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A theoretical model for organizing and understanding teacher learning opportunities to teach mathematics

Um modelo teórico para organizar e compreender as oportunidades de aprendizagem de professores para ensinar matemática

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Abstract

Understanding how teacher learning is constituted is an important research area in teacher education with many open questions to be investigated yet. Thereby, in this paper, we present a theoretical model whose main purpose is to enable the design of teacher education programs as well as to guide the understanding of professional learning opportunities for teachers. Based on our research outcomes on teaching and learning algebra and on a literature review, we describe how the model was elaborated and bring some illustrative situations to indicate the model "in action" throughout the teacher education process. The architecture of the program enabled teachers to experience professional learning opportunities linked to mathematical and didactical knowledge regarding patterns and regularities, as well as the opportunity to learn from and with each other, overcoming the isolation caused by daily work in their schools and leading them to explore practices close to their own school reality.

Keywords: Teacher learning; Professional learning tasks; Theoretical model; Teacher education

Resumo

Compreender como se constitui a aprendizagem de professores é uma importante área de pesquisa na formação de professores, com muitas questões em aberto a serem ainda investigadas. Assim, neste artigo, apresentamos um modelo teórico cujo principal objetivo é permitir o desenho de programas de formação de professores, bem como orientar a compreensão das oportunidades de aprendizagem profissional para os professores. Com base em nossos resultados de pesquisa em ensino e aprendizagem de álgebra e em uma revisão de literatura, descrevemos como o modelo foi elaborado e trazemos algumas situações ilustrativas para indicar o modelo "em ação" durante todo o processo de formação de professores. A arquitetura do programa permitiu que os professores experimentassem oportunidades de aprendizagem profissional vinculadas a conhecimentos matemáticos e didáticos sobre padrões e regularidades, bem como a oportunidade de aprender uns com os outros, superando o isolamento causado pelo trabalho diário em suas escolas e levando-os a explorar práticas próximas à própria realidade escolar.

Palavras-chave: Aprendizagem dos professores; Tarefas de aprendizagem profissional; Modelo teórico; Formação de professores.

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Introduction

How do teachers learn? How does this learning develop throughout their career? These questions are recurrent in studies about teachers, their knowledge and their practices (e.g. Webster-Wright, 2009; Russ, Sherin, & Sherin, 2016). Just as important as these questions are those related to teacher learning opportunities, such as what is meant by learning opportunities and how can we provide them to teachers? The idea of "learning opportunities" has been researched for a long time (Heyd-Metzuyanim, Tabach & Nachlieli, 2016), with emphasis on research involving elementary school students. In teacher education, the search for understanding how opportunities for teacher learning come about is much more recent and has focused on prospective teacher education (Tatto & Senk, 2011).

To understand what constitutes opportunities for teachers to learn, one must first understand how teachers learn. This article is based, on the one hand, on the understanding that teacher learning lies in daily practice, including moments in the classroom, but also when planning lessons, evaluating students, and collaborating with peers and others (Davis & Krajcik, 2005; Trevisan, Ribeiro, & Ponte, 2020); and, on the other hand, that the teacher's learning is distributed among individuals, as well as in artefacts, such as the tasks used for their education (Putnam & Borko, 2000). Thus, we consider that teacher learning

> involves the development and integration of a knowledge-base about content, teaching and learning; [the teacher] becomes able to apply this knowledge in real time to make teaching decisions; participate in the discourse of teaching; and become encultured (and engaged) in a variety of teaching practices (Davis & Krajcik, 2005, p. 3).

Based on these principles, the "Professional Learning Opportunities for Teachers (PLOT)" framework is a theoretical and methodological model for (1) organizing the design of teacher education processes that aim to promote teacher learning and (2) generating opportunities for teachers to learn during these educational processes, from three domains: (a) Role and Actions of the Teacher Educator (RATE), (b) Professional Teachers Learning Tasks (PTLT), and (c) Discourse Interactions Among Participants (DIAP). The characteristics of the different components of each domain are presented and discussed later. The purpose of this paper is to describe the construction of the PLOT theoretical model and to illustrate how this model was used for the design and implementation of a teacher education process that addressed mathematical knowledge for teaching algebra in basic education. We also discuss if and how this model can be used in prospective mathematics teacher education, as well as outside mathematics education.

The "Professional Learning Opportunities for Teachers (PLOT)" model

To study if and how a teacher education process can support teacher learning, the design of this process should be thought of for that purpose (Davis & Krajcik, 2005; Fuentes & Ma, 2017). The PLOT framework (Figure 1) can be used to identify and evaluate the role and characteristics of each domain in creating learning opportunities for teachers. Thus, when

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exploring the first potentiality of this model, used as a guide for the organization and enactment of the educational process, it is necessary to ensure that it contemplates the three domains, in an articulated manner.

This article presents a detailed account of the model and its components and seeks to provide useful elements to guide the design of the teacher education process, which ultimately aims to promote learning that enables teachers to grow (Loucks-Horsley, 1997; Feiman-Nemser, 2001), especially regarding the assumption of effective practices (Lampert, 2010). In addition, we understand that the model can be used as a basis to create an analytic framework that identifies if and evaluates how a teacher education process that contemplates the three domains (RATE, PTLT, DIAP) and their characteristics, makes it possible to unveil and understand what were the opportunities for teachers to further their professional knowledge to teach mathematics, and how they came about. So, with this, we explore the second potentiality of the model.

The structure of the *"Professional Learning Opportunities for Teacher"* model aims to break with a linear and compartmentalized logic for conceiving teacher education processes that aim to provide teacher learning (Goldsmith, Doerr & Lewis, 2014). The adoption of an interactive and interconnected perspective that considers the three domains, inserted in a specific context, can contribute to generate learning opportunities for teachers. In their study, Goldsmith, Doerr and Lewis (2014) understand the "need to develop shared structures for the study of teacher learning" (p. 23) and further suggest that

> The framework proposed by Clarke and Hollingsworth (2002) can, and perhaps should, provide a common framework for future studies. Its breadth can accommodate a variety of research focuses and allow for the accumulation of findings that further articulate the critical characteristics of individual domains and their interactions (Goldsmith, Doerr, & Lewis, 2014, p. 23).

Following this suggestion, the architecture of the PLOT model was inspired by Clarke and Hollingsworth (2002), whose work presents an "interconnected model of professional growth" (p. 951). However, the PLOT model is different in that it considers other domains as constitutive of teacher learning opportunities, and it seeks to understand teacher learning and growth at a more refined level of analysis, as suggested by Schoenfeld (2015).

The PLOT model (Figure 1) was conceived with the purpose of providing parameters for planning and developing teacher education processes that aim to effect learning opportunities for teachers. Therefore, besides being composed of three domains, it consists of three phases of operation:

(1) Planning: moments when the teacher educator elaborates the educational process (either in whole or in part) and designs the PTLT(s) and potential DIAP(s).

(2) Enactment: moments when the participants (teachers and teacher educators) begin interacting with each other, mediated by the use of PTLT(s) and the achievement of the DIAP(s).

(3) Finishing: when, by binding the three domains (RATE, PTLT and DIAP), the PLOTs

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are achieved.

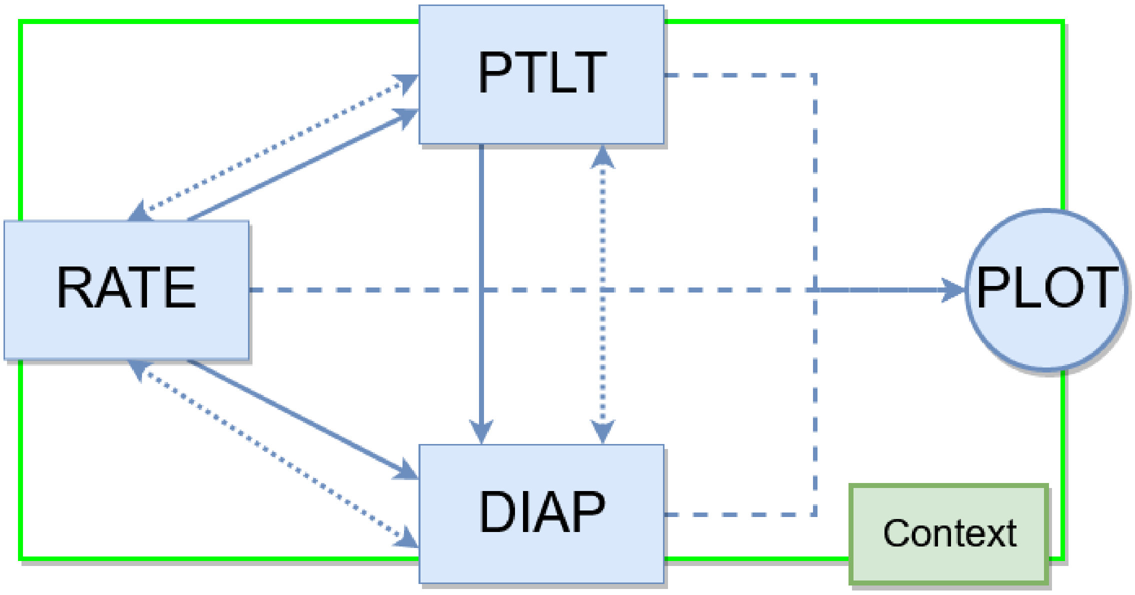

Figure 1 - "Professional Learning Opportunities for Teacher" framework (RATE: Role and Actions of Teacher Education; PTLT: Professional Teacher Learning Tasks; DIAP: Discursive Interactions Among Participants; PLOT: Professional Learning Opportunities for Teachers; Context in which the PLOTs is inserted)
Source - Elaborated by the authors

The following aspects of Figure 1 should be noted:

- the rectangles represent the three domains (RATE, PTLT and DIAP), which are distributed in a connected way, but following a continuity/flow logic.
- the arrows (continuous, dotted and dashed) indicate the continuity/flow of the process and, according to the direction (directional or bidirectional), represent interactive movements between the three domains, which change according to the operating phase of the model. Continuous arrows indicate movements in the planning phase; the dotted arrows, the movements in the enactment phase; the dashed arrows, as they come together (between the enactment and achievement phases) form an amalgam of the different domains, which eventually enable teacher learning opportunities.
- the circle represents the achievement of the creation of professional learning opportunities for teachers.
- the rectangle that surrounds the other components – the Context – represents the situated learning perspective that theoretically supports the model.

Theoretical considerations: principles and bases that support the PLOT Model

One of the main features of the PLOT model, as discussed previously, is that it considers, interactively and interconnected in a single system, the three different domains that

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compose it. By articulating these three domains in a single system, a theoretical-methodological tool is generated to plan and implement teacher education processes that enable learning opportunities for teachers who teach mathematics.

The literature on teacher education, in general and in the field of Mathematics Education, demonstrates that the research community is already studying the three domains that make up the PLOT model, but disconnected from each other. For example, the role and actions of the teacher educator was studied by Remillard and Geist (2002) and Bruce, Esmonde, Dookie and Beatty (2010). The professional teacher learning tasks are considered in studies by Ball and Cohen (1999), Smith (2001) and Swan (2007). Regarding the discourse interactions among participants, there are works by Ponte and Quaresma (2016), Craig and Morgan (2015) and Nemirovsky, Dimattia, Ribeiro and Lara-Meloy (2005). For this reason, we consider our proposal as a new and different way to design and develop teacher education programs.

The lack of integration of the three domains, however, is a gap that must be overcome in research on teacher education. Thus, the PLOT model was created, as shown in Table 1, with two dimensions (conceptual and operational) and four components (in each of the three domains). These elements characterize, on the one hand, the structure and the theoretical bases of the model (conceptual dimension) and, on the other hand, the way that guides its use (operational dimension). Thus, these two dimensions, taken together, have the purpose of organizing a teacher education process and/or identifying and understanding if and how professional learning opportunities come about for teachers.

Domains of Professional Learning Opportunities for Teacher (PLOT) model: dimensions and components

Role and Actions of the Teacher Educator (RATE): The components of this domain are Approximation and Articulation (conceptual dimension) and Management and Orchestration (operational dimension). These components consider: the need to bring academic mathematics closer to school mathematics (Elias, Ribeiro, & Savioli, 2019; Kilpatrick, 2019; Moreira & David, 2008; Schubring, 2019; Wasserman, 2018); the importance of considering the articulation between mathematics and didactics in and for teaching (Ball, Thames, & Phelps, 2008; Ponte, 1999; Neubrand, 2018; Rowland, Huckstep, & Thwaites, 2005); the relevance of building, through "classroom" management, inquiry-based teaching-learning environments (Jaworski & Huang, 2014; Ponte & Quaresma, 2016); the possibility of considering the orchestration of didactical and mathematical discussions when taking into account teacher learning (Stein, Engle, Smith, & Hughes, 2008; Borko, Jacobs, Seago, & Mangram, 2014).

Professional Teacher Learning Tasks (PTLT): The components of this domain are Professional Knowledge and Inquiry-based Approach (conceptual dimension) and Mathematical Task and Records of Practice (operational dimension). These components emerge from the relevance of considering the specifics of the teacher's professional knowledge, in order to promote the exploration of this knowledge regarding mathematical

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tasks proposed for students (Silver et al., 2007; Boston & Smith, 2011); a teaching-learning environment that favors mathematical exploration and research (Ponte & Quaresma, 2016; Zaslavsky & Leikin, 2004; Jaworski & Huang, 2014); the importance of using high-cognitive demand mathematical tasks with students (Boston & Smith, 2011; Smith & Stein, 1998); the role of records of practice (Ball, Ben-Peretz & Cohen, 2014) when composing vignettes, for example, with videos and their teacher education potential (Maarten, den Hertog, & Gravemeijer, 2002; Borko, Jacobs, Eiteljorg, & Pittman, 2008; Coles, 2013; Beilstein, Perry, & Bates, 2017).

Discourse Interactions Among the Participants (DIAP): The components of this domain are Mathematical and Didactical Discussions and Argumentation and Justification (conceptual dimension) and Language Mobilized and Dialogical Communication (operational dimension). These components promote mathematical and didactical discussions as a means to favor teachers' professional learning (Heyd-Metzuyanim, Tabach, & Nachlieli, 2016; Ponte & Quaresma 2016; Shilo & Kramarski, 2018); involve teachers in an environment that promotes reasoning and justification (Mata-Pereira & Ponte, 2017) when discussing mathematical tasks for students; encourage the use of correct mathematical language appropriate to the educational level of students (Adler & Ronda, 2014; Radford & Barwell, 2016); lead teachers to recognize the importance of dialogic communication between them and their students (Nemirovsky, Dimattia, Ribeiro, & Lara-Meloy, 2005; Craig & Morgan, 2015).

Table 1 presents the domains, in their conceptual and operational dimensions, with the components and their characteristics, in order to synthesize and make the PLOT model architecture visible and explicit.

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Table 1. Dimensions, components and characteristics of the PLOT model in its three domains

	Conceptual Dimension		Operational Dimension	
	Component	**Characteristic**	**Component**	**Characteristic**
Role and Actions from Teacher Educator (RATE)	***Approximation***	Favoring approximation between academic mathematics and school mathematics and vice versa.	***Management***	Following an inquiry-based exploratory teaching-learning environment, in its different phases.
	Articulation	Stimulating articulation between the mathematical and didactical dimensions from professional knowledge for teaching.	***Orchestration***	Preparing and developing the orchestration of mathematical and didactical discussions among all participants.
Professional Teacher Learning Tasks (PTLT)	***Professional Knowledge***	Exploring teachers' mathematical and didactical knowledge related to the tasks.	***Mathematical Task***	Proposing mathematical tasks with high cognitive level.
	Inquiry-based Approach	Using a structure that provides an inquiry-based teaching-learning environment.	***Records of Practice***	Involving different types of records of practice, organized in the form of vignettes.
Discursive Interactions Among Participants (DIAP)	***Mathematical and Didactical Discussions***	Articulating mathematical and didactical issues related to the tasks.	***Language mobilized***	Using appropriate mathematical and didactical language for the students' level.
	Argumentation and Justification	Involving valid mathematical and didactical arguments and justifications.	***Dialogical Communication***	Promoting dialogical and integrative communication among all participants.

Source - Elaborated by the authors

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Study context: researches that resulted in the PLOT model

The PLOT model has been developed along a longitudinal research agenda based on the methodological principles of *Design-Based Research – DBR* (Cobb, Confrey, diSessa, Lehrer, & Shaube, 2003; Ponte, Quaresma, Mata-Pereira, & Baptista, 2016) and, specifically a type of *DBR* that aims to help teachers – working together and collaborating with researchers – develop learning that enables them to follow innovative teaching practices in their classrooms (Cobb, Jackson, & Dunlap, 2016).

Data has been produced since 2016 and researchers have used (1) video and audio recordings; (2) collection of documents (protocols produced by students and teachers, lesson plans, among others); (3) observation of professional learning opportunities for teachers; (4) interviews with teachers and teacher educators; and (5) observation of lessons held by teachers in elementary schools. The research participants are (a) pre-service teachers, (b) in-service teachers in elementary school classrooms, (c) Master's and PhD students, and (d) researchers and university professors.

Given the iterative and cyclical nature of DBR research, the diagram below summarizes the different phases of the research schedule, with emphasis given to the one in which data collection took place whose formative process is discussed in this article (Figure 2):

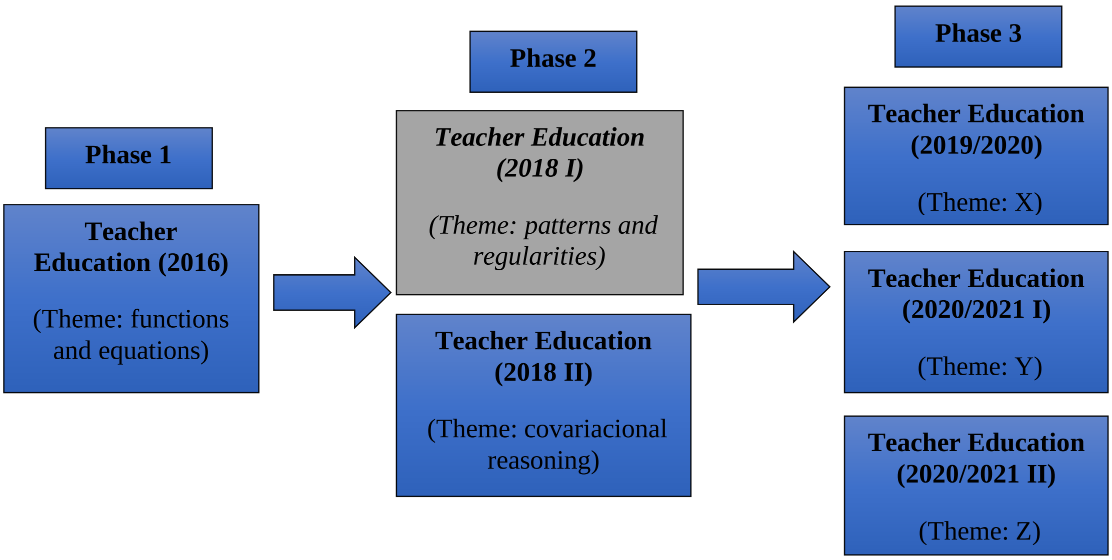

Figure 2 - Structure of the longitudinal study and its "phases".
Source - Elaborated by the authors

Thus, since the teacher education process discussed in this article is an integral part of DBR "Phase 2", as indicated in Figure 2, we now explain it below. It is worth emphasizing that, in phase 3, we will also carry out studies involving prospective mathematics teacher education.

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Context SP (Teacher Education 2018 I): This educational process was developed over 15 weekly 4-hour meetings. The meetings were organized by the researchers, as teacher educators, and combined moments of (1) individual work, (2) work in small groups and (3) work in collective plenary discussions. The work sessions included, in an interspersed manner, theoretical study moments (workshops, totaling 8 hours) and *hands-on* work moments, mediated by the tasks (PTLT) prepared by the teacher educators, five in total; discursive interactions among participants (DIAP); and the role played by the educators and their actions throughout the meetings (RATE). Most of the activities were held at the university, and three meetings were held at elementary schools. The teacher education process contained a total of five PTLTs were developed with the purpose of surveying the previous mathematical and didactical knowledge of the participating teachers on the subject *"Patterns and Regularities in School Algebra"* and, after formative encounters (Workshops), three other PTLTs were held with the teachers. The 3rd, 4th and 5th PTLTs were chosen because they were part of a work cycle called *PDR (**P**lanning, **D**evelopment and **R**eflection)* (Trevisan, Ribeiro, & Ponte, 2020)[3], incorporating lessons elaborated collectively by the group of teachers. They had the following format: (a) 3rd PLTL: ***P***reparation of high school lesson plans on the subject of patterns and regularities, and selection of a lesson plan to be used later; (b) 4th PLTL: ***D***evelopment of the selected lesson in a high school class by one of the participating teachers who volunteered for it; (c) 5th PTLT: ***R***eflection in a group, mediated by records of practice produced in the lesson developed in a high school classroom. Thirty-three mathematics teachers (MT) participated in the study (7 pre-service and 26 in-service teachers; of the 33 teachers, 5 had no classroom experience). For the enactment of the PTLT teachers were divided into 6 groups (4 to 6 participants), organized by the teacher educators so that in all groups there were (1) teachers with and without classroom experience and (2) teachers with a degree and undergraduate students. Jessica, who taught the class under analysis in this article, graduated 8 years before the study and, at the time, taught high school classes in a public school.

The next section brings excerpts from the teacher education process in order to exemplify the PLOT model "in action", as it underlined the organization and realization of the "Context SP" formative process. Due to space limitations, we could not explore all three domains, their conceptual and operational dimensions and their different components. We seek to illustrate aspects of how the PLOT model generally contributed to the design of the educational process and, at the same time, enabled teacher educators to develop the formative process, its first potentiality. Throughout the analysis we sought to identify teacher learning opportunities, the second PLOT model potentiality, regarding didactical and mathematical knowledge (Ponte, 1999; Ball, Thames & Phelps, 2008) about patterns and regularities.

[3] In this article the authors discuss extensively on the use of *PDR Cycle* in a study with in-service teachers (in a professional development process), about the mathematical knowledge and learning related to the concept of function in its covariacional aspect.

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The PLOT model in a teacher education process with mathematics teachers

In order to point out illustrative situations regarding the operationalization of the PLOT model in the “Context SP” educational process, we present excerpts of the three PTLTs that make up the *PDR Cycle*, while highlighting evidence extracted from the data collected during the teacher education process and which indicate to us the existence of PLOT for the participating teachers. We begin by illustrating, through excerpts from PTLT 3, 4 and 5, how the model was used to design the formative process, always articulating such illustrations to the model components, and also the PLOT that were evidenced.

The PLOT model and the 3rd PTLT (Planning lessons)

Before the encounter in which the 3rd PTLT was developed, teacher educators (TE) created a support script for the teachers’ lesson planning (Serrazina, 2017). By choosing to present the script for planning a lesson in PTLT format, teacher educators recognized their role as designers of the formative tasks and at the same time mobilized certain actions (RATE) in their choices to create the PTLT (Figure 3). For example, the ***articulation*** component between the mathematical and didactical dimensions of professional knowledge led teacher educators to contemplate, in the PTLT, opportunities for teachers to explore a mathematical task for students, taking into account both mathematical and didactical aspects (considering prospecting for future classroom management and reflecting on the strategies and difficulties students might have with the chosen assignment). At the same time, the teacher educators foresaw potential DIAPs that should arise in the development of the PTLT, since the ***orchestration*** of mathematical and didactical discussions was envisioned by the teacher educators, either by the way the PTLT would be developed (in small groups and in plenary), or by the inquiry-based teaching environment they proposed.

1st Step – Choose a suitable task and, at the same time, plan the knowledge you wish to promote with students.

2nd Step – From the task you have chosen and considering the knowledge you wish to promote, establish the goals you wish to reach with the class you have planned, including the grade and the duration of the class.

3rd Step – Try to anticipate the difficulties students might have and the possible strategies for solving the task.

4th Step – Anticipate possible questions from the teacher and answers from students.

5th Step – Define which resources (material and didactical) will be necessary for the class; use the questions as support for student learning; make the students work in groups.

6th Step – Prepare the evaluation, which must occur throughout the class (Remember what was proposed by Stein & collaborators (2008)).

7th Step – Prepare the students’ task.

Figure 3 - Excerpt from the 3rd PTLT, with guidelines for teachers.
Source - Survey data

The lesson plans prepared by the teachers were presented, discussed and selected at the subsequent formative meeting, which was mediated by the second part of the 3rd PTLT (which aimed to give support to teachers regarding the completeness and quality of the lesson plan that would be applied in the classroom). The lesson plan chosen was *“Investigating*

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Patterns through the Tower of Hanoi Game".

The PLOT model and the 4th PTLT (Developing the lesson)

The 4th PTLT involved the development of the previously planned lesson, which took place in a lesson taught by a teacher who belonged to the group (Jessica) that prepared the lesson plan, in a public school in the metropolitan region of São Paulo, Brazil, in a class of high school students (14-15 years' age). The lesson lasted 150 minutes and was attended by 33 students divided into 3 groups of 5 (groups 3A, 3B and 3C) and 3 groups of 6 (groups 3D, 3E and 3F). Two teacher educators and two teachers who participated in the teacher education process were present during the class, to film and observe it. The objectives of the 4th PTLT (Figure 4) were (1) to guide the observing teachers to take a more focused and detailed "look" at certain elements of lesson plan development and (2) to produce records of practice (Ball, Ben-Peretz, & Cohen, 2014) that could be used later in the 5th PTLT.

1o) Regarding the time management for the class:

a) Did the developed class allow students to participate and manifest their opinion, or did it prioritize the teacher's discourse?

b) Did the developed class enable students to understand and get involved with the activity?

c) Did the developed class enable students to discuss in groups and with everyone?

2o) Regarding the teacher's actions:

a) Were the guidelines given by the teacher necessary and enough for students to get involved in the class subject?

b) Did the teacher use questions and statements to help students understand the concepts that came up in class?

c) Did the teacher use suitable terminology (according to Mathematics and with the students' age group) and appropriate language to help students make the necessary connections?

d) Did the teacher, at the end of the class, enable the systematization of the mathematical knowledge that was part of the task?

3o) Regarding the students' discussions during class:

a) Did the teacher allow students to present different ways of solving the task (including possible incorrect strategies)?

b) Did the teacher consider the difficulties presented by students and intervene to solve them?

c) Did the teacher promote a debate between the different strategies presented by students and regarding their difficulties?

Figure 4 - Excerpt from the 4th PTLT, with guidelines for observers.
Source - Survey data

The script for the 4th PTLT ratifies the role teacher educators assume as designers of formative tasks and can illustrate other actions of the teacher educators (RATE) to build the PTLT and choose its components. Note here the ***articulation*** components (which encouraged observers about the mathematical and didactical aspects of the teacher's knowledge who taught the lesson); ***orchestration*** (so that observers could identify how the teacher conducted

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discussions throughout the class); and ***management*** (which directs the gaze of observers to the way the teacher manages the exploratory teaching environment provided for in the lesson plan).

The 4th PTLT, while subsidizing the observation of the classroom by teachers and teacher educators, allowed the production and collection of records of practice for later use in the teacher education process. For example, two episodes of lesson development were extracted, one of which related to discussions between the teacher and the students about a previously unscheduled task solving strategy:

> **S13F (Student 1 from group 3F):** Teacher, write this down: n equals na times 2 plus 1. [the teacher writes n = na x 2 + 1 on the board].
>
> **Jessica (Teacher Jessica):** And what does your graph look like? Ooh! I think something went wrong there and it's not right. Did you test it? Did it work?
>
> **S23F:** It worked! Do you want to do it? Just do it.
>
> **Jessica:** I do. So, let's do it. How are we going to do this? Discs and movements. [The teacher draws a table (Figure 2) on the board with two columns: discs and movements.].
>
> **Jessica:** Let's do it only for 3, 4 and 5 [discs]. What is this na you have?
>
> **S23F:** Number of previous movements.
>
> Jessica: What do I have to write here [points to the first line of the table on the 'movements' column]
>
> **S23F:** 3 times 2 + 1.
>
> **Jessica:** And what is this 3?
>
> **S33F:** The number of movements.
>
> **S23F:** No, it's the number of discs. 7 is the number of movements.
>
> Jessica: Here [on the line below] 4 times 2 + 1.
>
> **S33F:** No, teacher. The 7 goes there [in place of 4, 7].
>
> **Jessica:** What! But didn't you say that this was the number of discs?
>
> **S23F:** No. 7 goes there!
>
> **Jessica:** And why does 3 go here? [points to the line above, to number 3]
>
> **S23F:** Because to move 2 [discs] we make 3 movements.
>
> **Jessica:** So, this 7 is this 7 here. So 7 x 2+1=15. Put 15 here and make it 15 x 2+1=31.
>
> **S23F:** Yes!
>
> **Jessica:** Wow! It worked! So, it could be this, no?

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Another episode relates to the moment when the Jessica, when interviewed by a teacher educator, declares that it was remarkable for her to be surprised by the response of the 3F group:

> **Jessica:** Yes! And I hadn't noticed it. In class, I don't know if it was systematized [talking about her performance in class]. I don't think it was systematized. I think it was right and wrong. […]. [talking about what was missing in class] The part of exploring other solutions. I've played several times before, but at no point have I come up with another solution, and there may be another.

The PLOT model and the 5th PTLT (Reflecting about the class)

The 5*th* PTLT was organized by the teacher educators in order to value the inclusion of ***records of practice*** (one of the components of the PTLT domain in the PLOT model), collected in the class developed in the 4th PTLT. The teacher educators chose to organize the 5th PTLT around *vignettes* (consisting of student protocols, video recorded episodes, reports of the observations, among others) inserted in a script with questions to guide the discussions between teachers. The ***orchestration*** and ***articulation*** components of the RATE dimension are important, since the teacher educators were aware of the discursive interactions they wanted to promote among the participants at the collective plenary session, as well as the intention to promote the articulation between mathematical and didactical knowledge in the participating teachers. The development happened in two educational encounters and included moments of (1) small group discussion and (2) collective discussion. An important choice identified in the actions of the teacher educators was the availability, at the time of working in small groups, of a script and a computer with the class episodes. This choice favored that the ***management*** component (of the RATE) was maintained during the 5th PTLT, with the participants involved in an ***inquiry-based*** teaching environment. At the time of the collective discussion in plenary, the teacher educators led and streamlined the discursive interactions among participants (DIAP), ensuring that there was space and appreciation for ***dialogical communication*** among all, as well as favoring articulated ***mathematical and didactical discussions***. The teacher educators used the same script followed in the small groups and collectively watched all the selected episodes again.

Due to space limitations, in this article we restrict ourselves to the discussions that took place between the participants, during the final moment (plenary), where evidences such as the DIAP and RATE domains are contemplated and bring professional learning opportunities for teachers, especially regarding the didactical knowledge and discursive actions.

Mediated and stimulated by the dynamics implanted by the teacher educators, the teachers, when watching the episodes, were prompted to reflect and debate about the teacher's performance. We chose two moments to illustrate these events. In dialogue below, we observe that teachers explore the different mathematical strategies involved in the conversation between Jessica and her students, focusing on the Jessica's didactic actions. In the end, they end up validating the Jessica's choice of using a table representation as a way of

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favoring the organization and understanding of the generalization found by students.

> **T2 (Teacher 2):** I think it was clear, the strategy that they [the students] used, because she [Jessica] was writing it on the board. If it has only been explained verbally, I think I would not have been able to understand their reasoning.
>
> **T1:** It's because she leads! They explain, but she leads! This leading organizes the reasoning.
>
> **T4:** I even think that time when she let them get it wrong was important. Because they say it's 3, and they said it was the number of discs, and they realized by themselves that it was wrong and corrected it. I thought her action was important for them to realize the mistake they were making.

A second moment that we highlight is recorded in the reflections of Jessica in a reflective interview conducted by one of the educators at the end of the 5th PTLT. It shows that Jessica values the experience she had, by sharing a lesson taught by her with the other teachers (peers):

> **Jessica:** I felt the assembly pointed out important things, but the most important were the criticisms because it was constructive criticism and directed to improving the class. And I'm trying to change some practices in the classroom after what I saw in my class.
>
> [...]
>
> **Jessica:** I learned that planning the class is essential, but just as essential as planning is being ready to adapt the activities to the moment. At times I've missed teaching and learning opportunities that came up when students were asking questions, and I only realized this because the class was taped, and I could watch it afterwards. It got me thinking maybe, maybe I should have given more opportunities to some questions in detriment to others. But I only realized this later.

Conclusion

Although investigations about what is meant by "learning opportunities" have been around for a long time (Heyd-Metzuyanim, Tabach & Nachlieli, 2016), its understanding in teacher education has only recently been studied (Tatto & Senk, 2011). In order to contribute to fill this gap, in this article we present a theoretical model – which first ideas were discussed by Ribeiro and Ponte (2019) – whose main functions are to enable the design of teacher education processes as well as to serve the purpose of guiding teacher educators and researchers to understand if and how opportunities for professional learning occur for teachers participating in a formative process. With this in mind, the previous sections bring some illustrative situations to illustrate the PLOT model "in action" throughout the formative process, from the moment the teacher educators started organizing the professional learning tasks for teachers, to the development of the formative process and consequently identifying opportunities for participating teachers to learn about patterns, regularities, generalization and

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their teaching when addressing algebra in basic education.

Through the collective experience of the *PDR Cycle*, the participating teachers were provided with different moments of individual and especially collective work and reflection, thus generating the opportunity for them to learn from “classroom moments, but also from planning, assessment and evaluation in collaboration with colleagues and others” (Davis & Krajcik, 2005, p. 3). We conjecture that this was enhanced by the architecture of the teacher education process, in particular by the type of tasks available to teachers (Putnam & Borko, 2000), which were designed and performed by the teacher educators in an inquiry-based teaching-learning environment (Jaworski & Huang, 2014; Ponte & Quaresma, 2016), enhanced by the orchestration of didactical and mathematical discussions (Stein et al., 2008; Borko, Jacobs, Seago, & Mangram, 2014).

The choice of a high cognitive demand mathematical task (Boston & Smith, 2011; Smith & Stein, 1998), which allowed the emergence of different mathematical patterns as the students played the Tower of Hanoi game, which led teacher and students to think about different paths to generalization, was decisive in and for the quality of mathematical discussions in the classroom (Ponte & Quaresma, 2016) and for mathematical and didactical discussions in the formative process (Borko, Jacobs, Seago, & Mangram, 2014). In addition, a comprehensive and detailed lesson plan (Serrazina, 2017) indicates the emergence of a significant professional learning opportunity linked to the mathematical knowledge of patterns and regularities (Zazkis & Liljedahl, 2002), as well as the reorganization of the teachers’ knowledge of the subject (Branco & Ponte, 2014).

Teachers were given opportunities to learn from the way the PTLTs, especially the 5th, were organized and delivered by the teacher educators. This led the participating teachers to think and reflect on the teacher’s performance, the way she conducted the class, being a protagonist in some moments, but also enabling her students to take on this role. It was also possible, during the collective discussion in the 5th PTLT plenary session, to realize that the teacher did not value the students’ reasoning (as identified in dialogues above). However, it is noteworthy that the learning opportunities provided to teachers, due to the format of the educational process, led them to leave the isolation of their schools and experience practices generated within the group, close to their reality, especially when collective discussions that favored interaction and learning with each other took place (Ball & Cohen,1999; White et al., 2013).

The teacher education experience considered in this article illustrates the different domains of the PLOT model, with its dimensions and components, from the conception of the formative process by the teacher educators to its effective development. In particular, we highlight:

- The role of teacher educators (RATE) as designers of the formative tasks, by choosing to present teachers with a script to support lesson planning (Serrazina, 2017). To create the PTLT, teacher educators use the components ***articulation*** among the mathematical and didactical dimensions of professional knowledge; ***orchestration*** of

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mathematical and didactical discussions carried out throughout the formative sessions; as well as teacher educators' actions regarding the ***management*** of an inquiry-based teaching-learning environment during the development of different PTLT (alternating moments of working in small groups and the assembly).

- The professional teacher learning tasks (PTLT) contemplated and valued ***records of practice***, by bringing to the formative environment moments experienced in basic education classrooms; they enabled teachers to explore a ***high cognitive level mathematical task for students***, from both ***didactical and mathematical*** approaches, contemplating the component professional knowledge for planning the PTLT.
- The discourse interactions among participants (DIAP) favored ***dialogical communication*** among all teachers, and among them and the teacher educators, as well as enabling the occurrence of ***mathematical and didactical discussions*** in an articulated manner.

Thus, it is noted that the PLOT model made it possible to survey if and how opportunities for professional learning occurred for the participating teachers, discussed at the beginning of this last section, and was mobilized and contemplated during all phases of operationalization (as shown in Figure 1) of the teacher education process. This is what was expected, given that the PLOT model was designed from an interactive and interconnected perspective to address the "need to develop shared structures for the study of teacher learning" (Goldsmith, Doerr, & Lewis, 2014, p. 23). Naturally, the consolidation of the PLOT model as a theoretical-methodological tool for the conception and development of formative processes for mathematics teachers requires more studies.

In this sense, it is important to continue the development our DBR project to investigate if and how the PLOT model can be used in pre-service teacher education and to identify in what way some adaptations could be necessary. It is also important to research the applicability of this model to different mathematical contents and to other school subjects (e.g., science education), in order to improve the structure of the theoretical model, as well as to enable a refinement and a broader and longitudinal testing.

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