



Mathematics Teachers' Specialized Knowledge (MTSK) in the Web of Science until 2020

Conhecimento Especializado de Professores de Matemática (MTSK) na Web of Science até 2020

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Abstract

This work aims to map the production related to the Mathematics Teachers' Specialized Knowledge Model in the Web of Science database. For this, we carried out an analytical-descriptive research, with an exploratory-bibliometric approach. We found a corpus of 37 papers from 2013 to 2020 and analyzed: Indexing on the Web of Science; Distribution by year and types of documents; Authors; Development institutions and agencies; Countries and languages; brief Analysis of citations and Keywords. The results allow us to conclude that MTSK production is collaborative and global, involving at least 9 countries in Europe and Latin America, in a network movement that contributes to global dissemination and, in a way, is configured as an international validation test in different school cultures and contexts of teaching practice and training.

Keywords: MTSK; *Web of Science*; Review; Mathematics Teachers' Specialized Knowledge.

Resumo

O objetivo deste trabalho é mapear a produção relativa ao marco teórico *Mathematics Teachers' Specialized Knowledge* na base de dados *Web of Science*. Para tanto, realizamos um pesquisa analítico-descritiva, de aproximação exploratório-bibliométrica, na base de dados referida. Encontramos um *corpus* de 37 trabalhos de 2013 a 2020 e analisamos: Indexação na *Web of Science*; Distribuição por ano e tipos de documentos; Autores; Instituições e agências de fomento; Países e idiomas; Breve análise de citações e Palavras-chave. Os resultados nos permitem concluir que a produção MTSK é colaborativa e internacional, envolvendo ao menos nove países da Europa e América Latina, num movimento em rede que contribui para a disseminação global e, de certo modo, configura-se como um teste de validação externa em diferentes culturas escolares e contextos de prática e formação docente.

Palavras-chave: MTSK; *Web of Science*; Mapeamento; *Mathematics Teachers' Specialized Knowledge*.

Introduction

Mathematics Education has regularly analyzed its scientific production through mapping and synthesis studies (such as qualitative meta-analyses, systematic reviews, states of the art, among others), to understand the advances and trends in the various lines of the

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area. This has made it possible to know and (re)think research paths and possibilities aimed at improving education in the country (Crecci et al., 2017; Fiorentini et al., 2016; Moriel Junior et al., 2019; Paula & Cyrino, 2017).

In this perspective, we elaborated the present article as part of a broader project of mapping the production on teacher expertise, in the search to synthesize part of the scientific advances in mathematics education and bring them closer to the practice and teacher training (with state and institutional fomentation). The chosen theme is inserted in the knowledge base movement, which started in the 1980s, interested in describing the types of knowledge needed by teachers. Since then, different typologies on teacher knowledge/knowledge have been generated, and they have moved from the generic to the specialized perspective (Moriel Junior & Wielewski, 2017; Scheiner et al., 2019), that is, from the knowledge of the teacher of any area to that of the teacher of specific disciplines.

In this trajectory, stand out with greater emphasis in the first group the typology of Shulman (1986) and Tardif (2002), which only in the works cited have, respectively, more than 25 and 12 thousand citations in the Google Scholar base. When the focus is on the discipline of mathematics, the Mathematics Knowledge for Teaching model by Ball et al. (2008) stands out with almost 7 thousand citations and, more recently, the Mathematics Teachers' Specialized Knowledge - MTSK by Carrillo et al. (2014) with international recognition in the area (Kilpatrick & Spangler, 2015), but scarcity of synthesis studies for understanding its production. Given this, the objective of this article is to map the production related to Mathematics Teachers' Specialized Knowledge - MTSK in the Web of Science database until the year 2020.

The MTSK theoretical framework

Mathematics Teachers' Specialized Knowledge - MTSK is a theoretical model that describes the specific and specialized professional knowledge that a teacher possesses (or should possess) to teach mathematics (Carrillo et al., 2014; Carrillo-Yañez et al., 2018). It was developed considering the main characterizations, typologies, and models made by researchers in the field so far and advancing towards the limits detected in them (Escudero et al., 2012; Kilpatrick & Spangler, 2015; Montes et al., 2013; Scheiner et al., 2019). It is configured in a hexagonal format with two domains and six subdomains (Figure 1). At the center are teachers' beliefs about mathematics, its teaching, and learning, as they permeate the subdomains and give meaning to their actions. Below, we describe each subdomain based on Carrillo et al. (2014) and Carrillo-Yañez et al. (2018).

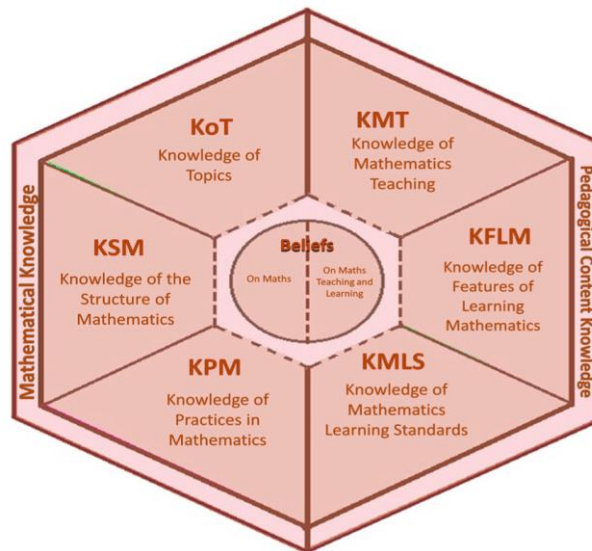


Figure 1 - Domains and subdomains of the MTSK.

Source: Carrillo-Yañez et al. (2018)

In the Mathematical knowledge domain, there is the Knowledge of Topics (KoT) subdomain that encompasses mathematical content to be taught (and its deep conceptual foundation) and includes definitions, interpretations, and properties of concepts, one or more demonstrations of a specific topic, justifications of algorithmic procedures, examples, and counterexamples, realistic models, application situations, and extra-mathematical uses. Examples are, knowing representational systems about division (Policastro and Ribeiro, 2021) or different algorithms (and alternative procedures) for dividing fractions and their justifications (Moriel Junior et al., 2019). It also includes inter-conceptual connections, as is the case with the concepts of relation and 1st degree function in which the definition of one is determined by the other (Vasco et al., 2017). In Knowledge of the structure of mathematics (KSM) are the inter-conceptual connections (between advanced and elementary, previous and future topics, from different mathematical areas, etc., except the inter-conceptual ones provided in KoT) that allow recognizing certain structures of mathematics, as well as, seeing it as a system of integrated elements. Examples include knowing that the concepts of limit, derivative and integral have in common an infinite subdivision process, or that the concept of a square root can be used to calculate the roots of a 2nd degree equation (Courant & Robbins, 1996). Knowledge of practices in mathematics (KPM) includes ways of proceeding in mathematics, including ways of creating or producing in the area (syntactic knowledge), aspects of mathematical communication, elements that structure a demonstration, ways of proving and defining, of selecting representations, of arguing, of generalizing from patterns and regularities (Campos-Cano & Flores-Medrano, 2019).

As for the subdomains of Pedagogical content knowledge, there is Knowledge of features of learning mathematics (KFLM) includes how learners learn mathematical content (formal or personal models and theories), the features of this comprehension process, common errors and their likely sources, difficulties, obstacles, and the language usually used

by learners when dealing with each concept. For example, knowing the APOS theory (Arnon et al., 2014) or the six stages of learning (Dienes, 1975) about a student's cognitive development in mathematics, knowing common errors of students when dealing with fractions (Ashlock, 2006; Bayoud, 2011), or knowing the learner interests and expectations for mathematics learning (Kaur, 2008). Knowledge of mathematics teaching (KMT) which concerns materials, resources, ways of presenting content, and their respective characteristics (limitations or potentials existing in themselves) that enable the teacher to choose a strategy to teach a given content (including organizing a series of examples or creating analogies and metaphors). For example, knowing the strategy of teaching fractions using a geometric figure (circular or rectangular, for example) or a model (such as pizzas or chocolates) and knowing that this is (more) suitable to develop the part-total interpretation (Moreira & Ferreira, 2008). It also includes knowledge (formal or personal) of theoretical elements about mathematics teaching such as, for example, types of instructional explanations (Charalambous et al., 2011) or approaches based on problem-solving or modeling (Biembengut & Hein, 2007; Krulik & Reys, 1998). Knowledge of mathematics learning standards (KMLS) refers to curriculum specifications involving the guidelines at the various educational stages on content and skills (conceptual, procedural, attitudinal, and mathematical reasoning), as well as the ways of assessment and progression from one year to another.

The construction of expertise comes both from academic-scientific sources - such as content and textbooks, scientific journals and articles, meta-analyses, curriculum parameters, among others (Becker, 2019; Courant and Robbins, 1996; Petit et al., 2010) - and from professional sources of school culture, such as experience reports and dialogue with other teachers.

Methodology

This is analytical-descriptive research, of an exploratory-bibliometric approach, given the purpose of monitoring and measuring the growth and development of scientific production in a given area of Mathematics Education concerning the MTSK (Araújo, 2016). In this sense, "we seek not only to quantify and verify but also to attribute [some] possible meaning to the data, qualifying them so that they can have better use [...] in broader contexts, regional, national or global" (Santos and Kobashi, 2009, p. 159, our emphasis), by mathematics educators, researchers or research groups. This paper is part of the efforts of the Teachers' Specialized Knowledge Research Group², coordinated by the author at IFMT, to keep up with scientific advances on the subject.

The data source is the scientific base Web of Science (WoS), which was accessed through the Periodicals Portal³ of the Coordination for the Improvement of Higher Education Personnel (CAPES) using "Remote Access via CAFE" by the author's profile as an IFMT

² Accessed at <http://dgp.cnpq.br/dgp/espelhogrupo/7032020622091895>

³ Access on <http://periodicos.capes.gov.br>

teacher. The type of information we analyze is the bibliographic records of the scientific production (books, journals, articles, etc.) indexed in the mentioned database. The universe of information available on the day of the search (June 6, 2020) totaled 73,217,951 records. The search was updated on August 5, 2021.

To select the material for analysis, we used the term "MTSK" in the searches in the source "Main Web of Science Collection", covering "All fields" and all years (1945 to 2020). The second step was carried out with the 66 resulting records, in which we applied the inclusion criteria (containing in the title the terms specialized or knowledge or mathematics) and the exclusion criteria (being from a different area than mathematics education, such as biochemistry, biophysics, biotechnology, microbiology, engineering, computing), arriving at 32 items. In the third step, we refined the search considering the articles that cited these 32 items and identified four more that met the criteria, totaling 37 records that composed the corpus of analysis (Appendix). For data collection, we saved a list within WoS with all the records of the corpus (our database), exported them to Endnote software, to text and spreadsheet files for later analysis.

In the data analysis, we used the WoS tools titled "Results Analysis" and "Citation Report" to generate consolidated and interactive data that allowed the detailing of the information found about the production. From these, we performed successive readings and new groupings, when necessary, by the similarity of content and frequency of data. Given the inconsistencies of registration inherent to this type of database (something common in all scientific databases), we standardized/cleared the names of authors, institutions, etc. whenever necessary. In the analysis, we used Descriptive Statistics and Content Analysis to explore the titles and abstracts of the corpus. Thus, the MTSK production was investigated covering the following categories: 1. indexing in the Web of Science (associated research areas); 2. distribution of production per year and types of documents; 3. authors involved; 4. institutions and funding agencies involved; 5. countries and languages; 6. brief citation analysis; 7. keywords.

Results

Our results indicate 37 MTSK productions from 2013 to 2020, which we discuss below, according to the analysis categories. We clarify that we kept the terms in the original language of the database (English) when it came to the names of categories, areas, events, journals and keywords used in the Web of Science.

Indexing production in the Web of Science (Research areas)

The scientific production on MTSK in this database is fully indexed to the "Research Area" named Education Educational Research and minority to Mathematics (Table 1). Knowing which are the areas in which the papers are classified in this database serves as a facilitating criterion in future searches for those interested in identifying studies on this theme and for future mapping updates.

Table 1 - Research Areas and Web of Science Categories of MTSK production (2013 to 2020)

Classification	Articles by Research area
<i>Education Educational Research</i>	37 (100%)
<i>Mathematics</i>	9 (24,3%)

Source: Data organized by the author.

Distribution of production by year and type of documents

The distribution of MTSK production by year indicates that the first publications indexed in Web of Science occurred in 2013 (Table 2). These are three papers published in the proceedings of the 17th annual Symposium of the Spanish Society for Research in Mathematics Education (SEIEM) held in Bilbao, Spain. Since then, the production has never been less than three papers per year in this base, reaching the maximum amount in 2018, a year that represents almost a quarter of the total period (2013 to 2020) and a cumulative 78% of the corpus. In our corpus, there are only two types of papers, and articles published in events slightly predominate (54%) over those published in journals (46%).

Table 2 - Distribution of MTSK production by year in Web of Science (2013 to 2020)

Job Type	2013	2014	2015	2016	2017	2018	2019	2020	Total
Event Article ⁴	3	3	4	3	3	4	-	-	20
Newspaper article	-	-	2	1	-	6	3	5	17
Total in the year	3	3	6	4	3	10	3	5	37
Total Cumulative	3	6	12	16	19	29	32	37	-

Source: Web of Science data organized by the author.

Previous surveys (Moriel Junior & Duarte, 2020) identify that the first production occurred in 2012 (Escudero et al., 2012), outside the temporal arc identified in WoS. We can say, then, that a phase of MTSK constitution occurred between 2012 and 2014, culminating in the publication of the book entitled *Un marco teórico para el Conocimiento especializado del Profesor de Matemáticas* (Carrillo et al., 2014). In it was presented the conceptualization of the model and its structure of beliefs, domains and subdomains, obtained until that moment. In WoS, there is also a paper in which the creators present the MTSK (Carrillo-Yañez et al., 2018) with some refinements (in terms of categories, for example) fruit of the ongoing theoretical-empirical development and validation of the model.

The production identified here is very important because it is a set of papers indexed in a database of high scientific impact and relevance. It is a select part of the entire MTSK production available in several other databases, such as Google Scholar (Moriel Junior & Duarte, 2020). Comparative and integrative studies of the production in different databases will be the focus of further work.

⁴ Translation of *Proceedings Paper*.

Authors

We identified 45 researchers involved with the production investigated. A third of them (16), approximately, have more than one publication indexed in WoS (Table 3) and these authors together participate in 31 papers, i.e., 86% of all production found. This suggests a collaborative scenario among researchers, and of internationalization given that the group that published the most has produced with the other authors, from various parts of the world (the two-thirds with only one publication).

Table 3 - Authors and participation in MTSK production on Web of Science (2013 to 2020)

Authors ⁵	Country ⁶	N. of papers	Percentage
Carrillo-Yanez, José	Spain	18	48,6%
Contreras, Luis Carlos	Spain	10	27,0%
Montes, Miguel	Spain	10	27,0%
Climent-Rodriguez, Nuria	Spain	8	21,6%
Munoz-Catalan, M. Cinta	Spain	7	18,9%
Escudero-Avila, Dinazar	México	6	16,2%
Flores-Medrano, Eric	México	6	16,2%
Aguilar Gonzalez, Alvaro	Spain	5	13,5%
Liñán Garcia, Maria Del Mar	Spain	5	13,5%
Zakaryan, Diana	Chile	5	13,5%
Espinoza-Vasquez, Gonzalo	Chile	3	8,1%
Vasco, Diana	Ecuador	3	8,1%
Barrera, Victor	Spain	2	5,4%
Flores, Pablo	Spain	2	5,4%
Pascual, Maria Isabel	Spain	2	5,4%
Ribeiro, Miguel	Brazil	2	5,4%
Arteaga-Martinez, Blanca (Espanha); Badillo, Edelmira (Espanha); Belmonte, Juan Miguel (Espanha); Borromeo-Ferri, Rita (Alemanha); Caceres, Maria José (Espanha); Cardenas, Janeth Amparo (Espanha); Carreno, Enrique (Peru); Carvajal, Christian Alfaro (Costa Rica); Codes, Myrian (Espanha); Delgado-Rebolledo, Rosa (Chile); Escudero-Dominguez, Ana (Espanha); Fernandez, Ceneida (Espanha); Ferretti, Federica (Itália); Prieto Gonzalez, Alba (Espanha); Huincahue, Jaime (Chile); Leon, Aurora Fernandez (Espanha); Martin, Juan Pedro (Espanha); Mena-Lorca, Jaime (Chile); Montoya-Delgadillo, Elizabeth (Chile); Moriel Junior, Jeferson Gomes (Brasil); Oliveros, Ingrid (Colombia); Pizarro, Noemi (Chile); Rodriguez, Muniz, Luis Jose (Espanha); Rojas, Nielka (Chile); Soto, Gabriel Valverde (Costa Rica); Verdugo-Hernandez, Paula (Chile); Lira, Marcela Muñoz (Chile).		1	2,7%

Source: Web of Science data organized by the author.

The collaboration among researchers is also expressed when we analyze the number of authors per production. There are only three papers published individually (Table 4), thus reinforcing the previous results. The characteristics identified here seem to contribute to the advance in the global dissemination and, in a way, it sets up a test of international validation

⁵ Format "Last name, First name"

⁶ Author's country of institutional affiliation according to WoS.

of the MTSK in different school cultures and contexts of practice and teacher training. Studies on other bases are being carried out to broaden the understanding of this panorama.

Table 4 - Number of authors per MTSK production per year in Web of Science (2013 to 2020)

Authors	2013	2014	2015	2016	2017	2018	2019	2020 ¹	Total
One author	-	1	-	-	-	-	-	2	3
Two authors	1	2	1	-	1	3	3	1	12
Three authors	2	-	1	1	2	4	-	2	12
More than three authors	-	-	4	3	-	3	-	-	10
Total number of jobs	3	3	6	4	3	10	3	5	37

Source: Web of Science data organized by the author.

Institutions and funding agencies involved

Our results show a total of 27 institutions involved in MTSK production in WoS (Table 5). Nine of them have more than one publication and are present in almost all the others (32 of 37). The institution with the most publications is the University of Huelva, Spain, something that seems consistent with recent history since the theoretical framework was developed there by the SIDM⁷ research group coordinated by Prof. Dr. José Carrillo.

The doctoral theses of SIDM members (Nielka Rojas, Jeferson Moriel-Junior, Miguel A. Montes, Eric Flores, Dinazar Escudero, Diana Vasco, Álvaro Aguilar, Mar Liñán) allowed the model to be developed, particularly its system of categories and indicators, in which the other members also participated (Carrillo, 2017, p. 8, our translation)

Table 5 - Institutions involved in MTSK production in the Web of Science (2013 to 2020)

Institutions	Country ⁸	Nº of papers	Percentage
Universidade de Huelva	Spain	27	72,9%
Universidade de Sevilha	Spain	10	27,0%
Pontifícia Universidade Católica Valparaiso	Chile	7	18,9%
Universidade de Oviedo	Spain	3	8,1%
Universidade Técnica Estatal de Quevedo	México	3	8,1%
Universidade Cardenal Spinola Ceu	Spain	2	5,4%
Universidade de Granada	Spain	2	5,4%
Universidade Autónoma de Puebla	México	2	5,4%
Universidade Estadual de Campinas	Brazil	2	5,4%

Instituto Federal de Mato Grosso – IFMT (Brasil), Universidade Autónoma de Barcelona (Espanha), Universidade Católica de Maule (Chile), Universidade Católica do Norte (Chile), Universidade Complutense de Madri (Espanha), Universidade de Alcala (Espanha), Universidade de Alicante (Espanha), Universidade de Costa Rica (Costa Rica), Universidade de Extremadura (Espanha), Universidade de Piura (Peru), Universidade de Salamanca (Espanha), Universidade Livre de Bolzano (Itália), Universidade Kassel (Alemanha) e Universidade Metropolitana de Ciências da Educação – UMCE (Chile)

Source: Web of Science data organized by the author.

⁷ Entitled Seminar of Researchers in Didactics of Mathematics

⁸ Author's country of institutional affiliation according to WoS.

As for support from research funding agencies, there are 5 explicit records in the database, namely: National Commission for Scientific and Technological Research - CONICYT of Chile, Ministry of Economy and Competitiveness of Spain, COIDESO Research Center of the University of Huelva in Spain and Sub-secretaria de Educación Superior PRODEP of Mexico. These data are in line with the previous ones, in that these three countries have the largest number of institutions with production in this base.

The results of this section converge with the previous ones, both in the collaborative aspect of MTSK production and in the presence of authors who produce more linked to the respective institutions. The data per country, below, also converge.

Countries and languages

The MTSK production comes from nine countries in WoS (Table 6). Of the papers found in this database, almost all (32 out of 37) have authors institutionally linked to Spain. This is also a reflection of where the theoretical framework was developed and disseminated from there. Of the nine countries found, only Germany has no members or collaborators in the Ibero-American MTSK⁹ Network. This means interest and dissemination within Latin America and in European countries. The continuity of the study in other databases may help to clarify not only if there are more productions in these countries, but also to unveil the existence of works in other regions of the world.

Table 6 - Institutional link of MTSK production by countries in Web of Science (2013 to 2020)

Country	Spain	Chile	Brazil	Ecuador	México	Peru	Costa Rica	Italy	Germany
Qty of associated jobs	32 86,4%	9 24.3%	3 8,1%	3 8,1%	2 5,4%	1 2,7%	1 2,7%	1 2,7%	1 2,7%

Source: Web of Science data organized by the author.

It is well known that international impact science communicates through the English language. However, in MTSK production the Spanish language predominates (Table 7). Although this may seem negative in terms of dissemination, it is the second most important language of international communication. We see this as a facilitating aspect for dissemination in Latin American countries (Table 6) where such language predominates and is also familiar to Portuguese-speaking countries such as Brazil and Portugal, belonging to the Ibero-American MTSK Network.

Table 7 - Distribution of MTSK production by year in Web of Science (2013 to 2020)

Language	Spanish	English	Portuguese	Total
Qty of jobs	26 70,2%	9 24,3%	1 2,7%	37 100%

Source: Web of Science data organized by the author.

⁹ Configured in 2016, it currently has 129 members from 10 countries (Argentina, Brazil, Chile, Colombia, Costa Rica, Ecuador, Spain, Mexico, Peru and Venezuela), collaborators from Portugal and Italy, and receives support from the *Asociación Universitaria Iberoamericana de Postgrado - AUIP*.

In any case, we find that the most impactful corpus article is in English, as we detail below.

Brief analysis of quotes

As for the dissemination of papers according to WoS, the set of productions of the investigated *corpus* has on the date of data collection an h-index¹⁰ equal to 4, an average of 10.63 citations/year and 2.3 citations/item, the sum of the number of citations equal to 85 and 66 articles that have cited papers from the corpus. The most cited paper, with an average of 7.25 citations per year, was published in 2018 in the journal *Research In Mathematics Education* entitled *The mathematics teacher's specialized knowledge (MTSK) model* (Carrillo-Yañez et al., 2018). This makes sense, as this article is particularly important as the creators of the MTSK make the formal presentation of the theoretical framework in a high impact scientific journal in the field (h-index = 18, SJR 2020 = 0.96; CiteScore 2020 = 2.1; Q1 Scopus; Q2 JCI WoS).

Keywords

All keywords, titles, and abstracts analyzed were in English, so we kept this language in the results, except when presenting mathematical content, given the equivalence between languages. The analysis of the MTSK productions indicated 146 keywords in total (approximately 4 per paper), which resulted in 78 terms after analysis of repetitions and similarity. From this group, it was possible to identify 16 keywords associated with mathematical topics, presented below in descending order: the concept of function (3), relative positions of lines (3), infinity, quadrilaterals (2), geometry, arithmetic (1), functions, rational numbers, line segment, real analysis, series, subtraction, systems of linear equations, linear algebra, matrices, and numbers.

The remaining terms (62) describe the theoretical and methodological aspects of the productions, which we present in the original acronym in descending order of frequency: Mathematics teacher's specialized knowledge (16), MTSK (15), Professional knowledge (8), Prospective teachers of mathematics (8), Mathematics teachers (3), Initial training of teachers, Mathematical Knowledge for Teaching MKT, Video analysis, Teacher (2), Analogies, Case study, Knowledge of Mathematics Topics KoT, Pedagogical Content Knowledge, Pre-service teachers, Prospective Primary teachers, University lecturer, Activity design (1), Analysis scenarios, Conceptions of teaching-learning mathematics, Curriculum, Definition, Didactic, Early childhood education, Evaluation, Feedback, Formative evaluation, High school student, High school teacher, Knowledge about mathematics learning, Knowledge indications, Knowledge of practices in mathematics KPM, Links and differences Mathematical modelling, Mathematical proof, Mathematics lecturer, Mathematics,

¹⁰ The h-index is based on a list of publications ranked in descending order by Times Cited in WoS. An h-index equal to 3 means that in the corpus there are 3 articles with 3 or more citations. For more details, we recommend you visit <https://clarivate.libguides.com/woscc/citationreport>.

Metacognition, MWS (referring to Mathematical Working Space), Narrative, Online forum, Online teacher education, Opportunities, Pedagogical practice, Pre-service primary teacher education, Problem solving, Reflection, Research methodology, Secondary school, Secondary teacher knowledge, Secondary, Self-questions, Specialized knowledge relations, Strength, Teachers' practices, Teaching of modeling model, Teaching, TEDS-M, Theoretical perspectives, Time measurement instrument, Time, To classify, To define, Trainee teachers mathematics knowledge, and Weakness.

The elements found in the keywords suggest that studies with MTSK have used a considerable range of methodological approaches to investigate the practice and teacher training (including distance learning), covering content from different levels of education (from kindergarten to university) and the two domains of MTSK, didactic content and mathematical, with emphasis on the subdomains Knowledge of Topics and Knowledge of Practices in Mathematics.

These results are a first approximation to understand the thematic and theoretical-methodological trends of MTSK production. These keywords will serve in future studies to analyze similarities with other bases, aiming at the integration of the mappings. Later, they will have an auxiliary role in the content analysis that deepens the qualitative perspective, including successive readings of the abstracts and texts.

Final considerations

In this paper, we map the production related to the theoretical framework Mathematics Teachers' Specialized Knowledge - MTSK in the Web of Science (WoS) database, whose results are:

- There are 37 papers on Mathematics Teachers' Specialized Knowledge in WoS, all indexed in the category/area Education Educational Research;
- There is production from 2013 to 2020 in WoS, in the format of articles in journals and event proceedings (slight predominance of the latter), with at least 3 papers/year and a peak of 10 in 2018, predominantly in the Spanish language;
- 44 researchers (in 9 Ibero-american countries and 27 institutions are collaboratively involved with MTSK research, so there is a core of 16 of them (from 4 countries and 9 institutions) that together participate in almost all the production, however, there is explicit information in the WoS of the fostering of only 3 countries;
- There are 146 keywords in the corpus, which we grouped into 16 terms linked to mathematical content and 62 to theoretical-methodological aspects, convergent with the categories emerging from titles and abstracts, which provide an overview of the MTSK research scenario in WoS;
- The most cited paper in WoS is an impact journal article in the field (Carrillo-Yañez et al., 2018) in which the creators of MTSK present the model including refinements (e.g., advances in categorization) considering the previously published book version (Carrillo et al., 2014).

Our main conclusion is that the MTSK production is collaborative and international, involving 9 countries in Europe and Latin America. It is a networked Ibero-american

movement that contributes to the advancement of global dissemination and, in a way, it configures itself as an international validation test of the MTSK in different school cultures and contexts of practice and teacher training.

Our results are limited only to the records indexed in the Web of Science database. Still, this work materializes an important part of the current scenario of MTSK production and takes a first step in tracking its growth and development to analyze impacts in the field. It may be useful not only for groups and researchers in the subject but also for trainers who intend to design courses and training activities that develop specialized knowledge of teachers focused on the practice of teaching and enabling them to learn mathematics. We also believe that the advances in research can help to subsidize reflections about public policies related to education and teacher training.

The mapping project will continue to integrate the production in other databases with the results of this article. With this, we will be able to understand how the results of this article behave concerning other databases. But, beyond this, a state of the art is being developed to deepen the qualitative perspective, including the analysis of the characteristics, scope, impacts, limits and potentialities of the scientific advances linked to MTSK. For that, besides the categories analyzed in this article, we plan to contemplate: the theme of the studies, other theoretical perspectives used in the MTSK production, objectives of the works, mathematical contents addressed, subjects involved in the research, contexts and levels of education contemplated, as well as main conclusions and scientific advances.

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Authors' contribution statement

JGMJ conceived the idea for this paper, collected the data, developed the methodology for analysis, implemented it, and wrote this article.

Conflict of interest declaration

This author, JGMJ, declares that he/she participates in the Ibero-american Network of MTSK Researchers and has no personal, commercial, scientific, political or financial conflict of interest in the manuscript.

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APPENDIX

Table. Summary of MTSK production records in Web of Science by year (2013 to 2020)

Year	References
2013	(Flores, Escudero & Aguilar, 2013; Garcia & Gonzalez, 2013; Montes, Contreras & Carrillo, 2013)
2014	(Carrillo, 2014; Escudero-Dominguez & Carrillo, 2014; Moriel Junior & Carrillo, 2014)
2015	(Escudero-Avila et al., 2015; Liñan, Montes & Contreras, 2015; Montes & Carrillo, 2015; Montes et al., 2015a; Montes et al., 2015b; Vasco et al., 2015)
2016	(Aguilar, Carrillo & Munoz-Catalan, 2016; Barrera et al., 2016; Climent et al., 2016; Espinoza-Vazquez et al., 2016)
2017	(Escudero-Avila, Flores-Medrano & Carrillo, 2017; Espinoza-Vasquez, Zakaryan & Yanez, 2017; Pascual e Montes, 2017)
2018	(Aguilar-Gonzalez et al., 2018; Badillo & Fernandez, 2018; Cardenas & Caceres, 2018; Carrillo-Yañez et al., 2018; Contreras, Carrillo & Climent, 2018; Espinoza-Vasquez, Zakaryan & Yanez, 2018; Huincahue, Borromeo-Ferri & Mena-Lorca, 2018; Montes, Contreras & Carrillo, 2018; Oliveros et al., 2018; Vasco Mora & Climent Rodríguez, 2018)
2019	(Carreno & Climent, 2019; Prieto-González & Aguilar-González, 2019; Zakaryan & Ribeiro, 2019)
2020	(Carvajal, Martinez & Soto, 2020; Delgado-Rebolledo & Zakaryan, 2020; Pizarro, Belmonte & Arteaga-Martinez, 2020)

List of records according to Web of Science (alphabetical order)

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