Green card for the Montessori Golden Beads

Cartão verde para o Material Dourado

Alan Marcos Silva de Rezende
Luciane de Fatima Bertini

Abstract

Does the project with the Golden Material exclude the discussion that the decimal numbering system is positional? It is this question that basically drives the writing of this article, in which a study of the appropriations about guidelines that Maria Montessori put into circulation in her works and the ideas presented by the authors of the article "Cartão amarelo para o material dourado", published in Nova Escola magazine in 2016, is presented. Thus, this paper presents a point of view about the uses of the Golden Material for teaching mathematics through the History of Mathematics Education. As a result, it was possible to highlight aspects of Montessori's proposals that surround the use of the material under discussion and that show the options, arguments and solutions presented by the author in relation to the limitations of the material, limitations that are also highlighted in current discourses such as those revealed in the cited article.

Keywords: Maria Montessori; Golden Material; Appropriation; History of Mathematics Education

Resumo

O trabalho com Material Dourado exclui a discussão de que o sistema de numeração decimal é posiciona12l? É essa pergunta que, basicamente, conduz a escrita deste artigo, em que é apresentado um estudo das apropriações acerca de orientações que Maria Montessori colocou em circulação nas suas obras e as ideias apresentadas pelos autores do artigo "Cartão amarelo para o material dourado", publicado na revista Nova Escola em 2016. Assim, neste trabalho apresenta-se um ponto de vista acerca dos usos do Material Dourado para o ensino de Matemática por meio da História da Educação Matemática. Como resultado foi possível destacar aspectos das propostas de Montessori que circundam o uso do material em discussão e que mostram as opções, os argumentos e as soluções apresentadas pela autora em relação às limitações do material, limitações essas também destacadas em discursos da atualidade como os revelados no artigo citado.

Palavras-chave: Maria Montessori; Material Dourado; Apropriação; História da Educação Matemática

Introduction

The "being a teacher" is marked by different debates and analyzing them from a historical perspective is a possibility that allows listing elements that can contribute to understanding the process of training teachers. One of the authors who deals with this is Antônio Nóvoa (1995), who points out that one of the great milestones for the recognition of this profession was the creation, at the end of the 18th century, of a license (or authorization) from the State, which dictated the profile of technical competencies that would serve as a

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1 PhD in Sciences from the Federal University of São Paulo (UNIFESP), São Paulo, Brazil. E-mail: alan.ufs@hotmail.com, ORCID: https://orcid.org/0000-0002-8642-7317
2 PhD in Education from the Federal University of São Carlos (UFSCar), São Paulo, Brazil. Professor at the Federal University of São Paulo (UNIFESP), Diadema campus, São Paulo, Brazil. E-mail: luciane.bertini@unifesp.br, ORCID: https://orcid.org/0000-0003-0948-4745
basis for the recruitment of teachers, for example, qualifications, age, moral behavior, etc. It was a move to take control of the most religious body of teachers away from the Church, and hand over to the State the role of determining the dynamics of the profession's functioning.

According to this author, over the years, institutions have been created to train these professionals with a body of knowledge and specific techniques, elements that change in the most diverse contexts and in each historical time. Besides physical spaces, Nóvoa (1995) points out that the different media played an important role in the process of disseminating guidelines for teachers, especially the written press.

Catani (1996) and Catani e Silva (2010) also highlight that printed materials such as pedagogical journals put in circulation guidelines for Brazilian elementary school teachers, in some cases under the responsibility of the State. Some had/have a partnership with the Government, such as the magazine Nova Escola. In this sense, the magazines are an important source for research from a historical perspective, since they allow access to the discourses in circulation in each historical time.

The magazine *Nova Escola* appeared in 1986 to support "the mission of educating" and is still today an important communication vehicle that circulates information about teaching and learning among Brazilian teachers. Until 2014 it had an agreement with the federal government for its monthly distribution to about 220,000 public schools.

In one of its publications, issue 291, April 7, 2016, the magazine published the article "Yellow card for the golden material", in which educators presented criticism of the use of the Golden Material in mathematics teaching. This title, as the authors themselves point out, is a reference to the world of soccer, in which "the yellow card is a sign of attention. For the golden material, the same principle applies: if the teacher chooses to use it, he needs to be aware of the limits of cubes and bars" (Hamine, Soares & Peres, 2016, p. 1).

Hence the title of this article: "Green Card for the Golden Material". Also in the world of soccer, the green card has already been used in history to represent the good behavior of the player on the field. Thus, the objective of this paper is to use the historical perspective to present a point of view about the uses of the Golden Material for teaching mathematics. To do so, we take aspects that Maria Montessori put into circulation in her works and, from that, we problematize the fact that her orientations, throughout time, were sedimented through use and transformation, which characterizes *appropriations* (Chartier, 1990), like the ones put in the article of *Nova Escola* magazine. As previously discussed, by putting in circulation pedagogical discourses and orientations to teachers in each historical time, pedagogical magazines are configured as an important space in the analysis of *appropriations*.

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3 For more information about this magazine go to https://novaescola.org.br/conteudo/4944/por-que-nova-escola-existe

4 For more information, access https://www.torcedores.com/noticias/2016/10/cartao-verde-e-aplicado-pela-primeira-vez
Given this, it is worth saying that, in this study, it is understood that the appropriations are characterized by different interpretations. What is meant is that it is not the intention to seek in the ideas put forth in Nova Escola magazine the same elements that Maria Montessori put into circulation in her publications. In this text⁵, above all, problematizations are made based on Montessori’s guidelines and the aspects put under discussion in the article cited.

This movement, as Chartier (2016) highlights, does not mean to say that history should be repeated, on the contrary, each historical time has its own functioning and remembering it can help in a critical understanding of the present. In other words, from the main objective of this text, the proposal is to highlight the contribution of the History of Mathematics Education to understand how the representations about the processes of teaching and learning mathematics were built, as well as to discuss the training of teachers today and the critical use of teaching materials and didactic-pedagogical proposals.

**Maria Montessori and the development of teaching materials**

Objects carry with them different meanings, cultures, and purposes, elements that change in each historical time. This is what Burke (2015) tells in his book *O que é história do conhecimento?* You see, at one time, the paintings exhibited in salons had the purpose, besides the visibility of art, to dictate models to be followed by artists. In another context, with the advent of paper, knowledge gained a new tool for its propagation: the letter.

These are small examples, among others, cited by Burke (2015) to highlight the fact that, over time, knowledge is elaborated and carries with it different representations, resulting from the processes of systematization and dissemination of knowledge in the most diverse contexts. Certainly, objects (texts, paintings, statues, TV, etc.) are adopted as one of these representations.

In another way, objects can be considered as the concrete representation of the systematization of knowledge, of years of studies and different contexts, they carry condensed knowledge. More than that, the author emphasizes that they can be taken as an important vector of investigation for historical research that intends to analyze the production of knowledge to better understand the functioning of these movements in a process of denaturalization of practices and understandings.

In the case of teaching, of course, the discussion about the use of materials is not new, since Rousseau, Froebel, and Pestalozzi⁶, in the 18th century, started a new pedagogical proposal that considered education as a natural process of child development and that valued the biological and psychological aspects of the student (Fiorentini & Miorim, 1990). A

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⁵ This work is part of the results of the thesis "Maria Montessori and the didactic materials: condensing professional knowledge of mathematics teaching (1900-1930)", supervised by Prof. Dr. Wagner Rodrigues Valente.

⁶ A detail of Froebel and Pestalozzi's proposals can be found in Durães (2011) as well as discussions on Rosseau's proposal can be deepened from Viñao Frago (2013).
In the context of the time, abnormal, were children admitted to psychiatric clinics with cognitive development problems and also with physical disabilities (amputation of limbs, mobility difficulties, etc.). Therefore, this word will be used here with this meaning, without the use of quotation marks, italics, or bold to highlight it. If, by any chance, another meaning wants to be passed on, it will be signaled in this text.
According to Pinheiro (2017), it was a period marked by the increase of studies that were interested in the organism and the mental development of children (pedology), characteristic of a new or renewed pedagogy that claimed to be scientific through experience. It was a time when scientific pedagogy began to establish itself, based on mathematized processes of treatment of educational issues, coming from experimentation laboratories and tests, a time when one began to rethink what to teach and how to teach.

It was in this context that Maria Montessori, a prominent character in this article, broadened her studies to deal with "normal" children. She founded in Italy, in 1907, the Casa dei Bambini, in which teaching was based on the freedom of the child, considering him as an active being in learning. According to Campos (2017), two years later, the educator and doctor taught her first teacher training course, based on the studies developed with "normal" children from 3 to 6 years old.

She published, in 1909, the work Scientific Pedagogy, in Italy, and in 1912 in the United States, "[...] where it became a best-seller, being the second best-selling non-literary book in that country. As a result, it was later translated into twenty different languages" (Campos, 2017, pp. 80-81), which contributed significantly to disseminating the Montessori method, as already stated by Lourenço Filho (1930), who highlighted the recognition of Maria Montessori worldwide, based on her productions and method, as one of the major figures in the renewed education. Such production put into circulation systematized knowledge, internationally recognized, based on the conception of freedom of the child, considering the sensory, motor and intellectual capacities, disseminating manipulative materials made for the development of the sensory phases (Montessori, 2013).

Later, in 1916, in an international course held in Barcelona, the materials and methodologies developed over five years for the teaching of children from 6 to 12 years of age were presented, with a focus on the areas of Grammar, Geometry, and Arithmetic. Later, in 1934, Montessori got support in Barcelona to publish three works in Spanish: Psicoaritmética, Psicogeometria and Psicogramática. These books "constitute a psychodidactic that displays both the transposition of knowledge through teaching materials and pedagogical lessons and the search for the elucidation of how the student learns" [emphasis in the original] (Campos, 2017, p. 89). Thus, it points to the recognition of the production of mathematics knowledge (read under the headings Arithmetic and Geometry), systematized by Maria Montessori, with a paradigm shift from "how to teach" to "how one learns," placing the child as the center.

It is based on one of these works, the Psicoaritmética (1934), that, in the next topic, the guidelines set by this doctor and educator are discussed and an examination is made of the appropriations conveyed in the Nova Escola magazine. In particular, the "Golden Material" is taken as the common thread for the discussion.
Appropriations of Maria Montessori's method

The works published by Maria Montessori represent a form of concreteness of the proposal of this physician and educator. But, for this moment, it is interesting to turn our attention to the one related to Arithmetic (Figure 1):

Figure 1 Cover of the book *Psicoaritmética*

Source: Montessori (1934)

One can assume that *Psicoaritmética* (Montessori, 1934) is the greatest expression of this systematization concerning the elaboration of knowledge related to the teaching of arithmetic. This assumption is not at random, it is supported by Montessori's statement, present on the back cover of this work, that it is the result of 25 years of experience. It is an Arithmetic developed, according to the guidelines indicated by child psychology, in a long period of work, summarized in a publication of 399 pages.

According to her, it is a work in which the organization obeys a logic that goes beyond orthodox conventions and places the child's psychic development above school subjects, which surpasses the usual limit by its reproduction and by the richness of the illustrations (300 color pictures).

She also highlights that one of the biggest problems in Arithmetic is the fact of how the child is prepared to deal with abstractions, often without considering the mental development and an elementary culture, which can cause the formation of "blocks", turning the subject into an arduous learning process. But, for her, if the individual is presented with "scientifically determined" material, then a concrete means of learning will be offered that

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8 A copy of Psychoarithmétique (1934) is available in the Digital Content Repository of the Federal University of Santa Catarina, which can be accessed through the following link: https://repositorio.ufsc.br/handle/123456789/191604
will allow the child to observe things through active exercise, the result of studies in experimental psychology.

It is important to recall, then, that in these 25 years of studies, Maria Montessori used different works to elaborate her work *Pedagogia Científica*, which became known as the "Montessori method". Some examples there, as seen, are Rousseau, Froebel, Pestalozzi, Itard, and Seguin. This directs to a reflection on one of the excerpts put in *Nova Escola* magazine, to justify why the Golden Material is so criticized today:

In the 1960s, supporters of the Modern Mathematics movement agreed with Montessori and related her thinking to Jean Piaget's (1896-1980) theory about the stages of child development. It was believed that it was necessary to offer manipulative materials so that children, between the sensory-motor and operative-concrete stages, could practically replace an object with a representation. However, decades later, researchers reinterpreted Piagetian theories and began to understand concrete as related to cognitive issues. "Concrete in Piaget is not always something tangible, material," explains Camilla Schiavo. (Hamine, Soares & Peres, 2016, pp. 3-4)

Can the "Montessori method" be reduced only to the Piagetian strand? In view of what has been said so far, it is reasonable to think that making inferences about the materials produced by Montessori, starting from only one perspective, that of the appropriation of Piaget's writings, seems to minimize the years of studies of this doctor and educator.

Continuing on the relationship with Piaget, the authors point out that

*the work with concrete objects, such as the golden material, would be much more linked to the characteristics of the objects themselves - what the thinker calls empirical abstraction - and, therefore, would not help children learn mathematics. The best way to ensure that children take ownership of the numbering system is to build on the class's prior knowledge. "Recent research shows that children already bring with them some assumptions about numbers, even without knowing what tens and hundreds are" says Priscila Monteiro, Math coordinator of the Our Network program, of the Chapada Institutes of Education and Research (Icep). With these references, they create hypotheses about the grouping of numbers. "It is a new way of looking at learning that respects the children's thinking and what they can build on this system," she says. [Emphasis added]. (P. 4)*

From what is cited, important elements articulated to the materials produced by Montessori seem sedimented over time and that, in fact, without them the use can be characterized as "the use for the use," or, as put, "the use of the objects would be much more linked to the characteristics of the objects themselves. See, according to Montessori (1965), materials are constructed from a set of different elements, such as color, shape, size degree of roughness, weight, etc. For example,

A set of tablets of various colors; a set of solids of the same shape but of graduated dimensions; other objects differing in their geometric shape, and still others of equal size and different weights. Each of these sets bears the same quality, but to a different degree: it is, therefore, a graduation in which the difference from object to object varies regularly and, when possible, must be established mathematically. (p. 103)
This is a discussion about the control of the sensorial phases of the child, about taking the focus off the material and putting it on the learning of mathematics, since, according to this author, the elaboration of the materials considered a graduation that determined what should be highlighted by each one, "[...] if we intend to prepare objects that serve to make one distinguish, for example, colors, it is necessary to build them all with the same substance, shape and dimensions, differentiating them only in color" (Montessori, 1965, p. 104).

Still, paying attention to children's prior knowledge is not a subject to be treated as a discovery of current research. as seen in the quote from Nova Escola magazine, on the contrary, it was already a subject addressed by the doctor and educator. For example, Montessori (1965) points out that children know the notion of quantity acquired in the family environment, they know how to speak large quantities, such as 100 and 1000, but they do not have in their minds a clear idea of representation of them. Unlike small numbers, which are associated with the fingers of the hand, the nose, the ears, the mouth and the eyes, for example. Thus, to help in the representation, she guides the use of materials such as the Golden Material.

Montessori (1965) indicates that children are constantly in contact with life, whether outside or inside the school. However, since for children information is often presented in a "messy" way, the school has the task of trying to better organize it, and the use of materials is one way to do this. For example, the Golden Material is given a generic form of representation as a starting point to later take on the role of different elements that can be found in the everyday life of a child and that are unlikely to be placed inside a classroom, such as cars, houses, trees... On the other hand, we have what the authors say in the journal:

> Another criticism is that the systematic use of this material creates an unnecessary intermediate step in learning. [emphasis added] "If a child needs to solve a problem with strawberries and jaboticabas, why convert that to wood and then convert it back to numbers? If necessary, the fruits themselves can be the manipulated material, you don't need an intermediary system in that process," argues Celia Carolino. (Hamine et al., 2016, p. 3)

The purpose of using the Golden Material, according to Montessori (1934), is for teaching the base-ten system, so how would the use of strawberries and jaboticabas help in understanding it? These are different purposes. The material was developed for specific situations and with predetermined goals, including the "intermediate step" itself, different from what happens with the use of "strawberries and jaboticaba".

These approximations and distancing between the quotes from the magazine and the speeches of the doctor and educator that characterize the appropriations, not only that, but they also point to the fact that Montessori (1934) presents elements for the teaching of the decimal system in her work Psicoaritmética that, if taken in isolation, may present gaps in the understandings. In addition to what has been seen, an example of this is in the guidelines for teaching units and ordering, using the prismatic sticks, which go from 1 to 10, each following unit has ten centimeters more than the previous one, 1 unit = 10 cm, 2 units = 20cm, and so on, until reaching 10 units = 100 cm. At each ten centimeter piece, there is an alternation of the colors blue and red, which would help in the perception that each following piece was increased by one unit (Figure 2).
Although the intention is to teach the notion of quantity and relative positions, Montessori (1934), points out that, with the composition and decomposition of the pieces, children end up performing additions and subtractions. However, he stresses a caution to be taken with the limitation of the resource. If it is proposed to perform combinations involving such operations, it is necessary to think of those that do not exceed 10 units as a result. Thus, when there are limitations, the author suggests the use of another material, which is perhaps why the average of 2.5 materials per chapter. Therefore, if the guidance for the use of other materials is not taken into consideration, it may lead to gaps, as said in the previous paragraph.

According to Montessori (1934), to overcome the problem with results that exceed 10 units, it is necessary to introduce a new group of classes. However, first of all, one must facilitate for the child the construction of the decimal system by itself, counting and calculating are things that are achieved as a consequence of learning. For this educator, the essential qualities for putting facts within the reach of students are simplicity and clarity of things. For this, she proposes the use of another resource, the "decimal system material" (Figure 3).
The single-colored, wooden material consists of loose pieces, small sticks of ten pieces, "squares," made up of ten sticks, which are 100 loose pieces, and finally the cube which is constructed of 10 "squares," placed one on top of the other. All the pieces, which are grouped together, are held on by wires (Montessori, 1934). The intention is to highlight the different types of pieces and their representations. About this, the author points out that

From a psychological point of view, it has been noted that in order to better reveal a certain quality, it will be necessary, as far as possible, to isolate the senses: a tactile impression is clearer when it is an object that does not conduct heat, that is, that does not carry with it, at the same time, sensations of temperature; the impression will be all the more perceptible if the person is in a dark and silent place where he cannot receive visual impressions, nor auditory, which could disturb his tactile impressions. (Montessori, 1965, p. 104)

Also with respect to the material of the decimal system, the proposal was not only to teach the representation of numbers. But, from the composition and decomposition of small and large numbers, work with the notion of representation with units from one to ten as when working with the prismatic sticks and with those of new classes (ten, hundred and thousand) that appear when having results greater than or equal to ten units. However, understanding the concepts of ten, hundred, and thousand is very different from truly taking ownership of the numbering system. The decimal base is only one of the elements that form the way we represent quantities. Other elements are symbols and the recognition of the value of digits depending on their position in numbers. Both of these aspects do not come into play in the manipulation of the golden material. [emphasis added] In the decimal system, each digit can take on several values. If written alone, the 2 can represent two units, but also 20, 200 and so on, depending on whether there are other digits to its right. (Hamine et al., 2016, p. 2.

Given this, it is important to say that these elements were also considered by Montessori (1934). The orientation was that the exploration of the decomposition of the numbers should take into account "both the effective quantities with the relative groupings of units according to the decimal system, and by what refers to the numerical symbols they represent" (p. 26). which indicates the knowledge of this doctor and educator about the limitation of the Golden Material being non-positional. For this, the material proposed by the author for the work with the decimal system included the objects (decimal system material) and the numerical ciphers (cards), as can be seen in Figure 4, in which she presents the "Exhibition of the Decimal System in its entirety", and in Figure 5 with the record of the representation of the number 958.
Figure 4: Decimal system material

Source: Montessori (1934, p. 21)

Figure 5: Representation of the number 958

Source: Montessori (1934, p. 22)
The cards were presented with different colors for units, tens, hundreds, and thousands, as in the following example of the number 1235 (Figure 6).

Figure 6: representation of the number 1235

Source: Montessori (1934, p. 26)

Thus, the cards would contribute to explore the positional characteristic of the decimal numbering system, since, according to the author, through the superposition of the cards each number would occupy "the place that corresponds to it" (Montessori, 1934, p. 25). From the notion of composition and decomposition built with the materials, the intention was to make the student perceive "both the effective quantities with the relative groupings of units according to the decimal system, and by what refers to the numerical symbols they represent" (Montessori, 1934, p. 26), going against the criticisms posted in the *Magazine*. So, it is worth saying: green card for the Golden Material!

**Considerations**

Since the main objective of this work is to use the historical perspective to present a point of view about the uses of the Golden Material for teaching mathematics, we took elements that Maria Montessori put into circulation in her works to problematize the appropriations put in the *Nova Escola* magazine.

It is reiterated that the intention was not to judge what is "right" or "wrong", on the contrary, it was to examine the closeness and distance of the representations put forward in the article "Yellow card for the golden material" to Montessori's guidelines, and from that to characterize appropriations, in the sense defined by Chartier (1990), based on their uses.

Anchored in the historical perspective, it was possible to highlight that, over time, Montessori's guidelines for the use of the Golden Material gained different representations, such as, for example, the non-indication of the use of the cards for the record in the manipulation. It is worth pointing out, however, that it is these representations that contribute
to the continued use of this material in mathematics teaching, even long after the publication of Psychoarithmetic (Montessori, 1934).

Likewise, the History of Mathematics Education allows a look at the representations that circulate and have circulated among teachers who teach Mathematics, which can help in the process of denaturalizing practices and the awareness that every material has limitations and, therefore, it is necessary to know them in order to seek alternatives to complement their uses. These are elements that help to understand the professional knowledge of the teacher who teaches Mathematics. An example of this is the criticism of the Golden Material, which argues that it is not positional and that it does not consider the student's previous knowledge, aspects that were questioned throughout this work.

Thus, in view of what has been presented, we defend the green card for the Golden Material. Not in the sense that it is the best, the only, or that it is indispensable and sufficient for teaching the numbering system, but in the sense that the proposal in its entirety offers elements for discussions and studies that take into account the characteristics of the decimal numbering system. Above all, it can be highlighted how the use of the History of Mathematics Education can contribute to the process of initial and continuing education of teachers who teach mathematics.

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