostar em uma cor, em qual delas apostaria? Por quê? Preto; $\frac{3 \text{ bolas pretas}}{5 \text{ bolas}} = \frac{3}{5}$; a probabilidade de sair a bola preta é maior do que a de saírem as das outras cores.

Figure 1 – Classic conception of probability: game context

Source: Collection G, 7º grade (2015, p. 237).


3 Fernando, Luana, Pedro, Alexandre, Izabel, Marta e Carla participaram de um torneio interno de xadrez na escola. Considerando que todos têm a mesma probabilidade de ficar em primeiro lugar, responda às questões a seguir.

a) Quantas são as possíveis combinações de colocação desses participantes no campeonato? 5.040 combinações

b) Qual é a probabilidade de Fernando ser o primeiro colocado? aproximadamente 14,28%

c) Qual é a probabilidade de Marta ser a segunda colocada, sabendo que Fernando é o primeiro? 2,38%

Figure 2 – Classic conception of probability: combinatorial context (permutation)

Source: Collection F, 9º grade (2014, p. 217).

In its turn, the approach to the frequentist conception of Probability is present from two biases: 1. *from statistical data*; 2. *from the realization of experiments* (bias observed in a smaller amount). Figures 3 and 4 illustrate these biases, respectively.

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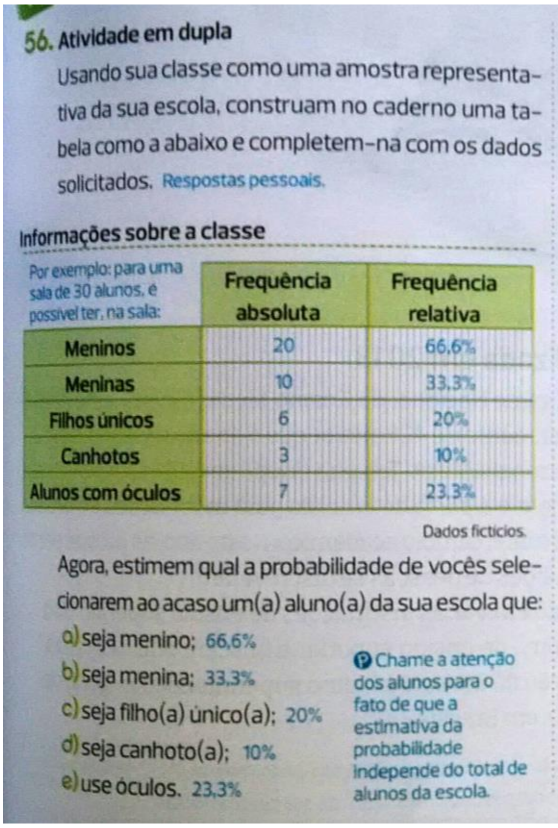

56. Atividade em dupla

Usando sua classe como uma amostra representativa da sua escola, construam no caderno uma tabela como a abaixo e completem-na com os dados solicitados. Respostas pessoais.

Informações sobre a classe

Por exemplo: para uma sala de 30 alunos, é possível ter, na sala:	Frequência absoluta	Frequência relativa
Meninos	20	66,6%
Meninas	10	33,3%
Filhos únicos	6	20%
Canhotos	3	10%
Alunos com óculos	7	23,3%

Dados fictícios.

Agora, estimem qual a probabilidade de vocês selecionarem ao acaso um(a) aluno(a) da sua escola que:

a) seja menino; 66,6%
b) seja menina; 33,3%
c) seja filho(a) único(a); 20%
d) seja canhoto(a); 10%
e) use óculos. 23,3%

Chame a atenção dos alunos para o fato de que a estimativa da probabilidade independe do total de alunos da escola.

Figure 3 – Frequentist conception of probability: estimation of probabilities from statistical data (real data)

Source: Collection E, 9º grade (2015, p. 291).

DOI: 10.20396/zet.v28i0.8656908

AÇÃO

sobre probabilidade

Qual é a chance?

São necessários 2 dados para cada grupo de 3 alunos.

Os dados devem ser lançados 20 vezes. Em cada lançamento, deve-se anotar a diferença (positiva) de pontos.

Ao final, deve estar preenchido um quadro como este ao lado.

Em seguida, com informações das tabelas de todos os grupos, o professor pode fazer uma tabela similar na lousa.

Diferença	Frequência	Frequência relativa (%)
0	■	■
1	■	■
2	■	■
3	■	■
4	■	■
5	■	■

Frequência: número de vezes que o resultado foi obtido.
Frequência relativa: porcentagem do resultado em relação ao total de resultados.

É quase certo que as frequências dos resultados não apareçam igualmente distribuídas. Esse é o fenômeno que precisa ser explicado.

Os alunos devem preparar um relatório contendo estes tópicos:

- Resultados obtidos pelo grupo.
- Resultados obtidos pelos vários grupos (a turma toda).
- Quais foram os resultados mais frequentes e os menos frequentes.
- Explicações para a distribuição de frequências obtida.

Para essas explicações, vamos dar uma sugestão:

- Há 6 possibilidades de se obter diferença zero ($1 - 1$, $2 - 2$ etc.).
- Há 36 possibilidades de diferenças (combine o número 6 do primeiro dado com todos os resultados do segundo dado; depois, combine o número 5 do primeiro dado com todos os resultados do segundo, e assim por diante você terá $6 \cdot 6 = 36$.
- Portanto, a probabilidade de diferença zero é $\frac{6}{36} = \frac{1}{6} \cong 0{,}166 = 16{,}6\%$.
- Que relação há entre a probabilidade de diferença zero e os resultados obtidos pelo grupo ou pela classe toda? Será que isso explica a distribuição de frequência das outras diferenças?

Figure 4 – Frequentist conception of probability: conduction of random experiments

Source: Collection I, 8º grade (2015, p. 18).

It is worth emphasizing that it is expected that activities of this nature will become increasingly present in mathematics textbooks, given the prescriptions presented by the BNCC (MEC, 2018). The realization of random experiments for the analysis and calculation of probabilities under a frequentist look is explained in this document and it has also appeared shyly in the analyzed collections. The other bias (articulation with Statistics) also tends to gain more strength, since the statistical data (whether these come from research published in the media or obtained from research to be carried out by students themselves –

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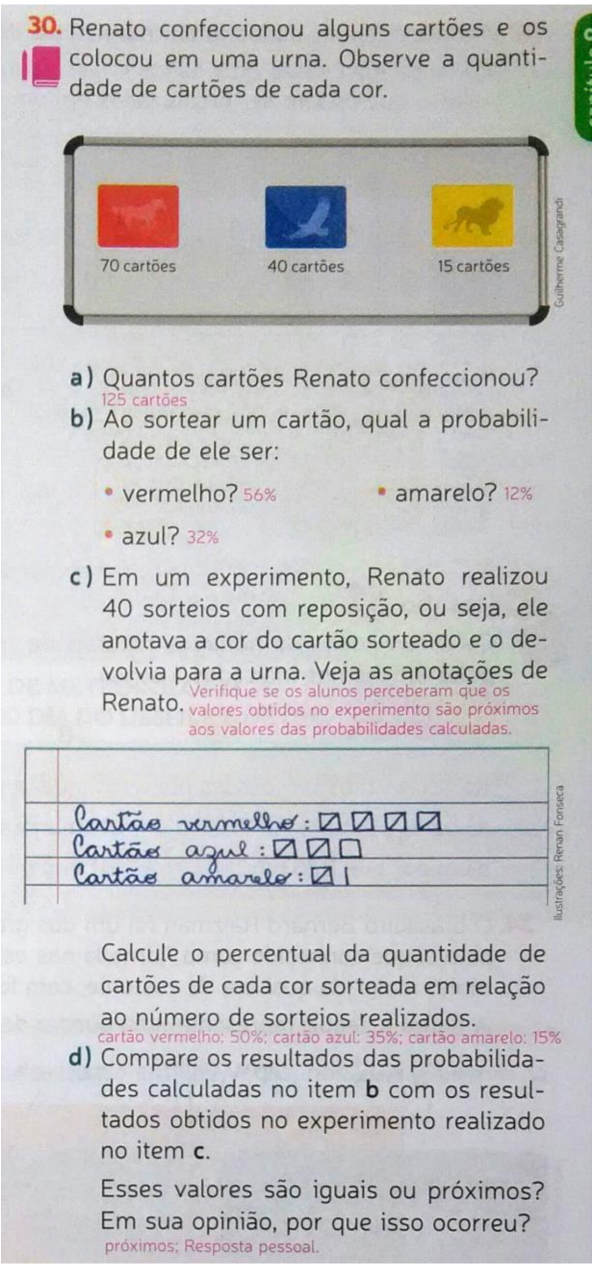
inside or outside their classrooms) have a great potential to allow a probabilistic look to be released to everyday information, for analysis and understanding of situations in which randomness is present.

Another skill to be developed in the Middle School, according to the BNCC (MEC, 2018) is related to the confrontation of results obtained through the classic conception and the frequentist conception of Probability. Only two examples of such approach were identified in two of the analyzed collections. One of these examples is shown in Figure 5.



30. Renato confeccionou alguns cartões e os colocou em uma urna. Observe a quantidade de cartões de cada cor.

70 cartões | 40 cartões | 15 cartões

Guilherme Casagrandi

a) Quantos cartões Renato confeccionou? 125 cartões

b) Ao sortear um cartão, qual a probabilidade de ele ser:

- vermelho? 56%
- amarelo? 12%
- azul? 32%

c) Em um experimento, Renato realizou 40 sorteios com reposição, ou seja, ele anotava a cor do cartão sorteado e o devolvia para a urna. Veja as anotações de Renato. Verifique se os alunos perceberam que os valores obtidos no experimento são próximos aos valores das probabilidades calculadas.

Cartão vermelho: ☒☒☒☒
Cartão azul: ☒☒☐
Cartão amarelo: ☒ |

Ilustrações: Renan Fonseca

Calcule o percentual da quantidade de cartões de cada cor sorteada em relação ao número de sorteios realizados. cartão vermelho: 50%; cartão azul: 35%; cartão amarelo: 15%

d) Compare os resultados das probabilidades calculadas no item **b** com os resultados obtidos no experimento realizado no item **c**.

Esses valores são iguais ou próximos? Em sua opinião, por que isso ocorreu? próximos; Resposta pessoal.

Figure 5 – Confrontation between the classic and frequentist conceptions of probability (item d)

Source: Collection K, 8º grade (2015, p. 201).

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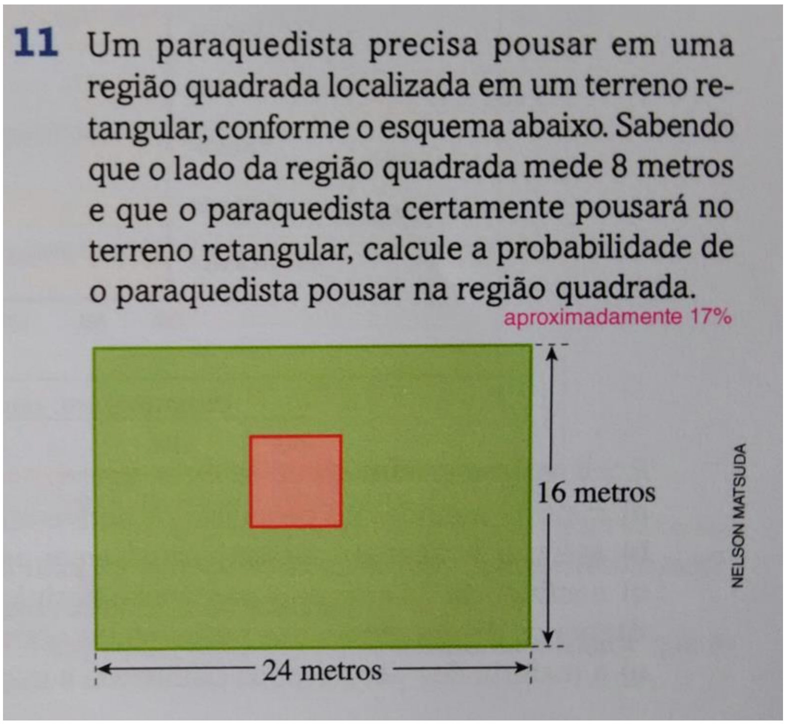
Finally, regarding the *geometric conception* of probability were identified problems that address the calculation of probabilities from the exploration of the concept of areas of quadrilaterals (Figure 6), as well as the concepts of angles, areas of circles, among others.


11 Um paraquedista precisa pousar em uma região quadrada localizada em um terreno retangular, conforme o esquema abaixo. Sabendo que o lado da região quadrada mede 8 metros e que o paraquedista certamente pousará no terreno retangular, calcule a probabilidade de o paraquedista pousar na região quadrada.

aproximadamente 17%

Figure 6 – Formal conception of probability (geometric): demand of knowledge of the concept of area of rectangular and square surfaces

Source: Collection H, 9º grade (2015, p. 105).

It is important to emphasize that problems like the one above truly explore Probability from Geometry, that is, under a formal conception, as pointed out by Godino, Batanero and Cañizares (1991): the probability, in this case, needs to be seen as a measure and there is no equiprobability involved. However, it was observed that most of the probabilistic problems of a geometric nature present in the analyzed collections, which are already few, propose a simpler approach, making great use of the idea of roulettes, as illustrated in Figure 7, below.

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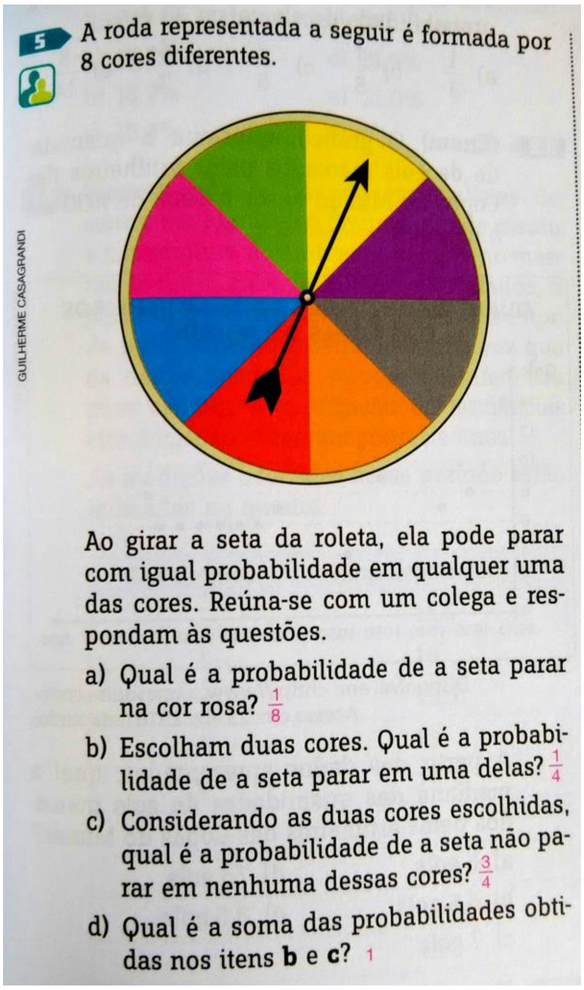

5 A roda representada a seguir é formada por 8 cores diferentes.

Ao girar a seta da roleta, ela pode parar com igual probabilidade em qualquer uma das cores. Reúna-se com um colega e respondam às questões.

a) Qual é a probabilidade de a seta parar na cor rosa? $\frac{1}{8}$

b) Escolham duas cores. Qual é a probabilidade de a seta parar em uma delas? $\frac{1}{4}$

c) Considerando as duas cores escolhidas, qual é a probabilidade de a seta não parar em nenhuma dessas cores? $\frac{3}{4}$

d) Qual é a soma das probabilidades obtidas nos itens **b** e **c**? 1

Figure 7 – Geometric probability: simple roulette (equiprovable)

Source: Collection D, 7º grade (2015, p. 177).

The problem presented (and the others identified that have the same approach), even though they are associated with a geometric probability (and were classified this way), are very close to the classic conception of probability as they do not necessarily demand the exploration of the idea of probability as a measure, since the sectors corresponding to each color have equal areas and the probability of each of them being drawn is, therefore, equiprovable. It is noteworthy, therefore, that the reduction of work with geometric probability only to that proposed in very simple problems and in the context of roulettes (which was observed in most collections), further reduces the diversity of concepts of Probability explored in the materials didactics analyzed here.

Now, some considerations from the main results discussed here are presented.

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Considerations

44 volumes that make up the collections of Mathematics textbooks for Middle School approved by PNLD 2017 (MEC, 2016) were analyzed. The importance of analyzing this didactic material is justified by the roles it plays in the teaching and learning processes.

The results found show that there is no homogeneous distribution of activities that explore Probability among such collections or between their volumes. Of the total of 875 activities identified, the highest percentage is present in the textbooks of the 7th grade (43%), while the textbooks of the 6th grade present the lowest percentage (only 5%). Corroborating the assertions made by different authors (Fischbein, 1975, Vergnaud, 1986; 1996, Bryant & Nunes, 2012, Campos & Carvalho, 2016), it is argued that the work with Probability should occur in a progressive manner, providing contact with its several concepts throughout the years of schooling, from different problems, so that the probabilistic reasoning can be developed properly, which goes against the observed result. In this sense, it is worth noting that the prescriptions present at BNCC (MEC, 2018) provide adequate guidance, by presenting different objects of knowledge to be worked on each one of the years that make up the Middle School.

It is also important to point out that most of the activities analyzed (81%) address the classic conception of probability - mainly from the context of games and sweepstakes, involving coins, data, among others. On the other hand, the frequentist conception of probability, which gains great strength in the BNCC, is addressed in only 10% of the activities analyzed, which mostly explore the relationship between Probability and Statistics - from the results of statistical research. Other biases, such as the performance of experiments in the classroom and the confrontation of theoretical and empirical results (classical conception vs. frequentist conception) need to gain space in this didactical material, in the light of the BNCC.

The analyzes conducted, based on the prescriptions present in the BNCC (MEC, 2018), allowed the expectations of changes that are going to occur in the textbooks in the upcoming years to be discussed, considering that it presents to teachers (and students) a translation of the official curriculum: indications of what, how and when to be worked in the classroom. In this sense, it is important that in the future, analyzes of the textbooks approved in the next PNLDs are conducted, with the objective of observing how these materials will transform in function of this role it assumes.

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