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Teacher training in Mathematics: a discussion on teaching probability

Formação do professor de Matemática: uma discussão sobre o ensino de probabilidade

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Abstract

The goal of this article is to present an analysis of the knowledge of future Mathematics teachers about teaching probability, in particular, about randomness. This investigation involved a training process in which the issue of the independent events was discussed based on the results indicated by Bryant & Nunes. This process included the participation of 11 students in the Mathematics Teaching Degree program on a campus at a public university in Sergipe. To analyze the data on teacher knowledge, we considered categories discussed by Shulman. Regarding teaching probability, our investigation was based on studies by Gal and Batanero, Contreras & Diaz and we used Zeichner for the reflective practice of teachers. The analysis revealed that the future teachers broadened their knowledge base for teaching probability, especially with respect to recognizing the need to overcome positive and negative recency in order to understand independent events. Moreover, they reconsidered a view contrary to what they had initially believed with respect to teaching in the initial years. Hence, we highlight the importance of training activities in order to provide participants with a reflective experience in learning situations that involve concepts of probability, through experiments and reflections.

Keywords: Teaching Probability; Initial Teacher Training; Reflective Teacher.

Resumo

Este artigo tem o propósito de apresentar uma análise de conhecimentos de futuros professores de Matemática sobre o ensino da probabilidade, em particular sobre a aleatoriedade. Esta investigação envolveu um processo formativo no qual foi discutida a questão da independência de eventos a partir dos resultados apontados por Bryant & Nunes. Este processo contou com participação de 11 estudantes de Licenciatura em Matemática de um campus de uma universidade pública de Sergipe. Para a análise dos dados, relativamente aos conhecimentos do professor, foram consideradas categorias discutidas por Shulman. Quanto ao ensino da probabilidade, esta investigação baseou-se nos estudos de Gal e Batanero, Contreras & Diaz. Sobre a prática reflexiva de professores utilizou-se Zeichner. A análise mostrou que os futuros professores ampliaram a base de conhecimentos para a docência de probabilidade, sobretudo em relação ao reconhecimento da necessidade de

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superação da recência positiva e negativa para a compreensão da independência de eventos. Além disso, reconsideraram a posição contrária, demonstrada inicialmente, ao seu ensino a partir dos anos iniciais. Destaca-se, assim, a importância de ações formativas para propiciar aos participantes a vivência de situações de aprendizagem que envolva conceitos de probabilidade, por meio de experimentações e reflexões.

Palavras-chave: Ensino de probabilidade; Formação Inicial de Professores; Professor Reflexivo.

Introduction

The goal of this article is to present a study that analyzed the knowledge of future Mathematics teachers about teaching probability, in particular, randomness, to primary school students. It is worth noting that the data used in this article are part of a research study carried out during the post-doctoral fellowship of the first author, linked to the Education Observatory Project at Anhanguera University of São Paulo. This project intends to establish a training and research center with the goal of investigating the teaching and learning processes of notions relating to Probability and Statistics.

We consider this topic to be relevant because learning concepts and procedures relating to probability should be a necessary step towards developing *probabilistic literacy* over the course of Basic Education. We agree with Gal (2005) that teaching probability is essential to preparing students for everyday life, since random events and chance phenomena occur regularly.

It is worth highlighting that probability is a pillar of the school mathematics curriculum on all levels of basic education in several countries: Australia (ACARA, 2010), Spain (MECD, 2014) and New Zealand (ME, 2007), for example. In the United States, the National Council of Teachers of Mathematics (NCTM, 2000) also publishes recommendations along these lines. In Brazil, the National Common Curricular Base (BNCC, acronym in Portuguese) uses Probability and Statistics as one of the guiding pillars of the Mathematics curriculum in all grades in Primary School. However, Lopes (2010) claims that, in Brazil, despite the inclusion of statistics and probability in the curricula, these topics are often placed at the end of the syllabus, so they are not always presented to students due to lack of time or lack of conviction by the teacher regarding the true importance of this pillar.

For the development of this study, we assume that the role of facilitating notions related to the topic of our study for students requires teachers to have a broad repertoire of knowledge, which enables them to make the adaptations necessary for the students' level of understanding and fosters connections between the notions to be used in different fields of Mathematics.

To this end, we organized a brief theoretical background on professional teaching knowledge, training reflective teachers and probabilistic literacy; the methodology used to collect and analyze the data; our perspective on the knowledge demonstrated by the student-teachers regarding probability and how to teach it.

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Theoretical background

It is worth noting that throughout the entire process of planning and analyzing the research data, whether to create the questionnaire to be applied initially, or to organize the learning situations and categorize the results, we assume the following ideas for our theoretical foundation: Shulman (1986, 1987), regarding professional teaching knowledge, Zeichner (1993), regarding the importance of reflective teacher training, and Gal (2005) and Batanero, Contreras & Diaz (2011), regarding probabilistic literacy.

In developing studies on how teachers teach, Shulman (1986) concluded that teachers need three types of knowledge associated to the content: specific content knowledge, pedagogical content knowledge and curriculum content knowledge. Shulman (1986, 1987) emphasizes that teaching necessarily starts with the teacher understanding what needs to be learned and how it should be taught.

Regarding specific content knowledge, which should include understanding the principles for organizing the subject to be taught and the fundamental ideas related to this subject, Shulman adds that

> [...] the teacher has a special responsibility in relation to content knowledge, serving as the primary source of student understanding of subject matter. The manner in which that understanding is communicated conveys to students what is essential about a subject and what is peripheral. In the face of student diversity, the teacher must have a flexible and multifaceted comprehension, adequate to impart alternative explanations of the same concepts or principles. The teacher also communicates, whether consciously or not, ideas about the ways in which "truth" is determined in a field and a set of attitudes and values that markedly influence student understanding. This responsibility places special demands on the teacher's own depth of understanding of the structures of the subject matter, as well as on the teacher's attitudes toward and enthusiasm for what is being taught and learned. These many aspects of content knowledge, therefore, are properly understood as a central feature of the knowledge base of teaching. (Shulman, 1987, p. 9).

Shulman (1987) emphasizes that it is not enough for the teacher to know that a determined notion, concept or procedure "is the way it is", the teacher needs to know "why it is the way it is" and should provide arguments that explain and justify the reasons for it being the way it is.

Another type of knowledge discussed by Shulman (1986, 1987) is pedagogical content knowledge, which is about teaching a determined concept or procedure. It necessarily includes the ability to select, organize and manage the components that can foster student learning by using representations, explanations, analogies and adequate arguments. Therefore, it would be very appropriate for teachers to provide a varied stock of examples, counterexamples and different approaches adapted to the students. Identifying when students have inadequate ideas on a topic and knowing strategies that can overcome them are also part of pedagogical content knowledge.

Curriculum content knowledge is related to teacher awareness of the curriculum

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recommendations for content development. Therefore, teachers should know how this topic is distributed throughout the grades and anticipate connections, if possible, between the content taught in a given year with other subjects studied in other disciplines (lateral curriculum knowledge) and teach aspects related to this content, but also pertinent to curricula from previous or subsequent years (vertical curriculum knowledge). (Shulman, 1986).

Regarding knowledge for teaching, Zeichner (1993) criticizes what he calls the dominance of conventional perspectives in the knowledge that teachers have of the subjects they teach, claiming:

> Encouraged by critiques such as those by Lee Shulman (1987), our attention is focused on the knowledge that teachers need to be able to transform into teaching content, in order to improve student understanding. There are recent studies that clearly demonstrate that knowledge of a given discipline is not, on its own, enough to be able to teach it. (p. 38)[4]

Therefore, teacher-trainers need to help future teachers internalize "the dispositions and abilities that enable them to reconsider their teaching strategies, making them responsible for their own professional development" (Zeichner, 1993, p. 55, our translation)[5]. For Zeichner (2003), reflective teaching is one of the aspects that should also be considered in initial training programs and become a practice over the course of educators' careers.

In this context, we defend that future teachers should experience teaching strategies in their initial training that enable them to ask questions and raise conjectures regarding decision-making in cases of uncertainty, in order to develop similar strategies with students in basic education and to promote probabilistic literacy.

Therefore, in planning and developing this training process, future teachers should be provided with experiences similar to those described in research results, such as those indicated by Batanero et al. (2011). These researchers suggest proposing examples of contextualized experimental situations to future teachers for their future professional work. Therefore, they consider the proposal of problems and experiments in this process to be indispensable. Along these lines, Lopes (2008), for example, also considers it important to carry out experiments.

> Probability provides a way to measure uncertainty and to show students how to do mathematics, and how to apply mathematics in order to solve real problems. To do so, we recommend teaching probabilistic notions based on a heuristic and active

[4] Original quote: "Com o estímulo de críticas como as de Lee Shulman (1987), centrou-se aqui a atenção no saber que os professores precisam de ser capazes de transformar em conteúdos de ensino, de modo a haver uma melhor compreensão dos alunos. Existem trabalhos recentes que demonstram claramente que o saber de uma dada disciplina não é, por si só, suficiente para ser capaz de a ensinar."

[5] Original quote: "as disposições e capacidades que lhes permitirão repensar as suas estratégias de ensino, responsabilizando-se pelo seu próprio desenvolvimento profissional".

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> methodology, by proposing concrete problems and carrying out real or simulated experiments (Lopes, 2008, p. 71)[6].

In the studies by Gal (2005, p. 51), we find references for analyzing probabilistic literacy. The author proposes five key classes of knowledge as the building blocks of probabilistic literacy, namely:

> 1. Big ideas: Variation, Randomness, Independence, Predictability/Uncertainty;
> 2. Figuring probabilities: Ways to find or estimate the probability of events;
> 3. Language: The terms and methods used to communicate about chance;
> 4. Context: Understanding the role and implications of probabilistic issues in various contexts and in personal and public discourse.
> 5. Critical questions: Issues to reflect upon when dealing with probabilities.

Regarding Mathematics, recent curricula recommend that teachers implement in their pedagogical practice the teaching of notions related to probability starting in the early years of Primary School. Therefore, we chose to analyze the discussions and positions of future Mathematics teachers regarding a learning situation inspired by Bryant, Nunes, Evans, Gottardis & Terlektsi (2012), which involves questions of randomness and independent events, among others. Its objectives are in line with the new requirements described in the National Common Curricular Base - BNCC (MEC, 2018).

Methodological aspects

This paper presents a description and analysis of a qualitative study, in the sense of Bogdan & Biklen (1999), regarding an investigation of the professional knowledge of future teachers regarding randomness and how to teach it. It is worth noting that the learning situation presented here was part of a sequence developed with the purpose of investigating the knowledge necessary for teachers to teach notions and procedures concerning probability in Basic Education.

Our study was carried out with the collaboration of a group of 11 students from the Mathematics Teaching Degree program on the campus of a federal public university in the state of Sergipe. This group participated for 15 weeks in training activities related to probability and teaching it. This group of students included young adults who intended to work as Mathematics teachers. The average age of these participants was 24 and they are mostly from public schools. All of them had already completed some kind of teaching activity in Basic Education, in the Supervised Internship courses or in Teaching Initiation programs.

In this context, we chose to use the *Design Experiments* methodology, following Cobb, Confrey, Disessa, Lehrer & Schauble (2003) because it allows us to carry out the

[6] Original quote: "A probabilidade proporciona um modo de medir a incerteza e de mostrar aos estudantes como matematizar, como aplicar a matemática para resolver problemas reais. Para isso, recomenda-se um ensino das noções probabilísticas a partir de uma metodologia heurística e ativa, por meio da proposição de problemas concretos e da realização de experimentos reais ou simulados".

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investigation in the same context of building and/or developing knowledge, in addition to accomplishing a dual purpose - teaching and research methodology. This methodology is a refinement of the initially elaborated proposal. Therefore, identifying inadequate ideas during the development of our experiment enabled us to continuously improve it.

The data presented in this paper are from answers given by the student-teachers to an initial questionnaire, which allowed us to identify experiences the group had regarding the study of probability and the application of learning situations that involve a discussion of randomness, which will be detailed over the course of the analysis and discussion of the data. This research also included audio recordings and a researcher who took notes on the group discussions in a field diary, in addition to reports prepared by the student-teachers at the end of the training process.

During the training process, we aimed to provide the future teachers with experiences, similar to those described in research results (Batanero et al., 2011), to be carried out in the classroom with primary school students, including those in the initial years. To do so, we created a sequence that underwent a cyclical process of analysis, revision and reinvention, with the goal of developing cognitive demands involved in the concept of probability: understanding randomness; building and analyzing sample spaces; quantification and comparing probabilities; and understanding correlations (comparing events). For this study, we present situations that took place during a session that discussed the cognitive demands for understanding randomness.

In this study, student-teachers are referred to as (A), (B), ... (K), in order to preserve their identities.

Analysis and discussion of the data

In the initial questionnaire, we asked the student-teachers to describe their experiences in relation to studying probability in Basic Education. All of them said they had not studied any topic related to this subject in Primary School and two of them had not had any contact with it in Secondary School either. However, when those who claimed to have studied probability in Secondary School were questioned further, they could not recall the topics discussed at this level of education, except for sample space and notions of probability.

Another aspect that stands out is that the same is true for Higher Education. Five of the six students who had already completed the courses Introduction to Statistics and Mathematics for Secondary School II, courses in which topics of probability are addressed, they could not recall the topics related to the subject they studied.

It was also in this questionnaire that the student-teachers unanimously said they were against teaching probability to children starting in the 1st year of Primary School, justifying that small children would not be able to understand content related to probability. In fact, Piaget and Inhelder (1975) concluded that, until the age of 8 to 9, children have no concept of randomness, but can start the process towards understanding it at this age.

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Since we believe that the future teachers needed to discuss the importance of including these topics in Basic Education for human development, we chose to begin the study with the participants analyzing the National Common Curricular Base (BNCC), which proposes teaching probability starting in the initial years.

Hence, we proposed that the student-teachers analyze the BNCC for the nine years of Primary School regarding teaching probability. In Table 1, we indicate the learning objects and skills from the first five years, for which teacher reflections were objects of the following analysis.

Table 1 - Learning objects and related skills

Grade	**Learning Object**	**Skill**
1st year	Notion of chance	(EF01MA20) Classifying events involving chance, such as "it will definitely happen", "it might happen" and "it is impossible to happen", in everyday situations.
2nd year	Analyzing the idea of randomness in everyday situations	(EF02MA21) Classifying results of random everyday events such as "not very likely", "very likely", "unlikely" and "impossible".
3rd year	Analyzing the idea of chance in everyday situations: sample space	(EF03MA25) Identifying all possible results in familiar random events, estimating those which have greater or lesser chances of happening.
4th year	Analyzing the chances of random events	(EF04MA26) Identifying, among everyday random events, those that have a greater chance of happening, recognizing the characteristics of more likely results, without using fractions.
5th year	Sample space: analyzing chances of random events	(EF05MA22) Presenting all possible results of a random experiment, estimating whether these results are equally likely or not.

Source: Created by the author based on data from the BNCC (MEC, 2018, p. 278-295)

Referred to in studies by Shulman (1986), who discusses the importance of curriculum knowledge for the professional teaching practice, we presented Table 1 to the group with the purpose of promoting discussions on the relations established between the skills and learning objects listed in the BNCC. We aimed to analyze the document with the participants with the goal of revealing the "organic and progressive [nature] of essential learning that all students must develop over the levels and modalities of Basic Education" (MEC, 2018, p. 7)[7]. In our case, we analyzed the first five years of Primary School with respect to learning probability. The following dialogue reveals the reflection made by the group of student-teachers about the learning expectations presented in the document.

[7] Original quote: "orgânico e progressivo de aprendizagens essenciais que todos os alunos devem desenvolver ao longo das etapas e modalidades da Educação Básica".

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Student-teacher C: Look, I didn't think it was so easy, when I saw that there would be randomness. It thought it was what we discussed in statistics classes, involving formulas and it's not as simple as it is there.
Researcher: Which formulas did you think would be seen at this school level?
Student-teacher C: I didn't think of any specific formula, but I noticed that what is proposed is feasible. Look...in the first and second year: looking at chance, things that might happen, what is impossible to happen, likely, unlikely and impossible events; that is easy.
Student-teacher H: And the degree of difficulty increases. From knowing if it is random to comparing which results are more likely to happen.
Student-teacher G: But, for this, you have to know how to do a sample space...
Student-teacher H: Look how it is all organized... it suggests that the teacher go slowly, step-by-step...
Researcher: This is an outline that we made so that you can learn about the document and reflect in a substantiated way on the inclusion, or not, of topics related to Probability and Statistics. Look at this example [referring to Table 1]... You can see the progression of what students are expected to learn in the initial years about randomness.

Given the discussions fostered by studying the BNCC, we proposed a learning situation that could be used in 4th, 5th or 6th grade classes, for example, with the intention of discussing randomness and the chances of an event happening, including estimating probabilities, the appropriate terms to communicate the ideas associated to chance and questions to provide the opportunity to reflect on the key points of the topic, as proposed by Gal (2005).

Initially, the professor had a black bag with a false bottom, placing 12 red blocks in one of the compartments and one yellow block in the other (false bottom), so that the group could not see how they were manipulated. Then, stating that he would draw one block from the bag, he asked the group what color it would be. In this session, in addition to the professor and group of students, another researcher also participated, who took notes on the group discussions (Field Diary).

Since most of the blocks were red, most of the student-teachers chose this color, while others, even though they knew the probability of drawing a yellow block was much lower, chose it because they argued that it could happen. Then, the teacher drew a yellow block. The students were surprised, but accepted it, because they understood that despite being very unlikely, it was still possible.

Then, the professor put the yellow block back in the false bottom, asked the students again what color would be drawn and simulated another drawing, taking out the yellow block. He repeated the same process two more times, always getting the same result, that of the yellow block.

For each new play, the future teachers were increasingly incredulous of the possibility of drawing the yellow block: "The yellow block was chosen three times, it is not possible for that to happen again" (Student-teacher D). This statement is representative of those collected and recorded in the field diary during the intervention.

Regarding this view, Bryant & Nunes (2012, p. 4) claim that: "A common mistake

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made by adults and children is to disregard the independence of successive events in a random situation". Among adults, the authors claim that what they call **negative recency** predominates, because in this age range, they believe "that, after a run of one kind of outcome, a different outcome is more likely the next time round". (Bryant et al. 2012, p. 4). Meanwhile, among children, it is more common for them to associate that an event that happened several times is more likely to happen again - **positive recency**.

Having chosen a methodology with a dual goal - teaching and research - and in the context of initial teacher training, we took the opportunity to explain that we used a false bottom with the goal of showing that an event can happen, even if it is not very likely. In this context, after our presentation, the group discussed the practice experienced in this activity, as shown in the following dialogue:

> *Student-teacher (B): I can't believe it, Professor. So, you tricked us?*
> *Researcher: But it was for a good reason.*
> *Student-teacher (F): I agree. If the event had been truly random, we would keep on trying...and we would probably not come to this result.*
> *Student-teacher (B): That's true. We really thought that after the yellow block came out several times, that the next color would be red.*
> *Student-teacher (H): The main thing was that, given this control over a random event, what she [the professor] wanted to appear, would appear.*
> *Researcher: I needed to discuss with you the issue of randomness and independent events. I wanted to point out that even an event that is not very likely can happen and that is why I used this strategy.*
> *Student-teacher (A): And this is why our planning is so important; we need to know well what we want in our classes.*
> *Student-teacher (G): And we had an example of simple activities that could be presented to the little kids, a sign that what is written in the base [referring to the BNCC] can work. In the early years, children can already deal with randomness.*

Based on Bryant et al. (2012), we observed that the group recognized that random events have certain characteristics, that is, they are not predictable events. They do not follow a pattern and are independent from one another. Given these results, we believe we can reflect on important aspects, such as, for example, the fact that, in order to identify and understand a sample space, one must develop combinatorial reasoning, because this type of reasoning allows for the interpretation of the possible results in situations of uncertainty (Godino, Batanero & Cañizares, 1996; Nunes et. al., 2011; Santos, 2010; 2015).

Analyzing this dialogue, we believe that the experience by the student-teachers led to reflections on the practice (Zeichner, 2003) and a discussion of aspects linked to pedagogical content knowledge (Shulman, 1986). This was due to the participants perceiving the possibilities of addressing initial ideas involving randomness through playful activities, as shown in the example of the learning situation they experienced during their training.

Therefore, based on the discussion resulting from the situation, we intended, during the training, to broaden reflections on positive and negative recency and on the concepts that can be advanced by using the learning situation in classes starting in the 4th year of Basic

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Education. We would also like to discuss the frequent mistakes made by students regarding aspects that facilitate or hinder learning and the use of different methodologies for teaching probability (Zeichner, 2003).

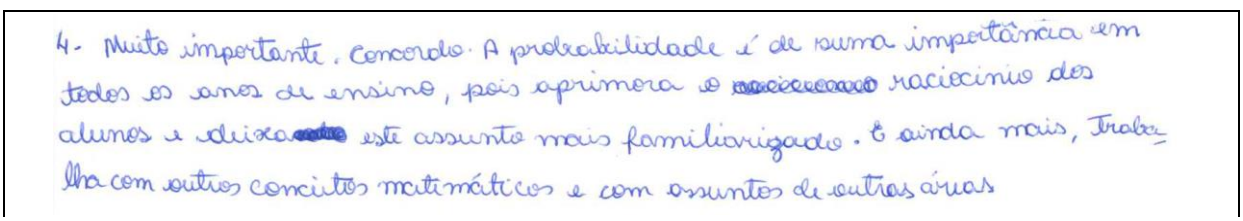
By analyzing what took place, we were able to identify that the proposed learning situation contemplates four of the five key classes described by Gal (2005). We observed that discussions about Randomness, Independence, Predictability/Uncertainty (Big Ideas) promoted the development of skills that enabled reflections on positive and negative recency (Critical questions), based on estimates of event probabilities (Calculating probabilities) and the refinement of vocabulary to communicate what happened (Language). In this activity, in particular, we did not activate the key class Context, though it is clearly important. It is worth noting that this class was the object of study and discussion for other activities in the sequence.

Therefore, we demonstrated that, during the learning situation, there were interactions between the key classes, which for Gal (2005), is necessary to develop skills towards "probabilistic literacy".

At the end of the training process, we observed that the participants broadened their professional knowledge base for teaching probability. Regarding the (re)definition of specific knowledge of the topic, when writing their impressions of the learning situation in the report, the future teachers were unanimous in recognizing that their knowledge had broadened. When asked if they had broadened their knowledge on some topic, we selected four statements:

"Yes, probability, likely, unlikely and impossible events" (Student-teacher I); "Yes, in fact, it did happen. The topics that I believe have been broadened, from the point of view of Mathematics, were: [...] randomness and probability" (Student-teacher A); "Yes. In relation to the sample space and the probability of various everyday situations" (Student-teacher H) and "the learning situation was very impactful in which the professor took the little bag and we had to know which color would come out, the possibilities, where the professor found a great way of showing us that what we thought was 'impossible' could happen". (Student-teacher K)

Moreover, they reconsidered a view against including topics of probability for students in the early years of Primary School. Upon asking the student-teachers to evaluate the importance given by the recent curricula to teaching probability and imbued by the discussions fostered with the goal of broadening their curriculum knowledge coupled with their pedagogical content knowledge, they claimed:


4- Muito importante. Concordo. A probabilidade é de suma importância em todos os anos de ensino, pois aprimora o raciocínio dos alunos e deixa este assunto mais familiarizado. É ainda mais, Trabalha com outros conceitos matemáticos e com assuntos de outras áreas

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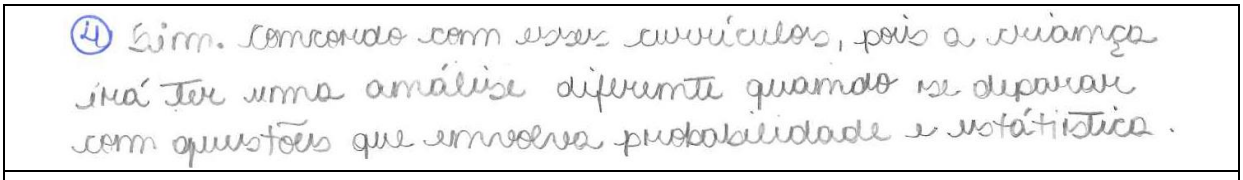
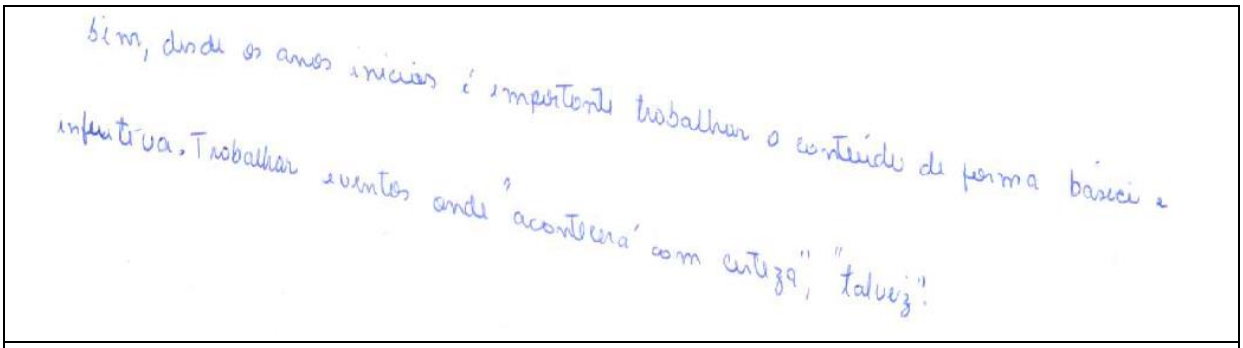
4. Very important, I agree. Probability is of the utmost importance in every year of education, since it improves student reasoning and better familiarizes them with the subject. Moreover, it works with other mathematical concepts and subjects from other areas. (Transcription)

Figure 1 - Excerpt from the report by Student-teacher G.
Source: Research archive



4. Yes. I agree with these curricula, because children will have a different analysis when faced with questions involving probability and statistics. (Transcription)

Figure 2 - Excerpt from the report by Student-teacher C.
Source: Research archive



Yes, from the initial years, it is important to work with this content in a basic and intuitive way. Working with events where "it will definitely happen", "maybe". (Transcription)

Figure 3 - Excerpt from the report by Student-teacher I.
Source: Research archive

Regarding the development of Pedagogical Content Knowledge, evidence was also found in other parts of the student-teacher reports, as we can see in the reflections presented below:

> The one with the little bag of red and yellow balls, only the yellow ones came out, leaving only the red ones. I was shocked, because it was possible to show that an unlikely event is possible to happen. (Student-teacher G)
>
> The activity [learning situation] with the bag can be used with various topics such as: greater or lesser probability, statistics and progression. In our case, since the bag was "preset", the questions asked were already expected. The possibilities of what could happen were already anticipated. (Student-teacher B)
>
> I really liked the activity [learning situation], because it is a way to show students that even when there is a very small probability of something happening, it can still happen. If we had done it without the [false bottom] bag strategy, it would not have been as fast and practical to demonstrate to the students or for them to understand. (Student-teacher C)
>
> I liked it a lot, because kids can learn probability and percentages without realizing it. The idea of making it happen is beneficial; it was very interesting and of the utmost

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> importance, because we need to show kids all of the possibilities and if we relied on luck, it would be less likely to happen in a short period of time. (Student-teacher J)

It is clear that, in general, the future teachers recognized the potential of the learning situation for discussing the results of random events as "not very likely", "very likely", "unlikely" and "impossible".

It is worth emphasizing that in these statements, we find indications of interrelations between the different categories of the professional teaching knowledge base by Shulman (1986), including curriculum, for example, when the student-teachers mention the topics (learning objects) that can be addressed during the learning situation.

Final Considerations

With the goal of identifying the knowledge of future Mathematics teachers regarding teaching probability, particularly randomness, in basic education, we saw that even among students in the Mathematics Teaching Degree program, near the end of the program, most did not take into account the issue of independent events and, after a series of repeated results, believed that a different result was more likely to happen (negative recency), which confirms the gap in training regarding topics in probability.

The learning situation presented here allows us to discuss, as defended by Zeichner (2003), the errors that Basic Education students may make when completing the learning situation (positive recency), based on the literature that addresses teaching and learning this topic. It is also necessary for the teacher to learn to use different activities, which can help provide justifications that facilitate student learning and contribute to them overcoming difficulties. In this aspect, the group recognized the potential of the false bottom bag strategy to obtain an unlikely result that enabled a discussion about randomness.

These results highlight the need to promote more discussions in initial and continuing training programs about the relevance of learning notions and procedures regarding probability, difficulties experienced by students and the importance of studying it in various stages in education. This is in addition to promoting reflections on methodologies that enable a (re)definition of specific content knowledge, pedagogical content knowledge and curriculum knowledge related to this topic.

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