



Energy consumption: analysis of a real situation for the mathematics functional study in high school

Consumo energético: análisis de una situación real para el estudio funcional de la matemática en la escuela secundaria

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Abstract

The current culture of information and technologies requires new approaches in education according to the demands of society. The current curriculum and the practices of studies in high school do not encourage the development of these key requirements for any citizen. In particular, in Argentina disappointing results are reported in relation to the performance of students in mathematics. In this work, based on the Anthropological Theory of the Didactic, an analysis of a situation for the functional study of mathematics is proposed. This proposal is in correspondence with the Argentinian high school renovation plan. In this work, a possible path of questions and answers around a real situation is presented. These questions are formulated in a strong sense, i.e., they demand not only information as an answer, but also the study of useful tools to address them and generate new questions.

Keywords: Mathematics, Anthropological Theory of the Didactic, High School.

Resumen

La cultura actual de la información y las tecnologías exigen nuevos enfoques en educación acordes a las demandas de la sociedad. El currículo actual y las prácticas de estudios en secundaria no favorecen desarrollar estos requisitos, necesarios para cualquier ciudadano. En particular, en Argentina se reportan resultados desalentadores con relación al desempeño de los estudiantes en matemática. En este trabajo, con fundamento en la Teoría Antropológica de lo Didáctico, se propone el análisis de una situación para el estudio funcional de la matemática. Esta propuesta se encuentra en correspondencia con el plan de renovación de la escuela secundaria argentina. En este trabajo se traza un posible recorrido de preguntas y respuestas en torno a una situación real, vinculada al ahorro energético. Se procura que se formulen preguntas en sentido fuerte, es decir que no solo demanden información como respuesta, sino del estudio de herramientas útiles para abordarlas y generar nuevas preguntas.

Palabras clave: Matemática, Teoría Antropológica de lo Didáctico, Secundaria.

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Introduction

Recent studies in Argentina show that the teaching of mathematics requires urgent attention because the Argentine students' achievements are not encouraging (Secretary of Innovation and Educational Quality, 2018). D'Amore, Godino and Fandiño (2008) identify students' distancing towards mathematics and indicate that this is not due to knowledge itself. This is due to the way mathematical knowledge is presented and the lack of interaction between the real world and the knowledge taught in the classroom. This immerses us in the need to modify the ways of thinking and proceeding in the classroom, seeking to involve students in a different, useful and functional education to 21st century citizens (Chevallard, 2017; Pochulu, 2018). In Argentina, a plan to renew high school education has been launched (Ministerio de Educación de la Nación, 2017). The purpose of this plan is to train citizens in an interdisciplinary study of knowledge, assuming as a principle that the problems of today's society need to be studied from the integration of interdisciplinary knowledge. In contrast to atomized and encyclopedic learning, integrated learning, seeks to achieve knowledge from project-based learning (Dirección de Educación Secundaria de la Provincia de Buenos Aires, 2017). The interdisciplinary study aims to overcome the fragmentation of teaching and learning, proposing dialogue, articulation and linkage between knowledge. This highlights the need to have feasible didactic devices to be implemented in the current high school that strengthen the linking of the disciplines. This implies modifications in the current educational system as the current plan of Buenos Aires province in Argentina is organized in disaggregated disciplines in charge of a single specialist teacher of the discipline. The new approach of high school requires modifications in teaching times, study organization and the role of the actors of the teaching system.

Project-Based Learning (PBL) proposed by the new high school approach is a pedagogical paradigm in which students are encouraged to plan, implement and evaluate projects that are correlated with the real world beyond the classroom (Blank, 1997; Dickinson et al, 1998; Harwell, 1997). One of the American precursors of PBL was William Heart Kilpatrick who traced the foundations of the project methodology at the beginning of the 20th century. Kilpatrick advocates an "experimental philosophy of education" in which knowledge is gained through experience (Kilpatrick, 1967b). Likewise, it criticizes the defragmentation of knowledge in subjects, course or areas, since learning in isolation means that "the student does not see or feel the usefulness or relevance of what is taught for any matter that interests him in the present, and therefore does not intelligently adhere to the current situation" (Kilpatrick, 1967a, p. 49).

The adoption of a PBL (and any other educational innovation) in public education is a complex task (Florensa, Bosch, Gascon & Winslow, 2018). Elementary and high schools are constrained to a compulsary state curriculum and expected to produce a uniform product. The usual practice is structured in specific temporary blocks and organized by issue. In this structure there is not enough room for teachers to support the study of problems that require longer times than usual, divorced from traditional teaching where problems require

immediate answers and do not invite further inquiries. The renovation plan of the Argentine high school (Ministerio de Educación de la Nación, 2017) advocates for a pedagogical paradigm, where the promoted educational objectives, the useful means, and the phenomena to which it reacts are generic, independent of the specific disciplines. Despite these changes, this proposal is not enough to address the problem of learning mathematics. Instead, a didactic paradigm which depends on a specific school discipline and a way of organizing its study process is required. In this work, based on the Anthropological Theory of Didactic (ATD) (Chevalard, 1999, 2007, 2013, 2017), the analysis of a situation is proposed for the functional study of mathematics in correspondence with the renovation plan of Argentine high school. The ATD proposes to introduce functional study processes into teaching systems, where knowledge does not constitute monuments that teachers teach students, but rather useful tools to study and solve problems. During the study, it is sought to formulate questions in a strong sense, that is, that they demand not only information as an answer, but also the study of useful tools to address them and generate new questions.

Framework

In this work, the Anthropological Theory of Didactic is adopted as a theoretical reference (Chevallard, 1999, 2013, 2017). Following the lines of research proposed by the theory, the need arises to introduce functional study processes into teaching systems, where knowledge does not constitute monuments that the teacher teaches to students, but rather material and conceptual tools, useful for studying and solving problem situations. This is characteristic of an emerging paradigm and opposed to the traditional one: instead of studying unmotivated knowledge, in response to questions whose origin is unknown or hidden, umbilical questions are formulated that require the study of material and conceptual tools, useful for studying and answering questions endlessly. The Study and Research Path (SRP) are devices that would allow facing the process of monumentalization of knowledge and bring to life What Chevallard calls teaching by research in the mathematics class. (Ladage & Chevallard, 2010). Carrying out the methodology that involves the SRP requires incorporating a set of didactic gestures, which imply radical modifications with respect to traditional teaching. Its educational objective is to create new postures towards learning, characterized by the attitude of problematization, associated with the herbartian, procognitive and exoteric character (Chevallard, 2013). That is, herbartian in the sense that the engine of learning is the receptive attitude towards the formulation of questions and unsolved problems; procognitive in the sense of considering knowledge to be discovered and not reviewing knowledge already discovered; exoteric in the sense of immersed in study because there is always room for new knowledge about a discipline; and finally the attitude of problematization is characterized by asking questions, so that some become problems for at least one people group.

The implementation of a teaching by SRP radically changes the relationship between the teacher, the students and knowledge. This implies changes in relation to didactic times, the way in which the study is organized and the place occupied by the actors of the didactic

system in the class. A teaching by SRP presupposes the study of questions that are agreed by all the members of the study community. This demands to distribute responsibilities and assign individual tasks, and then return to the group process of preparing responses. The works found or rediscovered to elaborate the answers, will be studied with a certain level of depth, to establish their relevance. In this way, new questions will also arise, which the study community will decide when and how to answer. Therefore, the responsibility of the study does not fall on the individual, but on the producing community, which sustains and validates the answers it generates collectively.

In this work, a possible set of feasible questions and answers is drawn from the analysis of a real situation, which has its origin in the analysis of energy consumption through the use of lighting lamps. The management of this proposal would allow to carry out some gestures typical of an education by SRP and in accordance with the education that is projected by the argentine high school.

Methodology

Researchers within the ATD framework use specific methodologies to design, manage and describe the knowledge of study processes. One of these resources is Didactic Engineering (DE). It is a research methodology to design study processes in which specific knowledge is involved with the purpose of observing, studying and analyzing phenomena related to the knowledge involved; as well as designing teaching proposals to overcome the unsatisfactory results observed in the practices (Artigue, 2013). DE allows researchers to systematically design, experiment and analyze study processes (Artigue, 2008). As a central tool to manage and describe the different phases of the ID methodology, the ATD proposes the notion of Questions - Answers Maps (Q - A Map). These maps are representations that in an inquiry process can be used to make explicit the questions derived and addressed by both the students and the teacher, as well as the answers obtained. The use of these maps has also been proposed as a model used by teachers to communicate and describe study processes (Winsløw, Matheron & Mercier, 2013). In more recent works, both teachers and researchers have used them in the design phase and in the management and evaluation of SRP (Barquero, Bosch & Romo, 2016; Jessen, 2014). In this work it is assumed that the ATD and in particular the DE methodology and the Q - A Maps can improve the initial selection of projects and provide tools for the management and communication of this type of study process.

In the following section, a possible Q - A Map is drawn around a real situation that originates from the analysis of energy consumption with the use of lighting lamps. The study begins with the analysis of a newspaper article that is available to any citizen. The investigation of this article leads to the study of different lighting devices, the way in which energy consumption is billed and the way of purchasing and paying for lighting fittings.

Energy consumption: analysis of a proposal for the study in high school

The situation begins with the analysis of a newspaper article that refers to a campaign by the Argentine government that promotes the use of LED lamps for energy saving (Annex I). One of the essential questions that can be derived from the analysis of the article is:

 Q_0 : How to light environments efficiently?

In this question, efficient lighting refers to the use of lamps that generate quality lighting at low cost, and with little damage to the environment. Q_0 is an open question with great generating power, which neither provides any numerical data, nor specifies which are the variables that should be taken into consideration to describe the system. The study generates a tree of questions whose answers are not linearly inscribed in school textbooks. The study requires an in-depth analysis of different media to provide an answer, including web pages, for example, of manufacturing lamps companies; retail lamps companies, electricity supply companies, etc. This makes students investigate and break with the traditional structures of the school, resorting to different media, favoring collaborative work and elaborating answers from their own inquiry activities.

Here are some questions that can be derived from Q_0 :

 Q_0 : How to light environments efficiently?

 Q_1 : How to produce light radiation?

 $Q_{1,1}$: How to control light radiation?

 $Q_{1,1,1}$: How does an incandescent lamp work?

 $Q_{1,1,2}$: How does a low consumption lamp work?

 $Q_{1,1,3}$: How does a led lamp work?

 $Q_{1,1,4}$: How does a halogen lamp work?

 Q_2 : How to calculate the energy consumption generated by lamp?

 $Q_{2,1}$: How to compare the cost of electrical energy generated by different lamps?

 $Q_{2,2}$: How do energy supply companies invoice users' consumption?

 Q_3 : How to minimize the costs in the acquisition of lighting?

 $Q_{3,1}$: What difference in costs does it imply to adopt different means of payment to buy lamps?

Briefly, these questions are proposed, which in turn can generate other questions. In particular, for the study of Q_0 three essential questions arose: Q_1 : How to produce light radiation?, Q_2 : How to calculate the energy consumption generated by lamp? y Q_3 : How to minimize the costs in the acquisition of lighting? From the study of Q_1 a tree structure of questions is created leading to an essentially physical-mathematical study, which requires

researching the different types of lamps that can be accessible to any user and analyzing their operation. This study leads to research how to compare the different luminaires, focusing the study on heir components, their impact on energy saving and the environment. Likewise, from the study of Q_2 a set of questions is created leading to an economic-mathematical study, which is linked to comparing the operation of the lighting and studying how energy supply companies bill users for their consumption. Finally, from Q_3 questions are derived related to an economic - mathematical study that involves comparing different payment systems for the acquisition of products in the market. In particular, the analysis of Q_0 can lead to the development of projects where the interdisciplinary study would occupy a prominent place. The projects could focus on inquiring about the type of lighting that the student homes have, what cost these homes must face for the consumption they make, what type of lamps would be convenient for them to replace the ones they have, when they will be able to perceive benefits of changes by analyzing energy cost savings and lamp replacement cost. This study could also lead to an analysis of ways of acquiring lamps, and the possibilities of replacement according to the conditions of the students. Moreover, projects can be generated to raise awareness among the population; such as of lamp replacements in terms of economic and environmental impact, etc.

In this work only some of the proposed questions will be deepened, emphasizing the situations where mathematics is useful to understand the system that is analyzed. In particular, the study around Q_2 , where, by considering different types of lamps, the following set of questions can be derived:

 $Q_{2,1,1}$: How to compare the cost of electrical energy generated by an incandescent lamp and by a low consumption lamp?

 $Q_{2,1,2}$: How to compare the cost of electricity generated by an incandescent lamp and a led lamp?

 $Q_{2,1,3}$: How to compare the cost of electrical energy generated by an incandescent lamp and a halogen lamp?

 $Q_{2,1,4}$: How to compare the cost of electrical energy generated by a low consumption lamp and by a led lamp?

 $Q_{2,1,5}$: How to compare the cost of electrical energy generated by a low consumption lamp and by a halogen lamp?

 $Q_{2,1,6}$: How to compare the cost of electrical energy generated by a led lamp and a halogen lamp?

The comparison suggested by the questions above requires considering lamps that have similar characteristics. For example, consider the question $Q_{2,1,1}$: How to compare the cost of electrical energy generated by an incandescent lamp and by a low consumption lamp? The study of this question fosters a functional algebraic modeling process in which the interplay between parameters and variables progressively takes on an essential role.

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As an example, the study is developed for incandescent and low consumption lamps, but the same can be done for the other lamps. In principle, the argentine state prohibited the commercialization of incandescent lamps and campaigns were carried out for their replacement by low consumption lamps. Although incandescent lamps are not currently marketed, it is not ruled out that they continue to be used in some homes. Therefore, it is of interest to analyze and understand why incandescent lamps are not on the market, since their commercialization lasted for more than 100 years. In particular, its energy consumption and environmental impact contrast with low consumption and LED lamps.

When consulting the market for different low consumption lamps, the variety turns out to be immense. Consider the following lamp for which the seller sets the following specifications:



Descripción

TIENDA OFICIAL PHILIPS

Lámpara Bajo Consumo Philips Marca: Philips Modelo: Mini Twister Consumo: 23W Equivalencia: Ilumina como 100W Lúmenes: 1455 Im Zócalo: E27 Conexión: 220V Requiere transformador: NO Temperatura de color: Cálida (2700K) Dimerizable: NO Vida útil: 10.000 h Temperatura de color: Luz Cálida (2700K-4000K):Este tipo de luz se utiliza para producir atmósferas traqnuilos, de descanso y relajantes, motivo por el cual resulta ser una excelente opción para iluminar dormitorios, livings, salas de estar, etc. Esta luz tiene un tono más amarillento. Luz Fría: (6000K-7000K): Este tipo de luz suele emplearse para generar ambientes dinámicos, mejorar las condiciones de visibilidad en el desarrollo de tareas específicas e incentivar la concentración. En virtud de lo anterior, es común observarla en espacios públicos y donde se realizan labores concretas. Se recomienda para oficinas, cocinas, baños, lavanderías, talleres y áreas de trabajo en general. Esta luz tiene un tono más azulado Philips Home Lighting

Figure 1. Characteristics of low consumption lamp **Source**: Argentine Free Market

On the other hand, let us consider an incandescent lamp with power characteristics similar to the low consumption lamp. Here we cannot count on the current value of the lamp, since they have disappeared from the market, being that by law 26473/08 their commercialization is prohibited. Figure 2 shows the essential characteristics of an incandescent lamp whose power is equivalent to the low consumption lamp detailed in Figure 1



Figure 2. Characteristics of incandescent lamp

Source: http://www.hansaindustria.com.bo/product/iluminacion/incandescentes/lampara-incandescente-fantasia-100w/839/

The study of the technical sheet of both lamps leads to formulate the following questions: $Q_{1,1,1}$: How does an incandescent lamp work? $Q_{1,1,2}$: How does a low consumption lamp work? This study leads to analyze the specifications indicated for each lamp by the manufacturer: power, luminous flux, color temperature, duration. This characterization is involved in the study of Q_1 and requires a physical analysis that exceeds this work. In particular, for the study proposed in this text, the analysis of $Q_{2,1,1}$: How to compare the cost of electrical energy generated by an incandescent lamp and by a low consumption lamp? To study this question, the energy consumption invoice of a particular city will be analyzed (Figure 3).

370	00487	0983	58001		3 3	Tarifa 1R	02/19		
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Figure 3. Energy invoice of a user in the city of Tandil **Source:** User's energy invoice

The invoice is made up of two parts: Concepts of Energy and Tax Charge. The former refers to the costs according to the thousand watts per hour (kwh) consumed by the user (Fixed Charge, Energy and Incr, Cost May. Purchase Distributor) while the latter is made up of: VAT, provincial taxes and municipal ordinances; each one results from obtaining a percentage of the cost of energy consumed by the customer. The cost of electrical energy can be calculated using the following expression, reconstructed from the analysis of the invoice and consulting the website of the energy supplier company:

$$C(t, w) = E(t, w) + I(t, w)$$
 (1)

Here t indicates the measurement period in days; w indicates one thousand watts per hour (kwh) consumed by the customer in the period considered; E(t,w) corresponds to the expression of a function that allows calculating the cost of energy and I(t,w) corresponds to the tax burden. In particular, E(t,w) turns out to be:

$$E(t, w) = \frac{P(R)}{30.5}t + k(R)w + qw$$
 (2)

This expression is made up of a set of parameters that depend on the kWh consumed by the user and, in particular, the value of some of them depends on a category *R* assigned to the client, according to the consumption of kWh in the period in which the measurement is made:

P(R) represents the fixed charge.

k(R) is the cost per kWh consumed in the period.

q represents the wholesale cost increase and established by a resolution of the Energy Ministry. This is a price difference that is allocated to the energy producer and to the invoice indicated in Figure 3 it corresponds to the value 0.3 according to a resolution of the Electric Energy Control Agency (Res. 216/18).

On the other hand, the tax burden expression I(t, w) turns out to be a function of the form: $i(x) = k_1 x + k_2 x + k_3 x + \dots + k_n x$ where k_i represents the percentage on the energy consumption of the different taxes established to the same. Thus, I(t, w) results from composing to E(t, w) whit i(x), namely, $I(t, w) = i^{\circ}E = i(E(t, w))$.

In particular, C(t, w) is a function of two variables of the polynomial type of degree 1. The independent variables turn out to be t and w but also the value of w determines the values of the parameters of the expression. This study turns out to be radically different from the typical one in high school. According to Ruiz (2010), Ruiz-Munzón, Bosch and Gascon (2011) in secondary education literal symbols play almost only the role of an unknown quantity (in equations) or the role of variables (in functional language). In general, the role of parameters is practically absent. This situation continues throughout secondary education and makes it extremely difficult to move from work with analytical expressions of elementary functions to the study of families of functions and the use of these families as models of systems in which relationships between magnitudes appear.

Finally, it turns out that the cost of electricity invoiced by the company according to the invoICE in Figure 3, is given by the following expression:

$$C(t, w) = 1.4560 \left(\frac{P(R)}{30.5} t + k(R)w + qw \right)$$
 (3)

To our case, we can confirm that the consumption of the energy invoice to 158 kwh and to a period of 31 days, turns out to be C(t, w) = \$ 893.04. This value was rounded to two decimal places, just like all the values indicated on the invoice. A small difference is detected in relation to the billing value, because if the calculations are made through the expression using a calculator or software, the operations are carried out with more than two decimal places. On the other hand, in the invoice all calculations are rounded to two decimal places. This type of task is interesting, since it requires the study of rounding and truncation of decimal numbers and the estimation of the error of the calculations.

In general, equation (3) can be written as follows:

$$C(t, w) = A\left(\frac{P(R)}{t_p}t + k(R)w + qw\right)$$
 (4)

 t_p indicates the theoretical measurement period by the energy company. While wconditions the parameters P(R) y k(R), since the value of the parameters is established for the range of watts consumed by the client. Finally, A will depend on the taxes that are established.

Expression (4) can be transformed in such a way that it allows comparing the cost of energy for each hour of consumption according to the lamp used, resulting as follows:

$$C(t, w) = A\left(\frac{P(R)}{732}t + k(R)wt + qwt\right) \quad (5)$$

In particular, for a user of category 1R, which is the category that corresponds to the user of the invoice in Figure 3, the tariff chart established by the energy supplier company turns out to be:

Cuadro Tarifario Vigente para consumos a partir del 1º de Octubre de 2018 Tarifa 1 - Pequeñas Demandas										
1R - Residencial -	Tarifa Plena									
Cargo Fijo 1			71,35	\$/mes						
Cargo Variable 1	(Consumo	≤ 100 KWh - mes)	2,6744	\$/KWh						
Cargo Fijo 2			109,18	\$/mes						
Cargo Variable 2	(100 < Consumo	≤ 200 KWh - mes)	2,8796	\$/KWh						
Cargo Fijo 3			148,90	\$/mes						
Cargo Variable 3	(200 < Consumo	≤ 400 KWh - mes)	3,0327	\$/KWh						
Cargo Fijo 4			184,83	\$/mes						
Cargo Variable 4	(400 < Consumo	≤ 500 KWh - mes)	3,1573	\$/KWh						
Cargo Fijo 5			296,41	\$/mes						
Cargo Variable 5	(500 < Consumo	≤ 700 KWh - mes)	3,3350	\$/KWh						
Cargo Fijo 6			479,86	\$/mes						
Cargo Variable 6	(700 < Consumo	≤ 1400 KWh - mes)		\$/KWh						
Cargo Fijo 7	•	•	585,77	\$/mes						
Cargo Variable 7	(Consumo	> 1400 KWh - mes)	3,8309	\$/KWh						

Figure 4. Tariff CHART of the energy supplier company

Source: Energy supplier company

In this way, the electrical energy cost function for an incandescene lamp $(C_I(t))$ and for a low consumption lamp $(C_{BC}(t))$ with the characteristics considered in Figure 1 and Figure 2 respectively, turns out to be:

$$C_{I}(t) = \begin{cases} 0.57t & si & 1 \le t \le 1000 \\ 0.68t & si & 1001 \le t \le 2000 \\ 0.78t & si & 2001 \le t \le 4001 \\ 0.87t & si & 4001 \le t \le 5000 \end{cases} (6)$$

$$1.11t & si & 5001 \le t \le 7000 \\ 1.51t & si & 7001 \le t \le 14000 \\ 1.77t & si & t \ge 14001 \end{cases}$$

$$C_{BC}(t) = \begin{cases} 0.24t & si & 1 \le t \le 4347 \\ 0.32t & si & 4348 \le t \le 8695 \\ 0.41t & si & 8696 \le t \le 17391 \\ 0.48t & si & 17392 \le t \le 21739 \\ 0.71t & si & 21740 \le t \le 30434 \\ 1.08t & si & 30435 \le t \le 60869 \\ 1.30t & si & t \ge 60870 \end{cases} (7)$$

 $C_I(t)$ and $C_{BC}(t)$ are two piecewise functions for category 1R. If the cost function for each lamp is compared, a greater slope can be observed for the use of incandescent lamps considering the same range of watts consumed. The variation in cost between the use of one lamp or another lies in the "speed" of consumption for the change of branch of the function according to the range of consumption. This causes the category of the consumer to vary, and in this way the costs of energy consumption are increased.

The expressions that allow calculating the consumption as a function of the time of each lamp correspond to direct proportionality functions, where the difference in the slope is noticeable. Thus, for each lamp it turns out that the watts consumed per hour can be calculated as follows:

$$L_{BC}(t) = 0.023t$$
 (8)
 $L_I(t) = 0.1t$ (9)

From these expressions it can be verified that if we have the low consumption lamp on 24 hours a day, after 181 days (4348 hours) the consumption will be greater than 100kw. While for the incandescent lamp after 1000 hours, the consumption of 100kw is produced, which is equivalent to having the lamp on 24 hours a day for 42 days. In this way, it can be verified that the consumption of the incandescent lamp turns out to be more than four times that of the low consumption lamp. Thus, for the change of category of the client according to their consumption, it turns out that after 1000 hours of use of the incandescent lamp, the consumption reaches 100kwt, which means that another branch of the function $C_I(t)$ has to be considered. On the other hand, this variation in the low consumption lamp occurs after 4348 hours of use. In addition, in this instance the incandescent lamp should be replaced because according to the manufacturer's instructions, the average lifespan of the lamp is 1000 hours. Thus, a great advantage is evident in relation to the use of low consumption lamps and incandescent lamps of equivalent power. For each lamp the consumption of watts per hour can be represented graphically as follows:

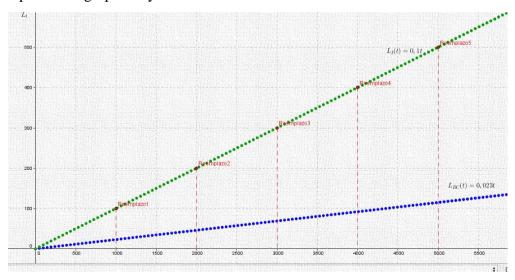


Figura 5. Representación gráfica de $w_{BC}(t)$ y $w_I(t)$ **Fuente:** elaboración propia.

The graph in Figure 5 represents the consumption in watts for each of the lamps as time elapses ($L_{BC}(t)$ and $L_I(t)$). The graph shows greater growth of $L_I(t)$ in relation to $L_{BC}(t)$, which shows greater consumption in the same period of time. Likewise, the instances in which the incandescent lamp should be replaced are highlighted in red, taking into account its time use. This implies an additional cost for the user who uses incandescent lamps, in relation to the one who uses low consumption lamps.

Conclusions

This paper outlines part of the analysis of a real and current situation that can generate a set of questions that could lead to the development of projects compatible with the renovation plan of the argentine high school. In particular, the activity focused on identifying a problematic situation and delimiting the system, formulating questions whose answers require a process of inquiry. These answers must be elaborated by the study community, since they are not inscribed verbatim in any media. Also, the study leads to select certain aspects of the system that are symbolized as variables and to establish mathematical relationships between the variables of the system. In particular, the development that has been proposed allows rediscovering the utility of different mathematical notions (polynomial function, function of one and two variables, piecewise function, variable, parameter, estimation by rounding and truncation, composition of functions), and linking them to study a real situation.

The map of questions and answers shows that the management of a study from the proposed situation facilitates the development of attitudes typical of a teaching in the world questioning paradigm. The study requires a receptive attitude towards the formulation of questions and unresolved problems. Knowledge requires inquiry and elaboration rather than the review of previously discovered inform.

It is highlighted that the management of the proposal requires that the teacher assume the role of facilitator, giving the students room to investigate, propose and study situations of their interest. Likewise, the management of the study will require the teacher to make the necessary decisions to prevent the initial situation from fading away, seeking to keep the problematization of it alive. Previous studies developed in the world questioning paradigm (Corica, 2018; Corica & Otero, 2019) indicate the necessary incidence of the teacher in this sense, since students must also be predisposed to change from the role that traditional teaching has given them as mere spectators to more study- committed one.

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Annex I

Lunes 28 de Wayo de 2018

Para pagar menos luz: habrá lámparas LED en cuotas y con Precios Cuidados

El Gobierno lanzó la campaña "Cambiá el foco" para impulsar el uso eficiente de la energía. Las lámparas LED estarán incluidas en el Ahora 12. Y hay una calculadora online para que cada usuario estime cuánto menos puede pagar en sus boletas.





En medio de la discusión que se está dando en el Congreso por el ajuste tarifario, el Gobierno lanzó la campaña "Camblá el foco" para impulsar el uso eficiente de la energía mediante el recambio de bombillas halógenas y de bajo consumo por lámparas LED, que producen un ahorro promedio del 80% respecto de las tradicionales y duran quince veces más.

Se trata de una iniciativa de los ministerios de Producción y de Energía y Minería para



promover un consumo responsable, favorecer la eficiencia y aborrar energía. Según informó la cartera que dirige Francisco Cabrera, la primera etapa del plan, inspirado en una campaña de Chile, incluirá el financiamiento para el recambio tecnológico con los programas Ahora 12 y Precios Cuidados, y una página web que les permitirá a los consumidores calcular cuánto ahorarian de energía si cambiasen las lámparas. Hoy apenas el 15% son LEO y todavía queda un 32% de halógenas en los hogares argentinos, informó Producción.

"Con esta iniciativa, estamos impulsando el consumo responsable de energía, y el ahorro en las facturas de luz a fin de mes. El uso de tecnología LED es tendencia en el mundo, no sólo por el ahorro energético que genera, sino también por sus beneficios para el medio ambiente", afirmó Cabrera, quien agregó que "la difusión del ahorro para los hogares, comercios, gymes y la baja de los costos del recambio son dos pitares fundamentales para lograr la eficiencia en el uso energético".

Para facilitar la compra de estas lámparas, la secretaría de Comercio Las incluirá en el programa Ahora 12 para compra en 3, 6 o 12 cuetas fijas con tarjeta de crédito. Además, el Gobierno acordó con Philips — proveedor de esta tecnología — que los productos LED estén Incluidos en Precios Cuidados y continúa negociando con las cadenas de supermercados para facilitar su venta.



Calculadora





La campaña también incluye una calculadora entine, disponible para que los consumidores puedan hacer su cálculo de ahorro. En la página se deben llenar los siguientes casilleros: cuántas lámparas se quiere cambiar; cuántas horas están encendidas al día; y qué tipo de lámparas tiene el hogar ahora (bajo consumo, incandescente o halógena).

A partir de estas preguntas, la calculadora estima el ahorro (mensual, anual y en 5 años) para cada hogar.

Según precisó el subsecretario de Defensa del Consumidor, Fernando Blanco Muiño, los hogares que cuentan con 12 lámparas, de cambiar el 100% a LED, podrían aborrar hasta 5 4,000 aeuales. "Una casa puede aborrar entre 10% y 15% de la factura al hacer el recambio total de sus lámparas", indicó el funcionario, al remarcar que pese a la suba de tarifas, el consumo de electricidad creció 8% en abril, por lo que es necesario avanzar en un "cambio cultural" que le permita a la Argentina tener consumidores responsables y tener una planificación más sería como país.

Fuente: https://www.cronista.com/negocios/Para-pagar-menos-luz-habra-lamparas-LED-encuotas-y-con-Precios-Cuidados-20180528-0090.html