



Ethnomathematics and problem solving as a methodological proposal to the Elementary Education

Etnomatemática e resolução de problemas como proposta metodológica para o ensino fundamental

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Abstract

This work aims to contribute with alternative methodological proposals for the teaching and learning process of Mathematics in school context of Basic Education, and aims to investigate the possibility of implementing a proposal for teaching Mathematics based on the teaching-learning-assessment methodology through problem solving, added to the principles of Ethnomathematics. The research followed a qualitative approach, characterized as a case study referring to the application of the previous methodology with students of the eighth grade of Elementary School, also showing some elements of action-research. The problem situations were contextualized to the students' sociocultural reality, based on the generator theme "water", chosen and defined with the participants. Data were collected and analyzed through questionnaires, interviews, audiovisual records and notes in a field diary. The analysis pointed out that such an application meant an advance, both in the knowledge of Mathematics and in those related to the theme worked on and that, for this, it was of fundamental importance the integration of mathematical knowledge from the school context with those existing in the community from which the students were part of.

Keywords: Basic Education, Ethnomathematics, Alternative methodologies, Problem Solving.

Resumo

Este trabalho visa contribuir com propostas metodológicas alternativas para o processo de ensino e aprendizagem de Matemática no contexto escolar da Educação Básica e tem por objetivo investigar a

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possibilidade de implementação de uma proposta de ensino de Matemática baseada na metodologia de ensino-aprendizagem-avaliação através da resolução de problemas, agregada aos princípios da Etnomatemática. A pesquisa seguiu uma abordagem qualitativa, caracterizada como um estudo de caso referente à aplicação da metodologia mencionada junto a alunos do oitavo ano do Ensino Fundamental, evidenciando também alguns elementos da pesquisa-ação. As situações-problema foram contextualizadas à realidade sociocultural dos alunos, a partir do tema gerador “água”, escolhido e definido com os participantes. Os dados foram coletados e analisados através de questionários, entrevistas, registros audiovisuais e anotações em diário de campo. As análises apontaram que tal aplicação significou um avanço, tanto nos conhecimentos de Matemática quanto naqueles relativos à temática trabalhada e que, para tal, foi de fundamental importância a integração dos saberes matemáticos oriundos do contexto escolar com aqueles existentes na comunidade da qual os educandos faziam parte.

Palavras-chave: Educação Básica, Etnomatemática, Metodologias Alternativas, Resolução de Problemas.

Introduction

In Mathematics Education, studies point out difficulties in teaching and learning processes, mainly related to the excessive, and exclusive, use of traditional approaches and to a teaching that is disconnected from the students' daily practices (Fiorentini & Miorim, 1990). Moreover, Teixeira (2004) emphasizes that mathematics is a procedural discipline, with abstract, deductive, and non-empirical concepts. Consequently, it has been reported by teachers as extremely difficult to propose alternative actions for teaching mathematics in school contexts

In Brazil, the National Curriculum Parameters (NCP) expressed the contributions of research in Mathematics Education that began to penetrate schools and teacher training courses, in the context of the educational reforms of the 1990s. The new conceptions emphasized the importance of mathematics as fostering creativity, inquiry, and problem-solving skills (Brasil, 1998). Such perspectives also highlighted the relevant role of cultural and social aspects of the contents of this subject, in an attempt to add ethical dimensions to the traditional curriculum, thus reformulating the objectives of its teaching (Pires, 2008).

We highlight the need to reconceptualize the school mathematics curriculum and generate greater meanings for students in the classroom. In this sense, this paper is part of a larger research and aims to investigate the implementation of an alternative proposal for the teaching and learning process of Mathematics, in the school context of Basic Education. This was based on the methodology of teaching-learning-evaluation through problem-solving (Onuchic, Allevato, Noguti & Justulin, 2014), associated with the principles of Ethnomathematics (D'Ambrosio, 1990), in its educational dimension.

To this end, we are guided by the following questions: (1) Is it possible to successfully and more continuously implement an alternative teaching methodology, based on aspects of Ethnomathematics and through Problem Solving, in the current conditions offered by the Brazilian public Elementary School? (2) What are the difficulties encountered in this implementation? (3) What can the model proposed in this methodology mean in terms of the mathematical learning of the participating students?

In this direction, in this article we will address a sample of the application of this proposal with eighth-grade students of a public school in São Paulo, through the scientific initiation

project of the second author, under the guidance of the first one, and with reflective contributions of the third author, to connect formal mathematics to the sociocultural context of the school community, in the light of the following theoretical references.

Theoretical assumptions

In this research, we adopted Ethnomathematics and the teaching-learning-evaluation methodology through Problem Solving as theoretical and methodological references. The combination of these perspectives occurred as we valued the cultural aspects of the school community and developed the mathematical contents based on problem situations arising from this cultural space. Thus, based on the reality and the school demands, we elaborated and executed intervention activities with the teacher, together with the participating class. Those were aggregated to a certain theme, in the effort that these problem-solving situations would be able to trigger the teaching and learning process.

Problem Solving in Mathematics Teaching

A problem can be understood as "any situation that makes the student think and that is challenging and non-trivial" (Onuchic, 1999, p. 215). In this way, it is characterized by being difficult for the one who proposes to resolve it, since he or she has no methods or tools to solve it beforehand. This definition is in line with our proposal to start the work in the classroom from a problem situation.

In this regard, Onuchic and Allevato (2005) highlight three ways of working with problem-solving in the classroom: teaching **about**, **for**, and **through** problem-solving. In the first one, the influence in the literature of Polya's (1978) book "How to solve it" (in Portuguese: "A arte de resolver problemas") is notorious, in which step-by-step strategies are proposed, being applicable to solving any problem. Teaching **for** problem solving, on the other hand, consists in providing some theoretical support to the student, who receives the mathematical knowledge transmitted by the teacher in order to apply it. In this way, the focus is on the utilitarian aspect of mathematics, approaching first the formal concepts and then using them to solve problems.

We will adopt the third conception of teaching and learning **through** Problem Solving, in which the knowledge to be taught begins with a problem. Under this perspective, "the problem is the starting point for the construction of new concepts; the students are co-constructors of their own knowledge and the teachers are responsible for leading this process" (Onuchic & Allevato, 2005, p. 80).

By teaching through Problem Solving, we believe to have the opportunity to provide a more intense dialogue between teacher-student and student-student, allowing a greater approach between them in the search for solutions to problems (Onuchic, 1999; Mandarino, 2002). This can promote a rich environment for learning Mathematics, besides favoring the construction of new concepts by the students themselves, in a more meaningful way (Onuchic & Allevato, 2005).

Onuchic et al. (2014) proposed the expression "teaching-learning-evaluation" to highlight the importance of evaluative practices occurring along with problem-solving. They also organized the methodology into ten steps: (1) Proposition of the problem; (2) Individual reading; (3) Reading together; (4) Solving the problem; (5) Observation and encouragement; (6) Recording the resolutions on the blackboard; (7) Plenary; (8) Search for consensus; (9) Formalization of the content; and (10) Proposition of new problems.

This script of activities is suggested to guide the educator and students during the development of the methodology. The proposition of a problem situation (step 1) should aim at the construction of new content and may be inserted in the context of some school project. For this research, we added that it should be related to the socio-cultural context of the participants' community, thus connecting two theoretical assumptions, as we will see later on. This last approach is inserted in the Ethnomathematics perspective, since the involved students had elected as a relevant social aspect for their local community, to verify if there were adequate treatments for the water consumed in the city and, also, for the sewage generated there. In other words, the raised theme was part of the students' sociocultural context and it was of their interest to investigate this in more depth.

The first five steps above are related to the resolution of the problem situation itself, in which the teacher mediates the construction of possible solutions by the students. In the others, these solutions are presented and discussed together with the collaboration of the teacher to reach a consensus. The teacher then takes on the role of promoting the formalization and synthesis of the concepts, and may use other evaluation tools after these stages.

Ethnomathematics

This work is also aligned with the theoretical assumptions of the Ethnomathematics Program, which proposes the study of the creation and transmission of mathematical knowledge in its cultural aspects, in its various forms and places in the world. Ethnomathematics understands mathematical procedures, ideas and practices as social products, linked to the cultural contexts of diverse peoples who use mathematics to explain, understand, comprehend and model the phenomena that occur in their daily lives (D'Ambrosio, 1990). In the school scenario, from this perspective, it is understood that the educational process must be linked to the socio-cultural context of the students.

In this sense, the principles of the Ethnomathematics program can be used in an educational dimension in mathematics teaching and learning processes in the classroom, contributing to a rethinking of mathematics as a school subject, which occurs from the insertion of socio-cultural components in this curriculum. Such pedagogical actions highlight the importance of teaching and learning mathematics in a more transdisciplinary and transcultural way, linked to diverse contexts and situations, in a process that raises awareness of the cultural differences existing in the school community (Wanderer & Knijnik, 2008; Rosa et al., 2016).

Thus, we argue that it is essential that educators discuss the existence of other ways of doing and knowing mathematics. The integration between the curricular mathematical content and

the community in which the students are inserted is indispensable, in order to value local contexts in which the students are inserted, beyond the school environment. To this end, the promotion of learning environments and pedagogical sequences that favor the use of inquiry and problem-solving strategies is an opportunity to reconceptualize the curriculum in this area of knowledge.

Such reconceptualization, while necessary, does not imply ignoring formal academic mathematics, but rather enhancing and humanizing it, in a movement toward valuing the historical and cultural aspects of this knowledge (D'Ambrosio, 1990). In this sense, we recognize the need to give social meaning to the mathematical knowledge we develop in the classroom.

Teaching based on Ethnomathematics should not occur through the use, repetition and reproduction by the student of a knowledge that is in the textbook, but rather in a dynamic process, in response to daily social and/or cultural problems/challenges. Thus, it is important that the educator can propose teaching and learning situations, in which the student moves towards solving and explaining general situations of his cultural environment (D'Ambrosio, 2008).

And in the case analyzed here, the students' choice of studies about water, among some suggestions given by the teacher of the investigated class, was considered as one of the most relevant for the local quality of life. They wanted to know where this water came from, how it was treated, and if the sewage generated in this community was treated itself, or, instead, if it was contributing to the pollution of nearby rivers, etc.

Methodological aspects of this research

We believe it is important that the work with Problem Solving in the classroom takes into account the socio-cultural context of the students and the community in which the school is inserted, so that the problem situations gain greater meaning within this context. In this sense, the school community has all the necessary elements for the development of a more meaningful mathematics curriculum for the student, because cultural differences coexist in this space (Rosa & Orey, 2017).

Rosa and Orey (2017) point out that the adoption of theoretical principles of Ethnomathematics in Problem Solving activities can help teachers and students in the teaching and learning processes of mathematics in the classroom, in order to articulate the knowledge arising from specific situations with those mathematically institutionalized.

We understand, as does Monteiro (2004), that Ethnomathematics is not yet constituted as a method of teaching mathematics and that "despite its contributions to the school context, due to its more philosophical character and the incipience of debates on the subject, the implementation of educational proposals in basic education is scarce" (Gonçalves, Bandeira & Araújo Júnior, 2013, p. 1). Hence, it is relevant to study and research in this direction, as we present in this article.

In this paper, we aimed to integrate the theoretical assumptions of Ethnomathematics and the methodological ones related to Problem Solving, from the adoption of these two perspectives simultaneously. We believe that the socio-cultural context of the school community can provide excellent problem situations, from which we can implement the teaching methodology of Problem Solving linked to the principles of Ethnomathematics. It is important to note that the literature in the area already reports some research that sought to integrate these two perspectives (Pereira & Bandeira, 2016; Gonçalves, Bandeira & Araújo Júnior, 2013).

Given the theoretical contributions and the discussion presented, this research is justified by the need to generate greater meaning in mathematics classrooms, in the school context of Basic Education, by approaching those two perspectives as a unified and pedagogically organized proposal.

Research Methodology

We used a qualitative research approach, with characteristics of case study and action research. The former is characterized by the researcher's look at some well-defined element of investigation, whether it is an individual or group of subjects, an institution, a program, or an event (Alves-Mazzotti, 2006). In addition, the case study allows for understanding the world from the participants' point of view (Fonseca, 2002). This research is also characterized as action research since it modifies the investigated environment and seeks to strengthen the relationship between the researcher and participating subjects (Thiollent, 2000). In this work, the researcher acted as a temporary teacher of the class during the application of the activities, that is, the official teacher of the classroom allowed the researcher to teach, and the proposed actions were always executed in agreement of both, under the official teacher's supervision. However, although this last one acted as an important collaborator in providing data on the class and the mathematical content to be developed, he was not involved in planning and execution of the didactic sequence analyzed here. Therefore, we consider that he did not constitute himself as a subject of this investigation.

The research subjects were eighth-grade students from a public school in the state of São Paulo. The participating class had about 23 students enrolled, in an age range of 13 years old, approximately. Data were collected and analyzed through questionnaires, interviews with the teacher to detail the characteristics of the class, the didactic resources that were regularly used, and the contents that were being developed, as well as audiovisual records and field diary annotations.

For the development and application of the intervention, the following methodological path was taken: (1) from an interview conducted with the class teacher, we raised the main subjects of mathematics curriculum in which there were difficulties for teaching and learning, and for obtaining positive learning results; (2) we applied the first questionnaire with the students aiming to raise information about their conceptions and difficulties regarding Mathematics; (3) we presented, in collaboration with the participating teacher, and in line with the answers provided in the first questionnaire, a proposal of alternative activities

in the light of the theories posed, aiming at greater participation and generation of meaning in Mathematics classes. At this moment we defined a generating theme: "water", detailed further on; (4) we applied the second questionnaire with the students, aiming to identify their previous knowledge about the theme and the Geometry subjects, chosen as content to be developed with the methodological proposal; (5) we applied the planned activities, which comprised the stages of exploratory investigation and Problem Solving; (6) we applied a third and fourth questionnaire with the students, to identify their progress regarding the Mathematics knowledge built.

According to Freire (1987), a generating theme should be raised from the problematization of the life practice of the students, and should be the starting point of the educational process.

In the case focused in this article, the generating theme is a problem, or concern, elected by the students participating in the research as socially relevant to be developed, from which the didactic sequence would be proposed, in order to generate new mathematical and social knowledge. This knowledge should be articulated in the contextualization of the main teaching activities proposed in this sequence, regarding aspects of the cultural reality in which the students were inserted. As previously mentioned, the issue of the origin, treatment and disposal of water consumed locally was considered, by the students, the most relevant to be studied.

Implementation and results

Initial diagnosis and definition of a theme

We performed an initial diagnosis based on the first four steps described above. Initially, the interview with the teacher highlighted his concern about the learning of Geometry, due, above all, to the difficulties that the students demonstrated in this particular topic. In addition, the definition of a theme for the activities took into consideration the answers given to the first questionnaire with the participating class, which aimed to understand the students' perception of Mathematics, and in which situations it is present in their lives.

The analysis of this questionnaire pointed to some suggestions from the students that were quite diverse and inconclusive, of which we highlight the inclusion of manipulative materials, the elaboration of models and the use of computers, as well as learning in some physical environment outside the school. Moreover, the mathematics content should be related to topics of Euclidean geometry.

Then, a discussion with the whole class was held in order to determine a guiding theme that could contemplate what was suggested by the students. Based on this, the teachers suggested the theme "water", which was accepted by the whole class. The students were excited to visit the Water Treatment Plant (WTP) near the school and to understand the calculation involved in water bills. This theme opened several possibilities for investigation, which included: the study of the characteristics, treatment and distribution of water, issues of individual consumption responsibility and sustainability, and the study of rivers in the municipality.

To identify the participant's prior knowledge about the theme "water" and Euclidean plane geometry subjects, a new questionnaire with open questions was prepared and answered by the students individually. The analysis of the answers showed that the students associated the theme mainly with health, energy, and hygiene. Few of them knew that the municipality has a water and sewage treatment plant, as well as the name of the company responsible for it. In addition, no students claimed to know how the water bill tariffs were calculated, but many showed great interest in learning this.

Regarding the Geometry curriculum topics, most of the class did not know how to correctly define the concepts of perimeter, area and volume, nor how to correctly make their calculations. Moreover, many confused perimeters with area, and others reduced this concept to expressions like: "base x height", besides misusing units of measurement.

We highlight some of these responses: "area is the part inside the figure" (student A), "area is the part that is inside the perimeter" (student B), and "area is the space inside the figure and the multiplication of the sides of the figure" (student C). The definition of the area as being "the part inside a figure" evidences an intuition of the concept, but brings problems when thinking about a non-planar, or open geometric figure. The mental image of the area as a multiplication of the values of its sides also shows an erroneous generalization of the concept with the case of the rectangle, which is usually the first to be studied in school.

Application of the activities: exploratory inquiry and Problem Solving

Given the above, we outlined paths for pedagogical action in order to provide the class with a didactic sequence capable of expanding the students' previous knowledge schemes, and engage them as protagonists in the construction of their knowledge.

We developed a sequence of activities divided into two moments: (a) a research-exploration of the theme in the school's computer room followed by a didactic visit to WTP, and (b) use of the teaching-learning-evaluation methodology through Problem Solving in the classroom.

In the first moment, the students did some research to get to know issues concerning the theme "water" in a deeper way, which was done on the internet, through a suggested bibliographic survey, and a visit to the WTP of the company responsible for water and sewage treatment in the city. From these activities, we hoped to collect information that would trigger learning and that could serve, later, as a starting point for teaching-learning-evaluation through Problem Solving.

Regarding the research, in order to enrich the whole process in terms of meaningful learning for the students and the quality of the material found, we created a website especially for this moment, making use of the Wix platform. The activities were performed in the school's computer lab, which had all the necessary infrastructure, with enough computers and internet access.

Each group was responsible for a theme, which dealt with: i) diseases associated with drinking water unfit for consumption; ii) research on the main rivers in the municipality and their importance for the city; iii) how the water bill is calculated and if we spend more than

we should; iv) where the water that reaches the city's residences is treated; v) where the sewage is treated and if this occurred in the city.

The students were asked to write a typed summary of what was researched. The analysis of the papers revealed that all groups answered the guiding questions and used the material provided, having understood the research proposal. However, they presented numerous graphical and grammatical errors, besides many doubts regarding the text formatting. They used the repeated practice of copying and pasting ready excerpts, being warned by the teacher and researcher about this, and also about the use of sources that were not on the website.

Despite these obstacles, we believe that the work, besides fulfilling its exploratory objective and allowing a deepening of knowledge about the theme, provided other skills and interdisciplinary competencies to the students, such as the development of writing on the computer, text elaboration (writing) and research activity, essential for the school of the new millennium, which has the challenge of preparing citizens who are able to insert themselves in a society that is increasingly complex, diverse and dynamic in its changes.

During the visit to the city's WTP, considered here, in light of the theory, as an interdisciplinary mathematical-pedagogical activity (Rosa & Orey, 2017), students had the opportunity to learn about the processes involved in water treatment, in addition to reflecting on the conscious use of this resource; there they also had contact with some spatial geometric shapes, such as parallelepipeds, cylinders and trunks of cones, and could associate them with the work developed by employees inside the WTP.

Elaboration and resolution of problem situations

Two problem situations were elaborated from the context of the visit to the WTP, about the cleaning of tanks and distribution of water. We mainly considered the large tanks that were shaped like parallelepipeds and from which the water, already in the final stage of treatment, is distributed. The parallelepiped was chosen because the topic had not yet been explored by the teacher responsible for the class.

The first problem covered the concepts of parallelepiped volume, percentage, rectangle area, and the use of related magnitudes and measures (meter, square meter, cubic meter, and liter). The second was based on the concept of proportionality employed in the context of the WTP through the flow. In addition, the problem situation involved representation by fractions and the conversion hours-minutes-seconds.

For this activity, the students were divided into four groups, each one recording on an individual sheet the answers they had collectively built. In addition to these, the researcher used an audio recorder. We point out in advance that all the steps proposed by Onuchic et al. (2014), except for the "proposition of new problems", were successfully implemented. The exception occurred due to setbacks at the end of the school term, which made it impossible for the teacher to give up the last few classes for discussion of future activities.

Following these steps, after the classes in which the actual problem solving occurred, we proceeded to the socialization on the blackboard, when the groups registered their answers,

and presented the paths taken and the strategies mobilized. The objective was to facilitate the later moments of discussion and plenary, creating a resolution panel, so that the groups could compare the answers, identify errors, and collectively arrive at the correct answer.

Initially, it was noted that some groups showed a lack of motivation to read the statement, as well as difficulties in interpreting the situation and using the data. It was also clear to the mediators that few had the necessary mathematical knowledge to solve the problems. These difficulties were gradually overcome in each group.

Regarding the stages of problem-solving, we reported difficulties in the class in interpreting the statements, in accessing prior knowledge, in detailing the resolutions, and in communication among peers. Such limitations are frequently reported in research in the area of Problem Solving, such as Zuffi and Onuchic (2007), and Alvarenga, Andrade, and Santos (2016).

To exemplify some of these, students showed difficulties in understanding the situation of tank maintenance described in the problem, in accessing concepts related to the area to develop the volume, in employing the correct unit of measurement, and associating the notions of proportion and fraction. We also identified the need for more detailed records of the mathematical ideas and procedures adopted. We also observed difficulty in performing calculations with large numbers that represented the actual measurements of the station's tanks. The exact measurements and the situations exposed are elements present in the daily lives of the employees of the company responsible for the WTP.

In this sense, the strategy of using examples related to contexts other than the problem situation facilitated the moments of help and mediation (Bean, 2001). The following dialog, in which P represents the professor-researcher and s1, s2 and s3 represent the students' speeches, exemplifies a situation in which the use of approximate questions was able to lead the students to arrive, by themselves, at the answer.

s1: "As we had said at the time, we did 32 times 4, which gave us 128. Then, we did it 12.8 times. Then, it was 1,638.4. Then, we saw that each cubic meter has 1,000 liters, so we did this value times 1,000. Then, it was 1,638,400.0 liters.

s2: "And there will be even more because it will expand by 20%. Then, we are going to do this value times 20".

P: "And what are you going to find out from this? When I do it times 20"?

s1: "We are going to find out that the value is going to get bigger".

P: "When you find 20% of that, the value will be bigger"?

[Everyone in the group said bigger too]

s1: "Wait, when I find 20%?"

P: "Will you find a value greater or less than the one you are multiplying"?

s1: "Greater".

s2: "Teacher, when I multiply by 20, does this 0 here add up, or does it continue"?

[Pointing to the record: 1,638,400.0]

P: "When I say 2 kilograms of soap powder or 2.0 kilograms of soap powder, is it the same thing"?

s2: "Same thing".

s1: "Teacher, I did 20 times 1,638,400 and it was bigger".

P: "How much did it give"?

s1: "32.768.000. Bigger, see."

P: "I agree it is bigger, but we want to know how much is 20 percent of a value."

s2: "Isn't that just multiplying by 20?".

P: "When I say 100%, I am considering the whole. If I say 20%, it's a part of that whole. Do you agree?"

[All answered yes].

P: "Do you guys remember the percentage calculation"?

s1: "More or less."

s3: "Do you have more"?

s2: "I think you need to divide".

P: "If it's 20 percent, 20 divided by 100".

s1: "That. dividing by 100, you want to see".

s1: "Teacher, it gave a smaller number!"

In general, it was quite difficult to mediate the process, because there were too many students to help, and they had little previous mathematical knowledge, as is the case of percentages in the previous example. From the mediator's point of view, it was extremely difficult not to provide the requested answer, especially for those groups that had little knowledge, and it was necessary to detach from the problem situation and use simpler data, in the cases of volume, percentage, flow, and hours, and use approximate questions.

Final Diagnosis

This stage consisted of applying the third and fourth questionnaires to the participating students, which included, respectively, four math questions with the same subjects worked on during the problem solving, and questions about the contributions to the learning processes provided by the intervention activities.

The figure below illustrates two records, from members of the same group, during the resolution of the problems (on the left) and after, when answering the third questionnaire (on the right). In this case, we can notice that the records, after the whole process of applying the methodology, were more detailed, organized and coherent with mathematical language.

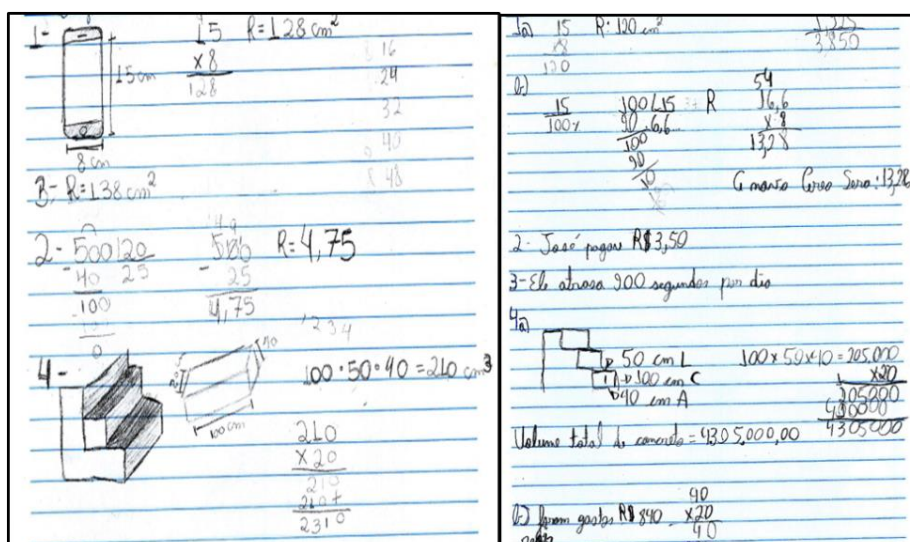


Figure 1 - Answers of two students after using the methodology

Source: authors' material collected during the research

Regarding the advances in the understanding of mathematical concepts, we identified that among the sixteen participants, thirteen answered correctly the first question about the concept of area, and also the fourth about volume, presenting the correct units of measurement. We emphasize that the class, in general, did not have a good understanding of the units of measurement during the resolution of the problems, as well as of the area. This reveals that the activities as a whole, with the subsequent discussions and theoretical syntheses, allowed the students to learn these concepts in a more meaningful way.

We emphasize, however, that this advance was not so expressive concerning proportion and percentage, since, of the sixteen participants, only four answered correctly the question about these concepts. This showed, especially for the teacher, the need for further work on these topics, in classes where the researcher was not present.

The last activity performed was the application of the fourth questionnaire, for which we will analyze some answers provided to the following question: "Throughout the year, we made a visit to the station, used the school computers to research, solved math problems, discussed with the whole class and, only afterward, the teacher used the blackboard to explain the concepts. In your opinion, was it easier and more attractive to learn mathematics this way? Why?"

The total corresponds to 20 responses, with 18 (90%) answering "Yes" and 2 (10%) answering "No". Regarding those who answered, "Yes", there were differences in the arguments, which consisted of three major justifications, referring to: "Research", with 3 answers (15%), "Space", with 3 (15%) and "Differentiated", with 12 (60%).

The arguments listed as "Differentiated" point out that it became more attractive and easier to learn mathematics due to the dynamic nature of the activities. The experience of visiting WTP was also pointed out as a motivating element by the students. Among the answers

provided, we highlight: "Yes, because we went to the site to see what we were working with" (student A).

Other answers, inserted in "Research", emphasized that the exploratory-investigative activities carried out in the computer room made the classes more attractive and meaningful, because, through them, they could get to know the themes better. For example, "Yes, it was easier because we used computers, blackboard, and the teacher explained very well and it was much more attractive" (student B). In an equal number, we identified answers that emphasized the importance of valuing a space for discussion, debates, and learning, in which the student is encouraged by the teacher and classmates to share their doubts. We highlight: "Yes because we can clear up our doubts and feel more comfortable to ask questions" (student C).

Regarding the two students who answered negatively to the first question, we highlight their answers: "No, because there are not enough classes. One teacher passes one thing, and the other one passes another" (student D); and "No, because I prefer that there is more explanation in the classroom because I can understand more" (student E).

The last answer seems to be related to the fact that students are not used to being protagonists in the construction of their knowledge and, whenever they have this opportunity, they feel uncomfortable, offering resistance to the proposition of alternative activities. The first, in turn, highlights that the research activities and the regular ones, offered by the teacher, when often interspersed, caused him/her some confusion.

The analysis of the questions showed that, for most students (18 out of 20), it was easier and more attractive to learn mathematics using the Problem Solving methodology and using the school computers for research. The visit to WTP was also listed as a positive factor to help with the activities, as the first didactic visit in this subject. In addition, the answers showed that the proposal generated a space for discussion and sharing of ideas in math classes, and the topics cited as having been learned were: parallelepipeds, volumes and measures of capacity, areas and perimeters, and how to calculate the water bill.

Another question on this questionnaire asked about working together to solve problems. Although many emphasized that the dialog and the exchange of information were essential, one student said that some members of his group were not interested in solving the problems, and that this made it difficult to work with peers. We also highlight that, even among those who answered positively to the question, there were reservations about the group participation.

This shows that making sure that all groups and their members are engaged in the activity is also a challenge for the educator. The group work method must be improved in face of the circumstances that arise during the classes, especially with more persistent use of the proposed methodology, since the reservations in the answers reveal that school discourses, in general, do not promote collective environments for sharing ideas and opinions. Thus, students are not used to changes in the didactic contract (Brousseau, 1988) in mathematics

classes, and a proposal like this one causes several instabilities with changes in roles and interactions in the classroom, which need time and continuity to be absorbed.

Final considerations

We conclude this article by answering the three research questions listed. We emphasize that the ethnomathematical assumptions were present in the activities, as we tried to highlight the value of the cultural element of mathematics, understanding it in a perspective of school contexts, but mainly in the social ones (D'Ambrosio, 1990). In this way, it was possible to propose and develop a pedagogical action articulating the assumptions of Ethnomathematics and Problem Solving by adopting these two perspectives simultaneously.

As to the question of what the methodology meant in terms of mathematics learning for the participants, we concluded that there were various advances, and that the articulation between the mathematical knowledge derived from the school context and that originated in the community's WTP was of fundamental importance. The data showed that the differentiated and dynamic character of all the activities made the mathematics classes more attractive and meaningful. This reinforces what has already been emphasized by D'Ambrosio (2000) and Rosa and Orey (2017), that it is necessary that students become aware of the reality in which they are inserted, and understand mathematics as a social and cultural product.

Our research also corroborates the work of Zuffi and Onuchic (2007), who investigated an experience of continuous practice of the teaching methodology through Problem Solving with high school students. We emphasize that the data allowed us to point out that the activities proposed here, with the association of two theoretical-methodological lines, although characterized as an initial experience for the students, with a reasonable duration, totaling thirteen weeks for good development of the activities, five of which to fulfill the stages of Problem Solving in Elementary School, provided the students with more significant learning of the focused concepts. The students' problem-solving records started to show more details of the procedures and mathematical operations, as well as better performance in the classroom in general, if we compare the questionnaires applied before and after the application of the methodology (second and third questionnaires).

On the issue of difficulties encountered in the implementation of this proposal, we observed that, in the exploration activities of the theme, many students did not know the text editor Word, which limited the writing work. In addition, despite having created their website with bibliographic indications, many students researched unreliable Internet sources, without following the orientation given by the researcher. We concluded from this activity that they did not have frequent experience with exploratory tasks, and that they were rarely stimulated to be protagonists in the construction of their knowledge. Despite this, the work with these exploratory-investigative activities allowed them to deepen their knowledge about the theme and to develop writing skills.

Another problem encountered during the application of the proposal was that, although most students belonged to the same neighborhood, their social interests and knowledge of mathematics were quite diverse.

Regarding the conditions provided for the implementation of this methodology in the public school system (question 1), we emphasize that structural and organizational factors did not interfere with the development of the activities. Inclusively, we counted on the help of several actors of the school community, for example, the Municipal Education Secretary, who paid for a public bus to take the students to the WTP, and the company responsible for the water treatment, who provided, free of charge, a guide employee. In addition, the school's computer room was used several times, and had all the essential resources for the good development of the activities.

On the other hand, we reported that the main challenges were related to the students' unfamiliarity with alternative proposals for teaching mathematics, as Correa and Maclean (1999) point out.

Regarding the reality of the São Paulo state public school, and considering the calendars and several external activities that often prevent the realization of a work of this kind, we emphasize that, despite the time allotted, it was perfectly possible to develop this work, and we understand that throughout the year, it is very plausible to develop similar activities in elementary or high school. Moreover, it was possible to contemplate several important topics of the school mathematics curriculum and promote interdisciplinary situations.

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