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Number sense in children: meaning of numbers, relative magnitude and numerical sequence

Sentido de número em crianças: significado dos números, magnitude relativa e sequência numérica

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Abstract

The present study examined, from a developmental perspective, three indicators of number sense: meaning of numbers, relative magnitude of digits in numbers and distance between numbers in the numerical sequence. A total of 50 children in the 1st and 2nd years of Elementary School performed three tasks, each related to one of these indicators. We used Grouping Analysis to define the participants' profile regarding their understanding of numbers, who were divided into two groups: children with a good mastery and those with a limited mastery. The most skilled children performed better than the least skilled children in assigning meaning to numbers and in relation to the notion of the relative magnitude of digits in numbers. However, assigning meaning to numbers was the most difficult task for the participants in both profiles. We should emphasize the role played by formal and informal experiences in the mathematical knowledge of children.

Keywords: Number Sense; Meanings of Numbers; Relative Magnitude of Digits; Numerical Sequence.

Resumo

O presente estudo examinou, em uma perspectiva de desenvolvimento, três indicadores de sentido numérico: significado dos números, magnitude relativa dos dígitos em números e distância entre números na sequência numérica. Cinquenta crianças alunas do 1º e 2º ano do Ensino Fundamental realizaram três tarefas, cada uma relativa a um desses indicadores. Utilizou-se a Análise de Agrupamentos para definir o perfil dos participantes quanto ao conhecimento que apresentavam sobre números que foram divididos em dois grupos: crianças com um bom domínio e aquelas com um domínio limitado. As crianças mais habilidosas tiveram um melhor desempenho do que as menos habilidosas em atribuir significado aos números e em relação à noção da magnitude relativa dos dígitos em números. Contudo, atribuir significado aos números foi a tarefa mais difícil para os participantes de ambos os perfis. O papel desempenhado pelas experiências formais e informais no conhecimento matemático de crianças é enfatizado.

Palavras-chave: Sentido Numérico; Significados dos Números; Magnitude Relativa dos Dígitos; Sequência Numérica.

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Introduction

Making children numerate is one of the major goals of mathematical education and a great acquisition from the cognitive point of view. This acquisition is broad, gradual and continuous, which, as claimed by Nunes and Bryant (1997), involves familiarity with situations in which numbers are inserted, understanding the logical rules that govern numbers and their relationships, as well as the domain of systems of representation. According to Spinillo (2006), becoming numerate is strongly related to the development of number sense.

Several authors recognize the difficulty in defining number sense. According to Greeno (1991), this difficulty arises from the fact that this term refers to a conceptual mastery of numbers, from which the individual acts in situations involving mathematics. Because it is a holistic concept, as mentioned by Maghfirah and Mahmudi (2018), it refers to a general understanding of numbers and operations, used in a flexible way to solve numerical situations.

Despite the difficulty in defining it, the authors converge in the direction of considering number sense as being a good intuition about numbers, their relations and properties, which develops from their uses and interpretations in diverse contexts, involving the individual's understanding of the use of numbers in the world, besides the ability to deal with them efficiently and flexibly, without being limited to the use of traditional algorithms and numerical accuracy (Godino, Font, Konic & Wilhem, 2009; Howden, 1989; McIntosch, Reys & Reys, 1992; Reys, 1989; Sowder & Shapelle, 1989; Yang & Wu, 2010).

Sowder (1995) comments that it is easier to recognize behaviours that express this term than to define it. Accordingly, from an educational or a cognitive point of view, it is relevant to identify these behaviours. Berch (2005), for example, lists 29 components of number sense. Other authors (e.g., Mohamed & Johnny, 2010; Spinillo, 2006; Yang, Hsu & Huang, 2004; Yang & Lin, 2015; Yang, Li & Lin, 2008) list a smaller number of indicators that, in a way, group and articulate the components listed by Berch.

Spinillo (2006) describes and exemplifies eight indicators, namely: performing flexible numerical computation; making quantitative judgments and inferences; using anchors; recognizing a result of a problem or an operation as adequate or absurd; recognizing the absolute and relative magnitude of the numbers; understanding the effect of operations on numbers, using and recognizing that one instrument or a support of representation may be more useful or appropriate than another; recognizing uses, meanings and functions of numbers in everyday life. This research focuses on three of these indicators: assigning meaning to numbers, recognizing the relative magnitude of numbers, in the case of digits in numbers, and using anchors in numerical sequence.

Meaning of numbers

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Numbers can take on different meanings in everyday life, as highlighted by Cebola (2007) and Spinillo (2014; 2018): a quantity (idea of cardinality), a measure (idea of quantification of dimensions), a position (idea of order) or an identification (idea of identity). For example, the number three can be the age of a child or the number of goals in a football match, however, it cannot be the license plate of a car or the number of a driver's license. The number 30, in turn, can be the age of a person, but it cannot be the number of goals in a soccer game. The ability to assign meaning to numbers is related to the numerical situations that the child faces in his/her daily life, in the most different social contexts.

The idea of quantity seems to be an usual and early notion. When the child understands that a number can mean the quantity of elements in any set, one can say that he/she has developed the notion of number as cardinal. The child will achieve this notion as he/she uses the numerical sequence to count elements and understand that the last mentioned number indicates the total of objects in that set of elements (Gelman & Gallistel, 1978). Counting and numerical sequence are also associated with the first notions of order when the child understands that the number can mean the position or place of something or someone in a sequence. For example, there is a fourth place because there is a third, a second and a first place that precede it.

Spinillo (2018) investigated the uses and meanings assigned to numbers by children aged seven and eight years old, students of the 2nd year of Elementary School in public and private schools. Through an interview, participants were asked to answer questions about numbers, operations and measurement. As for the meanings assigned to numbers, the key question directed to children was: "What are numbers for?" The provided answers were classified into different types. In general, the data showed a predominance of answers that referred to the number as a quantity, such as "... to know how many cookies you have in a package ..." (p.642); and "... it serves to count the things we have. To know how many things we have." (p.644). It is important to mention that none of the answers provided by the 40 interviewed children assigned to the number a meaning related to order and identity. Possibly, due to their age, the children were more familiar with situations involving quantities than with situations involving order and identity. From the aforementioned study, it is possible to conclude that some meanings are more easily assigned to numbers than others, and that this depends on the numerical experiences that children have in their daily lives in the most diverse social contexts.

Relative magnitude of digits in numbers

In general, this indicator of the number sense is related to the ability to compare quantities in absolute and relative terms, being able to discriminate these two instances (Sowder, 1995). Spinillo (2006) illustrates this ability in a dialogue with a child in which she understands that a person who spent R$ 2.00 out of the R$ 5.00 had spent more than a person who spent R$ 2.00 out of the R$10.00.

In symbolic terms, according to Siegler and Braithwaite (2017), the understanding of the magnitude of integers from 1 to 1000 happens slowly and gradually. For example, after

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knowing how to count from 1 to 10, the child still takes about a year or more to understand the relative magnitude of the numbers.

This question regarding symbolic numbers leads to another equally relevant question, which is the understanding of the location of digits in numbers. The understanding of the magnitude of digits depends on the integration of the numerical magnitude (non-symbolic referent) with the symbolic numbering. Children aged five do not yet integrate this information, but children aged six already do. Probably, this integration is related to the effects of formal education, taking into account that children in the 1st year are sufficiently exposed to symbolic numbers (digits), as commented by White, Szucs and Solt (2012) and Siegler (2016).

With the development of the number sense, children become able to understand that the subsequent number in the numerical sequence is always greater than the antecedents (Tracanella & Bianchini, 2017). Nevertheless, in order to decide on the magnitude of numbers composed of more than one digit, children do not usually recite the numerical sequence to decide which is greater. Actually, they make use of their knowledge of the decimal numbering system and decide on using, basically, two principles: (i) the number with the most digits will always be the greatest; and (ii) if the numbers have the same quantity of digits, the one with the first greater digit is the greatest; when the first digits are equal, the observation changes to the second digit; and so on (Lerner & Sadovsky, 1996). These principles apply to integral numbers and help children to understand that 65 is greater than 56 or that 198 is lesser than 201.

Estimating or specifying how much one number is greater or lesser than the other marks an advance in the understanding of the numerical relationship of magnitude (Maghfirah & Mahmudi, 2018). Knowing that there are 10 units of difference between 65 and 56 is to be able to go beyond the recognition that 65 is greater than 56. This gain in the understanding of the numerical magnitude has been associated with the development of arithmetic skills: the greater this mastery, the greater the knowledge of the magnitude of the numbers (Booth & Siegler, 2008; Siegler & Braithwaite, 2017).

Accordingly, understanding the relative magnitude of the digits in a number from the position that they occupy is an important acquisition. This is so true that this knowledge is evaluated in instruments that aim to test children's mathematical knowledge, such as the Numerical Knowledge Test proposed by Okamoto and Case (1996). In this test, there are items that specifically deal with the relative magnitude of digits in numbers, such as, for example, when asking "Which is the smaller: 51 or 39?" and "Which is the greater: 69 or 71?". Items similar to these are present in other instruments such as the Number Sense Battery developed by Jordan, Kaplan, Olah and Locuniak (2006) and the Zareki-R, which is the Neuropsychological Test Battery for Numerical Processing and Calculation in Children (Silva & Santos, 2011).

Zetetiké
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The distance between numbers in the numerical sequence

The knowledge of the numerical sequence is one of the most important tools for the notion of number. It is from this sequence that the principles of counting and the concept of unity are understood (Fuson, 1988; Gelman & Gallistel, 1978).

The first numbers in the numerical sequence are, generally, learned around the age of two. Nonetheless, the child still does not understand that there is an order in the recitation that must be obeyed. The repetition of the numerical sequence still does not have any numerical meaning, being understood only as words that follow one another. Around the age of four, the child starts to consider the order of the numbers and manages to perform the counting with few elements, always starting from the number 'one' (ascending counting). From five to six years of age, he/she has greater mastery of the sequence, being able to continue the counting process from any number (among those he/she knows) and in any direction (increasing and decreasing). However, full knowledge of the numerical sequence is reached around seven years of age, when the child is able to interconnect the notions of order, counting and cardinality (Fuson, 1991).

Serialization and sequencing skills are important for the understanding of the relationships between numbers in the numerical sequence (Piaget & Inhelder, 1983). Serialization is about knowing how to order the elements of a set based on some magnitude, such as size, for example. As for the sequencing ability, it is related to the identification of patterns, that is, regularities between the elements of a sequence. The notion of serialization allows to understand the logic of numerical succession (increasing or decreasing), and the sequencing allows to identify that there is the addition of a unit to the current number to form the subsequent number.

Although serialization and sequencing are consolidated around the age of seven, according to Piaget and Inhelder (1983), the understanding about the distance between numbers (one unit) can already be observed, in an elementary way, in children of four years old, who demonstrate understanding the distance to numbers ranging from 1 to 4. With the development and familiarity with counting situations, they understand that the numerical pattern is repeated for the other numbers in the sequence (Sanchez Júnior & Blanco, 2018; Siegler & Braithwaite, 2017).

A frequently used resource to assess children's understanding of the distance between numbers is the number line (Barth & Paladino, 2011; Booth & Siegler, 2006; Duro & Dorneles, 2019; Siegler & Braithwaite, 2017; Thompson & Opfer 2010). It is argued that the visual representation of the number line helps to create a mental representation about the order and magnitude of the numbers (Woods, Geller & Basaraba, 2018). In these surveys, in general, children are asked to estimate the location of numbers on a number line. Barth and Paladino (2011), for example, observed that children aged five and seven use different strategies to estimate the position of numbers on a 0-100 line. Five-year-old children use the lower (0) and upper (100) points as a reference, while seven-year-olds, in addition to the lower and upper points, also use the midpoint, thereby better adjusting their estimates.

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Duro and Dorneles (2019) identified five types of strategies used by children in the 2nd year to position the number on a number line: counting of unit markings on the line; reverse counting from the end to the beginning of the line; subdivision into 10 blocks containing 10 units; using the estimate previously made as a reference; and quick estimates. The counting of unitary markings was the most frequently used. The authors also observed that the accuracy of the location increases when the number to be positioned is closer to one of the reference points of the line, such as 0, 50 and 100 (for example, 49 which is closer to 50) or some previously used number. The reference points, as claimed by Whyte and Bull (2008), serve as important spatial clues in the accuracy of the estimate.

The distance between numbers in a numerical sequence is also the object of investigation in tests that evaluate the different facets of numerical knowledge, as is the case of the Numerical Knowledge Test developed by Okamoto and Case (1996), previously mentioned. In this test, there are items that deal with the distance between numbers, such as: "Which number is closer to 21? Is it 25 or 18?" and "Which number is closest to 28? Is it 31 or 24?". In these items, the distance between numbers is estimated by means of anchors or reference points that are provided to children.

Due to its breadth and diversity, number sense involves different components or indicators, making it necessary to conduct research that deepens the knowledge about these facets and their relationships, as the present investigation intends. Moreover, investigations on this topic are relevant due to the relationship between number sense and acquisition of mathematical knowledge that serve as a basis for the understanding of complex mathematical concepts. In light of the foregoing, this study investigates three indicators of number sense in children in the early years of Elementary School: meaning of numbers, relative magnitude of digits and numerical sequence. The objective is to examine which one would be the most complex and which would be the most elementary, seeking to identify a possible development trend in relation to these indicators in the scope of number sense.

In methodological terms, we used a paradigm compatible with the evaluation of number sense, which consists of asking the participants to make judgments about situations presented to them, without the need to perform any type of numerical calculation. An important differential in this research was the type of analysis used. While many investigations evaluate the participants according to the age group and school year, this study adopted another way of grouping them, which consisted of grouping the children according to their skill profile regarding the mastery of number sense. The aim is to offer a methodological perspective that allows the identification of the difficulties associated to each indicator in relation to the mastery of number sense.

Method

Participants

The investigation was attended by 50 Elementary School children, being 25 students from the 1st year, aged between 6 years and 1 month and 7 years and 3 months (M = 80

ZETETIKÉ

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months; SD = 4 months) and 25 from the 2^{nd} year, aged between 6 years and 9 months and 8 years and 1 month (M = 90 months; SD = 4 months). All participants were students from municipal public schools in the Metropolitan Region of Recife, without history of school failure, sensory limitations or any type of neurodevelopmental disorder. The children had voluntary participation and their legal guardians signed a FICF (Free and Informed Consent Form). This research was approved by the Research Ethics Committee of the Federal University of Pernambuco under number 526.504.

Instrument

The instrument was composed of three tasks related to different aspects of knowledge about numbers considered indicators of number sense. The task items were designed for this investigation based on the research by Jordan, Kaplan, Olah e Locuniak (2006), Okamoto e Case (1996) e Spinillo (2006; 2018). All items in the three tasks were multiple choice, with three alternatives each. These tasks are described below.

Task A: Meaning of numbers

The task had the objective of evaluating if children would be able to properly identify the different meanings that the number can take in their daily lives, namely: (i) identity, such as the number on the license plate of a car or a residence; (ii) quantity, such as the number of goals in a soccer game or the age of a person in years; and (iii) measure, such as a person's height or the distance between objects. The task was composed of 12 multiple choice items, with three alternatives. In four of the 12 items, the correct alternative referred to identity, quantity and measure.

In a summarized way, each participant was given the following instruction: "A number can be several things. It could, for example, be the person's weight, the quantity of toys that the child has, it could be even someone's phone number. We will show you some cards, each with a number. Each of these numbers can be a thing. You will have to find out what it is." Subsequently, a card was presented with a number that was shown and read aloud by the examiner, who then asked what it meant. Examples:

(Card with the number 3). "Do you think this number is: (a) the kilograms an adult weighs (measure); (b) the number of a person's identity card (identity); or (c) the number of goals in a soccer game (quantity)?"

(Card with the number 12). "Do you think this number is: (a) the litres of water in a pool (measure); (b) the number of shoes to sell at a shoe store (quantity); or (c) the student's number in the teacher's registration book (identity)?"

(Card with the number 300). "Do you think this number is: (a) the student's number in the teacher's registration book (identity); (b) the litres of juice in a jar (measure); or (c) the number of cars in the parking lot at a shopping centre (quantity)?"

(Card with the number 1988). "Do you think this number is: (a) the kilograms an adult weighs (measure); (b) the number of clothes in a person's wardrobe (quantity); or (c) The license plate number of a car (identity)?"

The number of digits in the numbers presented on the cards ranged from one to four, with three items with a single digit number, three items with two digits, three items with three digits and three items with a number consisted of four digits, as illustrated in the examples above.

DOI: 10.20396/zet.v29i00.8660951

Task B: Relative magnitude of digits in numbers

The objective of this task was to evaluate children's notions about the relative magnitude of digits according to their location in numbers. The task was composed of 12 multiple choice items, with two alternatives that were two numbers, printed on cards, that had two, three or four digits. In each item, two cards were presented, with a number in each, which were read aloud by the examiner.

In a summarized way, each participant was given the following instruction: "The numbers can be large and they can be small. Occasionally, there is a number that is greater than the other. For example, the number 21 (reads and shows a card with the printed number) is greater than the number 19 (same procedure). The number 5 (reads and shows the card with the printed number) is greater than the number 2 (same procedure). We will show two cards at the same time, each with a number, and you will find out which is the largest number."

In half of the items, the numerical pairs had the same number of digits. In these items, the digits were the same in the two numbers, but they occupied different positions (units, tens, hundreds and thousands). Examples:

(Cards with the numbers) "Which number is greater: 29 or 92?"

(Cards with the numbers) "Which number is greater: 460 or 604?"

(Cards with the numbers) "Which number is greater: 7901 or 1097?"

In the other half, the quantity of digits in each number was different. In these items, the number with more digits was formed by smaller digits and the number with less digits was formed by greater digits. Examples:

(Cards with the numbers) "Which number is greater: 102 or 97?"

(Cards with the numbers) "Which number is greater: 4210 or 899?"

(Cards with the numbers) "Which number is greater: 98 or 5024?"

In half of the items the correct answer was in the first alternative; and, in the other half, it was in the second alternative.

Task C: Distance between numbers in the numerical sequence

This task had the objective of evaluating children's intuitive notions of numerical sequence, specifically dealing with the distance between numbers. The task consisted of 12 multiple choice items with two alternatives. In each item, three cards were presented, with a number on each one, which were read aloud by the examiner. Of the three numbers, two were alternatives and one was used as an anchor or reference point for children's judgments regarding the distance between the numbers offered as alternatives.

In a summarized way, each participant was given the following instruction: "The numbers follow each other. For example, the number 5 comes before the number 9 and comes after the number 2. The number 2 comes before the number 3 and the number 5. We will show you some cards, each with a number. You will have to find out if the number that we are showing is closer to one number or if it is closer to another number that we are also going to show." Examples:

DOI: 10.20396/zet.v29i00.8660951

(Card with the number 4) “Is this number closer to the number 9 (the examiner reads and shows the card) or closer to the number 5 (shows the card)?”

(Card with the number 28) “Is this number closer to the number 20 (the examiner reads and shows the card) or closer to the number 30 (shows the card)?”

(Card with the number 900) “Is this number closer to the number 890 (the examiner reads and shows the card) or closer to the number 800 (shows the card)?”

(Card with the number 2509) “Is this number closer to the number 2500 (the examiner reads and shows the card) or closer to the number 2600 (shows the card)?”

In each item, the quantity of digits in the three numbers was always the same, and varied from one to four digits. In three of the 12 items, the three numbers had a single digit; in three items, the numbers had two digits; in three items, the numbers had three digits; and, in three items, the numbers had four digits, as shown in the examples above.

Considering the numerical sequence, in four items, the anchor or reference number was positioned before the two numbers that were presented as alternatives; in four items, the anchor number was positioned between the two numbers that were alternatives; and, in four items, the anchor number was positioned after the two numbers that were alternatives. In half of the items, the correct answer was in the first alternative; and, in the other half, it was in the second alternative.

Procedure

Children were interviewed individually, in a single session lasting approximately 30 minutes. The order of presentation of the tasks was randomized according to the six possible combinations of grouping for Task A, Task B and Task C. The order of presentation of the items in each task was decided by drawing lots for each child. If necessary, the task instructions were repeated and given in more detail.

Data analysis

The children’s skill profile regarding the mastery of number sense was defined by means of Grouping Analysis. We used Analysis of Variance (ANOVA) to compare the performance of participants from both school years in the three tasks, as well as to assess the degree of relative difficulty of the tasks. In order to examine the association between the mastery of number sense and the children’s schooling level, we used the Chi-Square test.

Results

The children’s profile regarding the number sense was defined by means of Grouping Analysis based on the scores obtained in the three tasks. Accordingly, two groups of participants were constituted: Group 1, with 46% of the participants (23 children), formed by those with smaller mastery of the notion of number; and Group 2, with 54% of the participants (27 children), formed by those with greater mastery of this notion, as shown in Table 1.

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Table 1 – Mean (M) and standard deviation (SD) of correct responses in each task according to the children's profiles

	Task A Meaning of numbers		Task B Relative magnitude of digits in numbers		Task C Distance between numbers in the numerical sequence	
	M	SD	M	SD	M	SD
Group 1 **Greater mastery**	0.54	0.13	0.87	0.10	0.61	0.14
Group 2 **Smaller mastery**	0.34	0.10	0.66	0.12	0.64	0.11

Source: The authors

The analysis of variance revealed a significant difference between the performance of the Groups [F (1.48) = 35.88, p <.01] in relation to the level of difficulty of the Tasks [F (2.96) = 107.85, p <.01], where these differences are better interpreted by examining the significant interaction between Groups and Tasks [F (2.96) = 16.86, p <.01]. Although the performance of the Group 1 was generally better for Task A and Task B, the same was not true for Task C. In this task, there was no significant difference between the groups. It is important to note that Task B (relative magnitude of digits) was expressively easier for children with greater mastery (Group 1) than for those with smaller mastery (Group 2).

The results also revealed that the level of difficulty among tasks varied according to groups. We observed that, for children with greater mastery (Group 1), Task B was easier than Task C, and this easier than Task A. For children with smaller mastery (Group 2), Task A was also the harder. Nevertheless, there was no significant difference in the performance of the Group 2 for Tasks B and C. We can observe that Task A (meaning of numbers) was the harder for children of both groups, especially for those with smaller mastery of the notion of number, who had difficulty in assigning meaning to numbers properly.

Table 2 shows the distribution of the groups according to schooling. As we can see, there was a significant interaction between the schooling of the children and the profile of skills that they presented (X^2 = 3.94, df=1, p=.05).

Table 2 – Number and percentage of children per school year in each performance group

Schooling	Group 1 Greater mastery (n=23)	Group 2 Smaller mastery (n=27)
1st year	8 (35%)	17 (63%)
2nd year	15 (65%)	10 (37%)

Source: The authors

It is possible to observe that the majority of the most skilled children regarding the notion of numbers (Group 1) was in the 2nd year, while the majority of the least skilled children (Group 2) were concentrated in the 1st year of schooling. This data suggest that there is a relationship between the school year and the child's skill profile in relation to the number sense.

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Conclusions and discussion

This study had the objective of investigating the number sense in children, students of the 1st and 2nd years of Elementary School. Three indicators of an intuitive understanding of numbers were examined: (i) the meanings assigned to them in different everyday situations; (ii) the relative magnitude of the digits that depends on the position they occupy in a number (units, tens, hundreds and thousands); and (iii) numerical sequence of numbers, using reference points employed to estimate the distance between numerical pairs.

With the analyses, it was possible to describe the children's profiles according to their skills regarding the notion of number based on these three indicators considered together. Accordingly, the children were grouped into two groups: one that had a greater mastery of the number sense and another with a smaller mastery.

The group with the greater mastery was predominantly constituted of children from the 2nd year of Elementary School, while the group with the smaller mastery was mostly formed by children from the 1st year. As mentioned, there seems to be a relationship between the advancement of schooling and the mastery of children with regard to number sense, so that children of the 2nd year have an intuitive notion about numbers more elaborated than those of the 1st year.

As each of the three indicators considered in this investigation revealed a specific facet of the number sense, it was important to examine which of them differentiated the groups of children who had different profiles as to the mastery that they presented in relation to the number sense, and which did not differentiate the two groups. From the obtained data, we found that the groups differed in relation to two of the three examined indicators: the meanings assigned to the numbers and the notions about the relative magnitude of the digits in a number.

The most skilled children, who tended to be the most advanced in schooling, more successfully identified the meanings assigned to numbers than those least skilled. However, despite this difference, the proper identification of the meanings assigned to numbers was difficult for children of both profiles. In other words, despite being more skilled and the majority being in the 2nd year, the children continued to have great difficulty in assigning meanings to numbers in an appropriate way. This result was surprising, since it was thought that, due to the fact that it is a kind of knowledge that can be acquired from informal out-of-school experiences, this would be an easily developed notion. Another intriguing result was that neither education nor a greater mastery of skills were enough for the development of this indicator of number sense.

Another difference between groups was regarding the notion of the relative magnitude of the digits from their location in the numbers. This indicator was more elaborate among the most skilled children who, in turn, were, in their majority, those who attended the 2nd year of Elementary School. It is important to comment that this indicator is associated with the notion of positional value of digits, a notion of a school nature, especially among students in

Zetetiké

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the 2nd year of schooling, when arithmetic operations are addressed in a more systematic way, including by means of the use of their algorithms. Accordingly, this result was already, in a way, expected.

Moreover, it is important to comment that, despite these differences between groups, for both the most skilled and the least skilled children, the ability to assign meanings to numbers was the most difficult indicator. This difficulty may be related to the fact that the meanings that can be assigned to the number are associated with the social contexts in which the numbers emerge. In other words, this ability depends on the child's social knowledge derived from the use of numbers in different situations. It is possible that, due to the age group, the social contexts that the children in this study encounter are little varied, thereby not favouring contact with a wide variety of meanings that the numbers take in their everyday life. For example, quantification is part of mathematical activities experienced by children in the family environment from an early age (Benavides-Varela et al., 2016; Cankaya & LeFevre, 2016), which can contribute to the notion that the number means a quantity. Conversely, the notion that the number can mean an identity may not be part of children's mathematical experiences as much as quantification. What is relevant to emphasize is that the meaning of numbers is strongly associated with social experiences and that these experiences may prioritize certain meanings to the detriment of others.

In light of the foregoing, an aspect that deserves to be explored in future research is the fact of knowing in which of the meanings assigned to the number (measure, quantity and identity) the child has more difficulty. The present investigation did not address this issue, but it certainly needs to be examined by comparing the children's performance in the items referring to each of these meanings.

However, regardless of the social experiences lived by the children, the school can play an important role in the development of this indicator of number sense, thereby creating didactic situations in which the different meanings assigned to the numbers are presented in a variety of contexts, which is explicitly mentioned and discussed with the students.

Despite the difficulty in assigning meaning to numbers, the relative magnitude of digits was an indicator easily understood by children grouped in the two skill profiles. According to Lerner and Sadovsky (1996), since the age of six, children have had a certain mastery of the decimal numbering system and this knowledge seems to help them to discriminate the magnitude of multi-digit numbers. However, schooling enhances this knowledge. It is possible that, as discussed by Booth and Siegler (2008) and by Siegler and Braithwaite (2017), there is a correlation between learning arithmetic and knowing the magnitude of digits in numbers; this knowledge is also associated with the systematic exposure to numbers provided by formal education, as commented by Siegler (2016) and White, Szucs and Solt (2011). In this case, the formal instruction in working with the concepts of absolute and relative value of digits in numbers may have contributed to the children's good performance in relation to this indicator.

ZETETIKÉ

DOI: 10.20396/zet.v29i00.8660951

On the other hand, the knowledge about the distance between numbers was the only aspect that did not vary due to the children's skill profiles and, consequently, their schooling level. Apparently, recognizing and comparing the distance between numbers is a complex activity whose mastery extends beyond the early years of Elementary School. Recognizing and comparing the distance between numbers, as examined in the present investigation, requires more sophisticated knowledge than understanding the sequence of numbers on the number line, as it offers a visual support that was not considered in Task C. It is worth commenting that offering a number as a reference point to judge the distance between numbers was not enough to assist in the resolution of this task. It is important to mention that visual representations such as the number line help to create a mental representation about the order of numbers as highlighted by Woods, Geller and Basaraba (2018), for example. It is possible to suppose that if the procedure adopted in Task C included the presentation of the number line, the children could have performed better. From an educational point of view, it could be relevant to explore the use of the number line initially as a visual support to help in the understanding about the sequence of numbers. Subsequently, when the children had developed an understanding of this sequence, the teacher could highlight numbers on this line, which could serve as reference points to judge the distance between two other numbers, as done in Task C.

The data obtained in this study shows that the concept of number sense involves several facets that, although related, do not develop as a whole. What has been observed is that the indicators that constitute it have a certain independence and that, therefore, they develop at different rates. As it is not a single construct, studies about the development of number sense require the investigation of each of its indicators.

The development of number sense is a long and wide path that involves both informal and formal experiences with mathematics in and out of school. This development should not be understood as something referring only to students in the early years, but also as a type of knowledge to be developed in students in earlier years of schooling, as emphasized by several authors (Akkaya, 2016; Bütüner, 2018; İymen & Paksu, 2015; Yang & Hsu, 2009; Yang & Lin, 2015). This is justified because studies show that there are relationships between number sense and acquisition of mathematical knowledge in general (e.g., Cekirdekci, Sengul & Dogan, 2016; Jordan, Glutting & Ramineni, 2010; Yang, Li & Lin, 2008) and highlight the importance of number sense to understand children's learning difficulties in mathematics (Corso & Dorneles, 2010).

In light of the foregoing, it is important to conduct intervention studies that develop the number sense such as those held by Castro e Rodrigues (2009), Cebola (2007), Yang e Hsu (2009), Yang, Hsu e Huang (2004) e Yang e Wu (2010), for example. However, it is necessary to accomplish intervention studies with younger children, students attending early years of Elementary School. Equally relevant is conducting research targeted at the training of teachers in terms of ways of teaching mathematics with the objective of developing a number sense in students, as emphasized by Kaminski (2002) and Serrazina and Rodrigues (2021).

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Finally, developing the number sense is, ultimately, thinking mathematically in situations involving numbers. From a psychological point of view, it is necessary to understand the different manifestations of number sense, which leads us to consider their indicators, as held in this investigation in relation to some of them. From an educational point of view, it is necessary to teach mathematics in such a way as to develop the number sense in relation to all mathematical concepts dealt with by the school. This is particularly important in the early years so that students, from an early age, become numerate.

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