



The mathematics of geometric drawings in the Resende Costa's Handicraft Weaving

A matemática dos desenhos geométricos presente na Tecelagem Artesanal de Resende Costa

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Abstract

This research sought to investigate the textile handicraft developed in the municipality of Resende Costa, Minas Gerais, Brazil, and the drawings present therein. In this way, questions were raised about the proximity of these drawings to school geometry and which aspects of the dimensions of Ethnomathematics formulated by D'Ambrosio are observed in the weaving. The study was carried out through an ethnographic approach with observations, interviews and recording of images, audios, and videos of weaving. The results indicated the presence of counting elements, operations, symmetry and plane geometry and specific characteristics associated with the dimensions of the Ethnomathematics Program.

Keywords: Handicraft weaving; Aesthetic Elaborations; Dimensions of Ethnomathematics.

Resumo

Esta pesquisa buscou investigar o artesanato têxtil desenvolvido no município de Resende Costa, Minas Gerais, e os desenhos nele presentes. Desta forma, questionamos sobre a proximidade desses desenhos com a geometria escolar e que aspectos das dimensões da Etnomatemática formuladas por D'Ambrosio são observados na tecedura Resende costense. O estudo foi desenvolvido sob a perspectiva etnográfica com a realização de observações, entrevistas e registro de imagens, áudios e vídeo do tecer. Os resultados indicaram a presença de elementos de contagem, operações, simetria e geometria plana e características específicas associadas às dimensões do Programa Etnomatemática.

Palavras-chave: Tecelagem artesanal; Elaboraões Estéticas; Dimensões da Etnomatemática.

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Introduction

It is known the difficulty of teaching geometry in basic education in Brazil, particularly regarding ways to make it closer to students' daily lives, "it is noticed a valuation of mathematical knowledge as a strictly mental elaboration, supported by logical deduction. Such valuation materializes in the large number of tasks with these characteristics, as in the content organization through sequenced definitions, theorems, propositions, and observations" (Cury, 2019, p. 18). We see that this difficulty is rooted in teachers' education, which occurs according to the adopted curricula that, in most cases, consider the theoretical basis knowledge as more relevant.

Thus, we have the following challenges for the teaching of geometry in Brazil: "many teachers do not have the geometric knowledge necessary to carry out their teaching practices" (Lorenzato, 1995, p. 3), "if it is visible the abandonment of geometry teaching, it is observed, however, among mathematics teachers, a great uneasiness regarding to it" (Pavanello, 1993, p. 7), "it is pointed out by teachers the absence of time to prepare different activities and for studying to improve the teaching practice. Consequently, the teacher teaches geometry through formulas, theorems, graphs, specific exercises aimed to fix the content" (Hartwig, Pereira, Machado & Miranda, 2016, p. 251). On the other hand, geometry is advocated as indispensable knowledge for the students' development in school according to the Brazilian national curricular base, the BNCC (Base Nacional Comum Curricular).

The teaching practice "proves that the presentation of abstract and difficult topics should not be done from the beginning. The teaching of Design must follow the History of Science; one cannot start an instruction with the most general ideas and abstract formulations" (Montenegro, 1991, p. 158). According to this author, it is essential to start from the historical, familiar, and concrete context in the process of geometric forms creation for understanding their logics. For this, it is important to "seek alternatives that enable challenging the student to create relationships between Geometry and its expression in the world around" (Hartwig et al., 2016, p. 251). This perspective has been investigated in the Ethnogeometry field, which had the researcher Paulus Gerdes (*In Memoriam*) as its main precursor, who mentioned that:

Geometry was born as an empirical or experimental science. In the "confrontation" with his environment, ancient Stone Age man arrived at the first geometrical knowledge. The process of acquiring abstract images by working out the spatial relations between physical objects and their parts first proceeded in an extremely slow way (Gerdes, 2012a, p. 29).

From this point of view, we can assume that there difficulties would not happen in the teaching of geometry, because its historical and anthropological basis indicates that geometric thought was built slowly and has been present since ancient times in the daily lives of societies. Studies in ethnogeometry suggest that the geometries present in various cultural contexts should be analyzed and may contribute for the teaching and learning in which the

construction of geometric concepts occurs without detachment from the "world" experienced by students.

About weaving, particularly, since "geometric knowledge and its concepts are widely applied in the creation of tactile and visual effects on surfaces. The manipulation of geometric knowledge allows creative applications on surfaces" (Menegucci, Martins & Menezes, 2016, p. 80), the aesthetics of surfaces applied to textile materials has the possibility of diverse categorizations that encompass the structures for crossing threads. Therefore, for the formation of drawings and fabrics texture, we note the existence of aesthetic characteristics and techniques for interweaving threads expressing cultural identities, given that "there is 'hidden' or 'frozen' mathematics. The craftsman who imitates a known production technique is usually not doing much mathematics. But the craftsman who discovered the technique, did mathematics, developed mathematics, was thinking mathematically" (Gerdes, 2012b, p. 72).

The present research was conducted in the town "of the loom" – Resende Costa, where a "craft weaving" intangible and secular cultural asset was developed and has been preserved, with its origins from the times of Portuguese colonization of Brazil, particularly in the south region of the actual state of Minas Gerais. Historically, "in the middle of the eighteenth century, the traveler who circulated through the paths of Minas, fulfilling the route between the dynamic São João del-Rei and the old mining region, spotted the Arraial da Lage, built on an imposing granite rock" (Resende, 2016, p. 15). According to Martins (2016), the settlement and the hamlet were formed first, then a village was built and, finally, the city. According to Castro (2016), the village of Lajes gained its emancipation as a municipality in 1912 and received the name Resende Costa, thus honoring two inconfindentes, the father and the son who lived in the village and participate of the group and movement against the colonial domain intitled "Inconfidência Mineira", which took place in the captaincy of Minas Gerais in 1789. Today, the town lives predominantly from textile craftwork.

The textile manufacturing of Resende Costa is integrated to cultures and techniques that range from the construction of the loom carried out by carpenters to the art of weaving itself. The activity is valued by the community, since the quilts and rugs produced, specifically, have several meanings and needs, such as memory and culture preservation, source of income, economic maintenance of the town, and others.

This article focus attention on the geometric patterns of the Resende Costa's weaving. Our investigation aimed to identify the drawings used in the making of textile handicrafts manufactured in the municipality and to answer the following research questions: what relationships may be established between the weaving designs made in the town and school geometry? What aspects of the Ethnomathematics dimensions formulated by D'Ambrosio are observed?

The Ethnomathematics Program

The Ethnomathematics Program was founded by a group of mathematical researchers at Unicamp (State University of Campinas), with Professor Ubiratan D'Ambrósio (In Memoriam) at the forefront. "Ethnomathematics owes the beginning of its development as an area of Mathematics Education to Ubiratan D'Ambrósio, who, in the mid-1970s, presented his first theorizations about this field of studies" (Knijnik, 2004, p. 20-21). Similarly, Gerdes (2010, p. 17) mentioned that "Ubiratan D'Ambrósio is internationally considered the 'father of ethnomathematics', founder of a whole profound program of reflection and research on the development of mathematical ideas in the most diverse historical, cultural and educational contexts". Currently there are many academic works on Ethnomathematics, which shows a growing interest for the field.

According to D'Ambrósio (1998, p. 7), "Ethnomathematics is a program that aims to explain the knowledge generation, organization, and transmission processes in diverse cultural systems and the interactive forces that act in and among the three processes". He described the program by means of a fundamentally holistic perspective, that is, mathematics is not seen in a fragmented way. Accordingly, for Vergani (2007, p. 25), Ethnomathematics is a "holistic socio-culturally contextualized education", which "recognizes the inevitability of resorting to anthropological thinking, as a look resting on the totality of man".

As Ferreira (2007) points out, the Ethnomathematics Program is the scientific educational program that is mostly concerned with the students' social issue, and it is vigorously political also. This author argues that works such as those made by Knijnik, Powell, Frankenstein and Gerdes demonstrate this, and mainly emphasize political action, but this does not mean that this concern is absent in other studies.

Therefore, the Ethnomathematics Program is a field of research in which the mathematical doing has suggestions for the didactics of mathematics in order to mediate changes in the educational field, as well as in the social and political arenas. Furthermore, D'Ambrosio (2018) organized the Ethnomathematics studies according to six dimensions: conceptual, historical, cognitive, epistemological, political, and educational, which are often interconnected and allow to reflections and pedagogical actions. These dimensions were formulated in his book "Etnomatemática – elo entre as tradições e a modernidade" (*Ethnomathematics – link between traditions and modernity*) (D'Ambrosio, 2018), which was first published in 2002. The studies undertaken in this research were based on the 2018 book edition and the dimensions are presented below.

The conceptual dimension

The conceptual dimension of Ethnomathematics, that is, its general and abstract representation is constituted as a reaction to the impetus of survival and transcendence which are natural and located at the origins of mathematical principles. D'Ambrósio (2018) explained that man and animal are different related to the need for evolution, since the human species is always progressing in creating better survival techniques, and the mathematical and technological knowledge is involved on it. The theories applied to reflect reality and generate examples of understanding space and time for immediate solutions transform the individual

behavior, which is, most of the time, aiming to improve routinary activities ideally. This knowledge is disseminated by community and preserved, resulting in harmonious group culture, as there are shared knowledge and compatible behavior in a culture.

The historical dimension

For D'Ambrósio (2018) the historical dimension of Ethnomathematics corresponds to the study of human's development over time and space. In this context, the analysis and procedures integration techniques for instruction developed by modern science are justified by a historical perspective, such as the knowledge of the Egyptians, Babylonians, Jews, Greeks and Romans, traditionally related to the foundation of the current mathematical logical bases. It is known that the quantitative reasoning of Babylonians opened space for the typical qualitative reasoning of Greeks. Therefore, there is a pulsation to understand the qualitative reasoning also.

The cognitive dimension

The cognitive dimension of Ethnomathematics, that means the human beings ways of thinking, is the research object of many scholars. It aims to study models of thought of communities and searches for understanding how mathematical ideas are manifested, as "the mathematical knowledge can be considered a set of knowledge and actions accumulated by the members of these groups, which is composed of mathematical ideas such as comparing, classifying, quantifying, measuring, explaining, generalizing, modeling and evaluating, which are inseparable and in permanent evolution" (Rosa & Orey, 2014, p. 89). It is, then, a dimension inserted in the scientific, social, and anthropological contexts.

Throughout history, according to D'Ambrósio (2018), human species has developed strategies and capabilities to ensure survival in daily practices, which have being conveyed through communication. Behavior and knowledge are individual, but they are transformed by the existence of the other, by interaction within the group. Thus, the concept of culture is established as there are shared knowledge and compatibilized behavior composing each group.

The epistemological dimension

The epistemological dimension of Ethnomathematics refers to that which investigates answers for humankind questions of survival and evolution in each group and how its knowledge mechanisms are formed. Real quotidian problems, which refer to its transcendence, are the knowing and doing practices of a given culture. To understand the relationship in the use of mathematics by these groups over time synthesizes the controversy between the empirical and the theoretical, that is, the realm of lived experience versus scientific knowledge.

The epistemological dimension of Ethnomathematics was presented by D'Ambrósio (2018) as a cycle that values the subject's action correspondence to reality, his/her behavior, and knowledge as a whole in the course of the method. By this process, the subject is validated as a member of society in a harmonious cycle in which the production,

systematization, and propagation of knowledge returns to those who produce it, resulting in understanding the knowledge cycle in an integrated way.

The political dimension

The political dimension of Ethnomathematics as science of the organization manifested in the world globalization process, in which Western knowledge is predominant in societies, derives from the historical itinerary of civilizations involving conquerors and conquered, with occupations and impositions of methodologies, ideologies, social and political organizations. In short, the elimination of historicity and the exclusion of the dominated is the materialization of this invasion.

According to D'Ambrósio (2018, p. 41- 42), "negative and perverse results are usually noted, which manifest themselves, above all, in the exercise of power and the elimination or exclusion of the dominated". From a political perspective, Ethnomathematics recognizes and respects the value of other cultures and their history, reinforcing these sociocultural roots. In this process, it proposes a reflection on the excluded and the subordinated possibilities to access, seeking for educational strategies that favor the students' autonomy through their dignity restoration. In D'Ambrósio's (2018) thought this is the most important dimension of Ethnomathematics.

The educational dimension

The educational dimension of Ethnomathematics, or the perspective of teaching-learning processes, does not disregard academic mathematics, since this is the current knowledge that is present and drives much of modern society routine. Therefore, "it is not a matter of ignoring or rejecting modern knowledge and behavior. But, rather, to enhance them, incorporating values of humanity synthesized in an ethic of respect, solidarity, and cooperation" (D'Ambrósio, 2018, p. 43).

It is a more humanized mathematics that comes from the incorporation of values such as solidarity, cooperation, respect, ethics, flexibility, and honesty in the methodology of teaching-learning mathematics, that is, it proposes to fortify cultural roots supported by qualitative reasoning. In the educational context, Ethnomathematics involves holistic and multicultural features.

Methodology

This research was qualitative in nature with an ethnographic bias. In this sense, "the methodological procedures of ethnography provide the insertion of the researcher in the field, as a participant observer, permanent and reflective, listening, seeing what happens in this environment" (Fritzen, 2012, p. 59). However, according to Ghasarian (2008), the idea of participant observation is paradoxical because, for him, if there is too much participation by the researcher there is the risk of increasing closeness, and if there is too little participation there may be a failure to understand the insider's point of view. Thus, during the research trajectory, we sought to conduct moderate participant observation; the author clarifies that,

"ideally, every 'good field' combines the views of insiders and outsiders, and ethnographers go back and forth between observation and participation, depending on the situation" (Ghasarian, 2008, p. 15).

The group chosen for the investigation was formed by people who live in the city of Resende Costa and make weaving drawings using handlooms, which are similar to school plane geometry figures. Initially, we consulted the local library to understand the history of this handicraft and made conversations with local people to identify places where quilts and rugs were produced and, more specifically, who knew to make drawings using the handloom. Next, we visited the handicraft selling points and the ateliers for the weaving work photographic documentation.

After this explorative step, we prepared a script containing ten questions for listening 18 craftspeople, which were audio recorded. The questions were organized into categories as personal data, activities, level of education, school mathematics content related to weaving according to the interviewed, and reports about the know-how-to-do fabrics with drawings on the loom.

Next we followed the making of a "beak rug" by two weavers, with audiovisual recording. One of them participated in the previous conversations. Then, the "listening" was transformed into seeing and participating as an insider, that is, as someone who enters the group to learn. The data collection took place in person, that is, at the artisans' workplaces. Therefore, the interactions involved 19 people, 16 women and three men, whose participation was voluntary. The data analysis was done by means of the theoretical foundations of the Ethnomathematics Program and academic and school mathematics.

History, materials, geometric designs, and weaver identity

In our research about the history of weaving and what happens in the ateliers, we identified the use of the manual treadle loom for the elaboration of textile handicrafts. The artisans told us that this production was once made with wool threads brought by travelers to the "Arraial"; there was also washing this wool, its coloring, and the production of threads. This activity was not highly valued as "cultural assets which don't belong to the elites were relegated to oblivion for a long time" (Corá, 2014, p. 1098). Without being attached to social, economic, and political issues, and keeping the focus on the memory of the old weaving way, historian Flávia Silva stated that:

After the preparation of the raw material, the artisans started the warp, with the help of a rustic warp machine; they then did the complicated mathematics of dividing and "passing" the threads through the "four-leaf clover" to create different design patterns. Very creative names were given to the various "repassings", such as dice, cross, buckle, hug roses, Solomon's crown, and broken orange. Even today we can find in some houses the little notebooks in which the repasses were noted with delicacy and care (Silva, 2016, p. 180).

This quote mentions the mathematical skill in the way the designs of the handmade rugs and quilts were made, suggesting relationships with geometric shapes since the old

times. Figures 1 and 2 show two images of quilts made with wool threads on old looms in Resende Costa and the designs made on them.



Figure 1 - Drawing made on a wool quilt made in Resende Costa/MG on the old loom.
Source: research data (2019).



Figure 2 - Another drawing made on a wool quilt made in Resende Costa/MG on the old loom.
Source: research data (2019).

Currently, the weaving is done with twine threads and often with laminated fabrics remnants from clothes manufacturing that are twisted to make threads, which are then enrolled to form spherical balls to facilitate the movements in the loom. There is a combination of colors and shapes in the weaving, and each pattern and texture relates to an epoch and its trends. Thus, the colors, the threads density, and the quantity of hanks used determine the appearance of the drawings and the surface of the fabrics.

Therefore, the elaboration of geometric drawings in Resende Costa's weaving has crossed generations. We are referring to a cultural heritage transmitted by the threads crossings in the looms, a reproduction of geometric drawings that has passed through several generations and by changes with time. In figure 3 we show a rug being woven and in figure 4 an example of a rug done with fabric remnants threads:



Figure 3 - Photograph of the warp and weft of a fabric - Resende Costa/MG

Source: research data (2019).

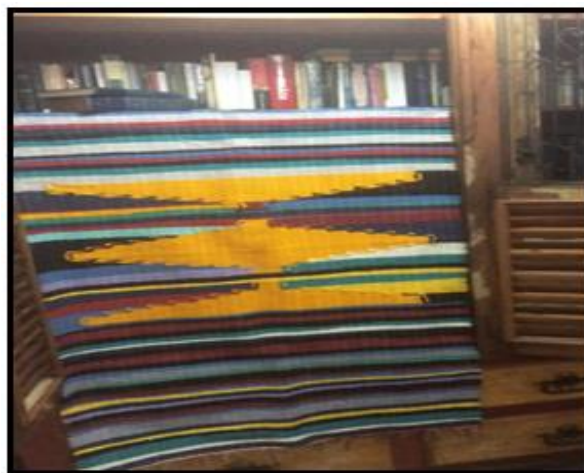


Figure 4 - Photograph of a rug recently made in the municipality of Resende Costa.

Source: research data (2019).

In figure 5 we present some images registered by photographic recordings made in the initial fieldwork involving stores and ateliers and their corresponding decorative patterns sketches. The images show that the aesthetic patterns observed correspond, in most cases, to closed polygonal lines. The association of these figures to "polygons" can be observed in the images where there are "sides", that is, simple lines that are formed by segments of straight lines. This characteristic is defined by the properties of two-dimensional figures. Furthermore, "a simple polygon can be concave or convex. A polygon is convex if the line that contains any of its sides leaves all other sides in the same semi plane (in one of the two semi planes which it determines)" (Iezzi, Machado & Dolce, 2010, p. 178) and "a polygon is concave if there is a line that contains one of its sides and leaves part of the other sides in one semi plane and part in the other semi plane (semi planes determined by it)" (Iezzi et al., 2010, p. 178). Therefore, from this perspective and through an understanding of the sketches presented, we argue that there are similarities of the drawings with the convex and concave polygons of school mathematics.

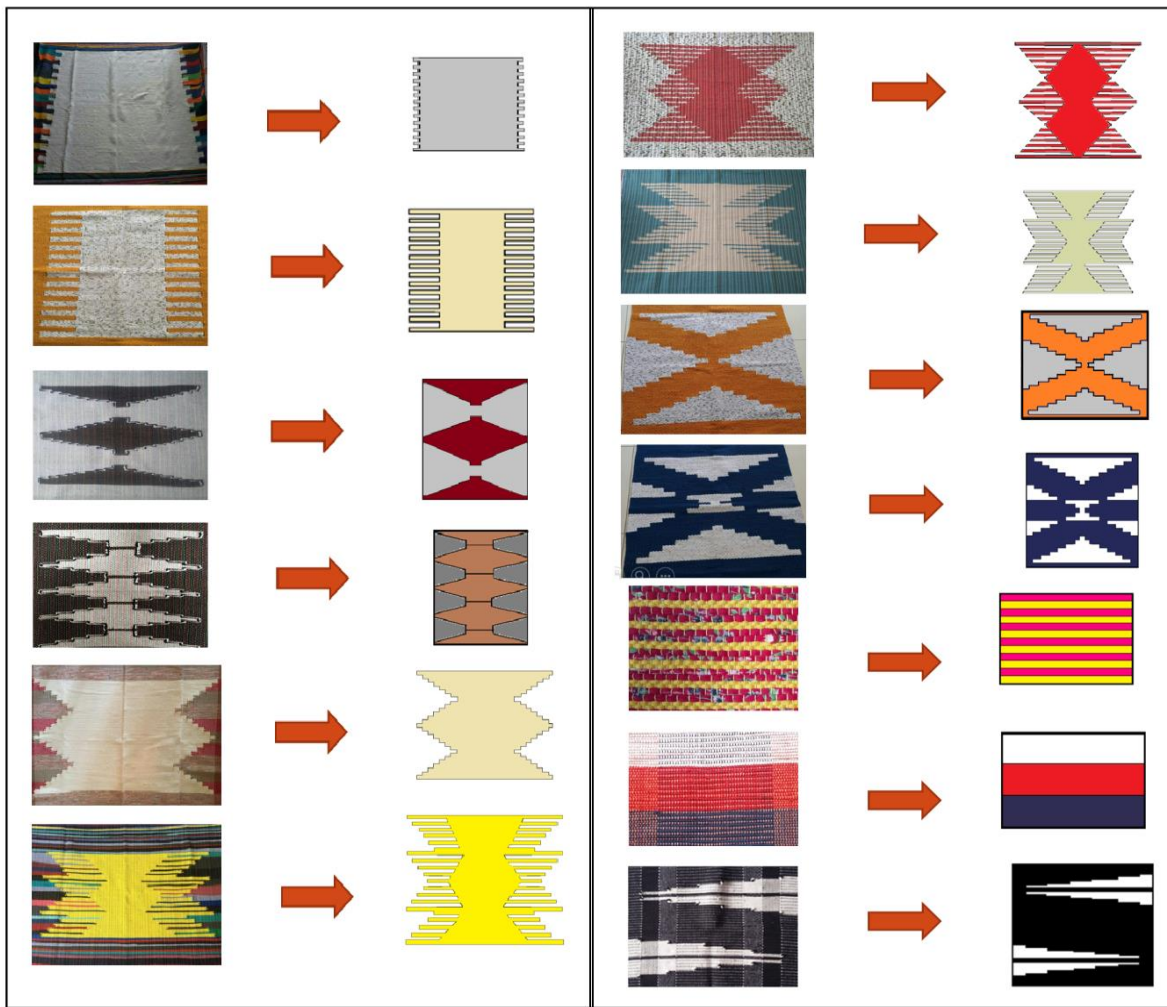


Figure 5 - Patterns of aesthetic decoration in quilts and rugs made in Resende Costa/MG.
Source: research data (2019).

The knowledge of the weaving community

In the proposal to understand the geometry of the Resende Costa's textile crafts, focusing on the quilts and rugs, we identified that mathematics was associated mainly with: "counting, we have to count the threads", "we have to count the little squares of the drawings", "we count the threads to put in the right place the changes of the drawing", "go counting the threads from one point to the other", "it is symmetrical", have to "measure very well", "we have to count the threads", "if you don't count", "the beak gets crooked, it has to be the same amount on each side". From these utterances, we observed an awareness of the basic operations use, like counting "threads and squares", and measuring to guarantee the symmetry.

The weavers associated some names of geometric figures, probably learned in school, to the figures they make in the rugs. The following utterances exemplify these particularities:

The most common that we do here is the pyramid, also the triangle and the rectangle sometimes [Weaver 2].

I know the frame one, but I never weaved this one, my mother weaves the frame one, what I used to do was the triangle [Weaver 4].

The same as I weave the bedside rug, I consider the beak as the triangle [Weaver 15].

Triangle, rhombus and square. Actually a combination of several, because, for instance, there is one drawing that is the leaf, which has the square and the triangle in the same drawing [Weaver 16].

We noticed that one of them mentioned the "pyramid", revealing no differing between two-dimensional and three-dimensional figures, although it really looks like a pyramid, but in our view, it is a polygonal figure. Moreover, other names also appeared, such as "frame", "beak" "star" and "leaf", as well as the drawings may have the "junction of several". Therefore, we identified that these names given to the drawings are part of the weaving culture in Resende Costa and have relation to the geometry content of school mathematics.

Regarding the way of making the drawings, the explanations were similar. We reinforce the presence of counting processes, the measurements, and their association with symmetries by the weavers, involving tape measure, the palms of the hands, the fingers, and the mind. The following narratives exemplify this knowledge:

First we make a flat piece on the carpet. You put the color of the drawing at the beginning of it, and then you have a certain number of "cabrestilhos" that count for a start. Then, for making the beak, we go up adding and then subtracting until we finish the drawing and do the other edge. Always the same counting for increasing and for decreasing to form the drawings. Our rugs have two meters and one and a half meters long [Weaver 2].

It is programmed for you to do that; it follows a sequence. You change the sequence, and we change the drawing, that's basically it. There is a counting, for example: two "cabrestilhos" to the right and two to the left. For those who start in the other side, a total symmetry, it goes downhill and then comes back [Weaver 7].

It is all counted and all measured, you measure and count. For example: if there are three "cabrestilhos" on one side, you have to put three on the other side, and to get down the drawing, you go down from "cabrestilho" to "cabrestilho". Then you go counting them. There is a right counting. There is a time to start and a time to finish. You have to finish just the way you started it [Weaver 8].

It is manual, hand and foot movement. There is a threads counting to make the drawings. To make a quilt you need 38 to 40 balls, this quilt is also used as a rug, there are three balls on the loom, and you count from six to six to form the drawing [Weaver 10].

We observed that the quilt and rug which have the "beak" drawing are well known and frequently produced by the weavers and that is why we decided to delve into their elaboration.

The making of a "beak rug"

The step-by-step weaving of a "beak rug" was observed and video recorded, which is shown below, in parts and by screen capture in the figure 6.



Figure 6 – The “beak rug” making steps recorded by screen capture.

Source: research data (2019).

To make the "beak", initially the weaver chose the colors and the number of hanks. She chose two different colors for the fabric remnants, one darker and one lighter, to create a contrast between the drawing and the rug. Then, with good will, she began to manage the loom and show the details. She introduced a ball of the fabric threads into the loom, hit the “queixa” (the wooden part used to hold the comb and to beat the weft of the fabric) with her hands, grouping the threads by the comb, and used her feet on the “pisadeiras” (pedals that help to change the warp arrangement) to open and close the “cala’s” overture (the warp overture through which the ball was passed). When she opened the “cala” it was to give passage to the thread of the fabric threads, and when she closed it was to hold it in the warp (set of textile threads stretched along the length of the loom). Thus, it happened the crossing between the fabric remnant thread and the warp threads for the fabric formation. The procedure can be seen in the video accessed by the QR code below:



Source: research data (2019).

Next, the weaver began to weave the "beak" by counting and dividing the warp threads by the “cabrestilhos” (set of divided threads) to position the fabric thread balls in the weaving. She then inserted two balls of the same color, on each sides, and inserted a ball of a different color in the center of the rug. Now she began to work with three balls at the same

time. She made a counting from the right side, from three to three up to 12 threads, and did the same counting on the left side to define the point of introduction of the balls on the loom and to put the "beak" in the center of the rug, that is, to position the base of the design symmetrically on the rug. During this course, she emphasized that her countings were made "by memory". Thus, she passed the three threads of fabric inside the "cala" overture but not all at the same time, one at a time and in a certain place of the "warp", repeating the process of changing the "pisadeiras" and beating the "queixa" each time she threw the balls from one side to the other into the "cala's" overture. Then we noticed that there was a change in the arrangement of the balls on the loom and a beginning of narrowing of the "beak". The weaver explained:

Now, here, four "caroçinhos" (she counted four bulge points in the rug). Now I count more six for inside (she counted six threads in the warp). And here the same thing, I count six threads to inside (she counted six threads in the warp in the other side of the rug). Now, here, one more "caroçinho" (she counted one point of protuberance in the rug) and six more to inside (counted six threads in the warp) [Weaver].

The protuberance in the fabric, the "caroçinho", appears as the result of the crossing of two fabric threads, a "knot", that is, this is necessary for the go and come movements of the fabric balls inside the loom's "cala", performing a "back and forth" and by steps.

Based on the weaver's words and replacing them by elements of geometry, the crossing of the threads in the fabric are the "points", the threads are the "lines" and the fabric made is the "plane". These mathematical concepts of point, line, and plane refer to basic notions of plane geometry that can be associated with the elaboration of the "beak". Figures 7 and 8 illustrate the process of making a beak showing these elements.

The warp threads are positioned vertically along the loom, as shown in Figure 7. They are fixed according to a series of parallel straight lines in this position, and the fabric threads are then weaved with the first ones in the horizontal direction. The fabric is then formed by the crossing of perpendicular straight lines, in which a horizontal line is passed through several vertical lines, sometimes below and sometimes above. This inversion of the "warp" threads is possible by the act of changing the pedals of the "pisadeiras". In this way, a plane or woven fabric is done.

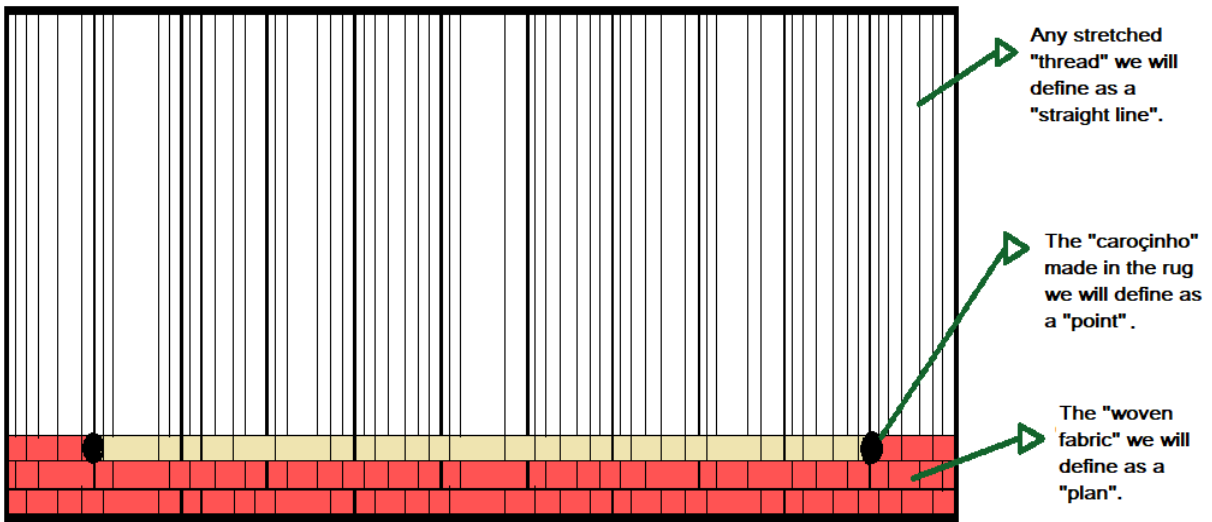


Figure 7 - Schematic of the initial elaboration of a "beak rug", with replacement of the words used by the weaver with terms from plane geometry.

Source: Research data (2020)

In order to draw a simplified outline of this rug, the stretched threads of the warp were divided into groups of six threads (straight lines). Therefore, this division in equal parts of straight lines and the dots refer to increases and decreases of the sides of this design. Figure 8 shows a part of the weft being done, with the reduction of the sides of the "beak" and its final formation.

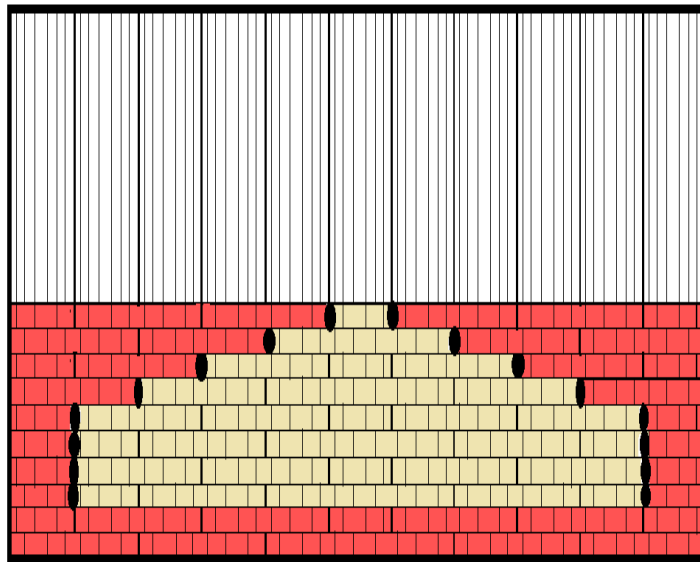


Figure 8 - Final beak ("bico") formation

Source: Research data (2020).

For the formation of a beak in the center of the rug, it is necessary to have an increase or decrease in the threads counting until the polygonal lines are closed, and when the weaver weaves "further on the rug beaks in reverse", what is seen in Figure 9.

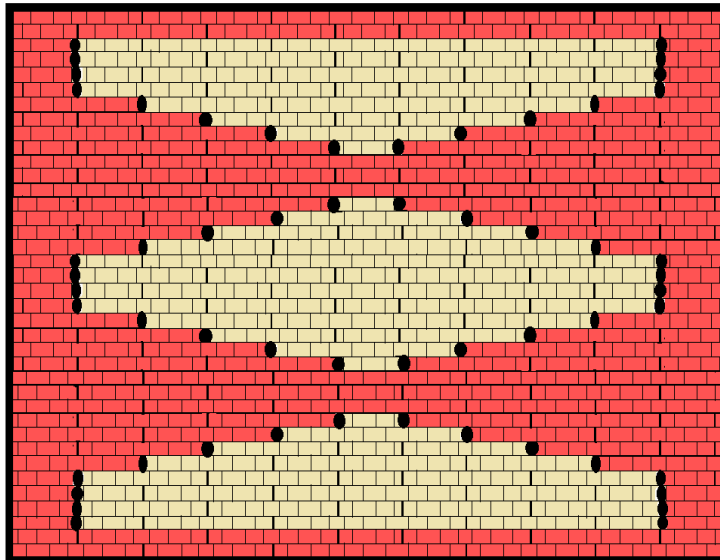


Figure 9 - Drawing of a "beak rug" in "reverse direction"

Source: research data (2020).

Therefore, we notice the use of mirrored figures in the “beak rug”. Since “the symmetry can be explained as the repetition of a form in an orderly manner and with a certain rhythm. There are several types of symmetry as of Reflection, Rotation, Translation and Sliding Reflection, which demands two movements” (Menegucci, Martins & Menezes, 2016, p. 83). In this case, we have the inversion of figures, that is a Reflection symmetry.

The Ethnomathematics dimensions in the Resende Costa’s weaving

In this research we noticed that a considerable part of the Resende Costa citizens develop the weaving work as a form of subsistence, either in the manufacturing or in selling the pieces, which reveals a reason for its cultural preservation. This activity occurs frequently in the garages or in an adjacent space of the weavers’ houses. The work atmosphere usually is joyful, there are friendly relationships among the artisans but also tensions associated with the future of the labor market. Due to technological development, they fear the activity’s end, because of the belief that in the future it will be done mainly by machines.

Concerning to the Ethnomathematics Program dimensions formulated by D’Ambrósio (2018), we consider that this last-mentioned aspect is associated to the political dimension. The author argued that this subordination relationship to who wields the power is a way of "colonization", that is, through the relationship between dominator and dominated the “roots” are cut. This dimension helps to explain the extinction of the weavers' long-time making ways, as nowadays the market has influences over people. Thus, the capitalist system favors the production updating and the speed of this process, since more one produces, more one earns, and in exchange the cultural roots are lost with time.

However, we argue that weaving is a source of pride and identity for the community, as there is a sharing of knowledge, a desire for its preservation and an incitive work in the municipality for its continuity, which is related to the epistemological dimension of

Ethnomathematics, once "knowledge systems are sets of responses that a group gives to the pulses for survival and transcendence, which is inherent to the human species. They are the knowledges and doings of a culture" (D'Ambrósio, 2018, p. 37).

Regarding the mathematics of the weaving, according to the reports gathered, most participants stated that counting was necessary and that they could see the relevance of mathematics on it. Some of them mentioned that they perform these counts mentally and as result of the practice; others used the palms of their hands or the tape measure. These responses highlight the presence of cognitive aspects in this weaving, revealing the cognitive dimension mentioned by D'Ambrósio (2018).

In the same way, we understand that these "doings" correspond to the Resende Costa's weaving identity and to the historical dimension as nowadays there is a continuity and a historical interpretation of the weaving process and of the methods' changes over time, as well as the respect for the weaving culture and tradition. Besides, as we are going through frequent social and economic transformations, it becomes fundamental to defend the preservation of cultural roots, as to recognize from where they came from and what baggage they carry is a perception of the individual origins. Therefore, in this activity we also have a historical dimension.

For D'Ambrósio (2018), this individual behavior feeling of compatibility within a group integrates its culture. Consequently, by the preservation and sharing identity and local culture, we come closer to the conceptual dimension underlined by this author, as in this way knowledge acquires value and is not undervalued. Furthermore, we observed a creative abstraction in the quilts and rugs weaving, responding to impulses of transcendence.

Through this view, we apprehend that this activity also fits into the educational dimension of Ethnomathematics, since, reiterating the geometry teaching and learning difficulties in Brazil, we suggest some pedagogical activities that involve this weaving to favor the learning of geometry, which are based on the inclusion of this know-how in Mathematics Education in Resende Costa. The works of weaving observed in the city may be presented in the classroom and dialogues between students and the weaving community can take place. Some practical activities as the weaving observation and audiovisual documentation can be held by the students. It is also possible to make a small scale "beak rug" in the classroom, using a wooden frame with the following dimensions: 20cm x 30cm, a small hank of twine, three small hanks of fabric remnant threads (two with the same colors and one different), school scissors and a comb. In these activities, the teacher can explore the associated contents of plane geometry (Silva, 2020).

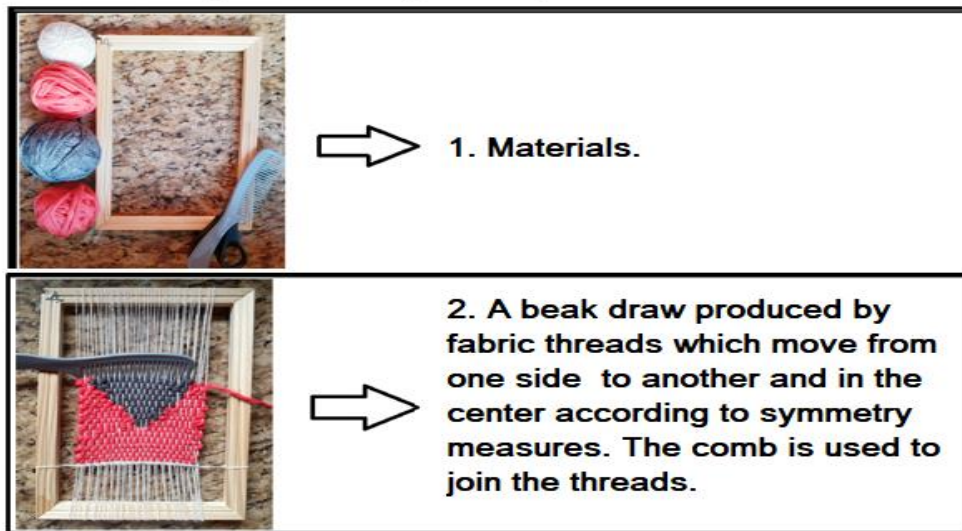


Figure 10 - Pedagogical activity: a small scale “beak rug” for classrooms

Source: Research data (2020).

The complete process of this pedagogical activity is described in the Master degree thesis intitled "Study of Mathematics Present in the Handmade Weaving of Resende Costa, MG" (Silva, 2020, p. 96 - 97).

In practice, activities and purposes may vary according to the didactic goals. Thus, we believe that geometry can be better known and explored to collaborate for a transdisciplinary teaching which values and includes the local culture.

Final considerations

In this research we identified the observed craftwork similarity with the mathematics and geometry theories. They are not the same, but throughout the inquiry we recognized and understand the presence of geometry in a different cultural context. After all, "although mathematical ideas may be very different in many cultures and in different social and cultural contexts, it is still possible to discover common aspects" (Gerdes, 2010, p. 160).

Thus, according to the interpretation of the results, we understand that the mathematics used in the Resende Costa's weaver activity “involves counting numbers, number line, basic operations, measuring, symmetry, mirrored figures, basic notions of geometry and plane geometry” (Silva, 2020, p. 100). We verify that there is an affinity between the know-do and this mathematical knowledge.

Through observation and interpretation, we realized that the weaving activity has its own characteristics “which correspond to the conceptual, historical, cognitive, epistemological, political, and educational dimensions, within the Ethnomathematics Program research perspectives” (Silva, 2020, p. 100), where there is heritage and identity preservation. The pedagogical dimension was highlighted, as we considered the possibility to make simple and concrete activities in basic education.

Regarding the difficulties pointed out in the basic level teaching and learning geometry, we understand that they can be minimized by improvement of teachers' education in undergraduate courses. To accomplish this, we suggest that the plane geometry content be based on observation of materials present in everyday life and culture. In this way, the perception of the principles of geometric shapes would happen before their theoretical presentation. Thus, we consider that geometry does not necessarily need to be approached by definitions, theorems and propositions only, given that the knowledge of measures and symmetries has been used throughout history by different people even without the mathematical deductions and is present in cultural manifestations, such as those of the weavers from Resende Costa. Therefore, we expect this experience may contribute as an example for teacher's education.

We conclude this research contemplated and valued the knowledge involved in the "artisanal weaving" of the municipality of Resende Costa, whose tradition presents elements for interaction with the curriculum of geometry. We note, however, that its insertion in the classroom requires responsibility for knowledge transference, aiming to preserve it closer to what it is in the connection with school approaches to mathematics.

References

- Castro, A. M. (2016). Tramar, destramar e retramar: a arte nas montanhas das Minas Gerais. *Caderno Espaço Feminino*, 29(2), 286-303.
- Corá, M. A. J. (2014). Políticas públicas culturais no Brasil: dos patrimônios materiais aos imateriais. *Revista de Administração Pública*, 48(5), 1093-1112.
- Cury, F. G. (2019). Análise de um Livro Didático de Geometria Plana Apoiada na Hermenêutica de Profundidade. *Zetetiké*, 27, 1-21.
- D'Ambrósio, U. (1998). *A Etnomatemática. A arte ou técnica de explicar e conhecer*. São Paulo: Ática.
- D'Ambrósio, U. (2018). *Etnomatemática – elo entre as tradições e modernidade*. Belo Horizonte: Autêntica.
- Ferreira, E. S. (2007). Programa de Pesquisa Científica Etnomatemática. *RBHM Especial* (1), 273-280.
- Fritzen, M. P. (2012). O olhar da Etnografia no fazer pesquisa qualitativa: algumas reflexões teórico-metodológicas. In M. P. Fritzen & M. I. P. Lucena (Orgs.), *O olhar da etnografia em contextos educacionais: interpretando práticas de linguagem* (pp. 55-72). Blumenau: Edifurb.
- Gerdes, P. (2010). *Etnomatemática – Da etnomatemática a arte-design e matrizes cíclicas*. Belo Horizonte: Autêntica.
- Gerdes, P. (2012a). *Etnogeometria – Cultura e o despertar do pensamento geométrico*. Belo Horizonte: Reedição: Instituto Superior de Tecnologias e de Gestão (ISTEG).

- Gerdes, P. (2012b). *Etnomatemática - Cultura, Matemática, Educação: Coletânea de textos 1979 - 1991*. Belo Horizonte: Reedição - Instituto Superior de Tecnologias e Gestão (ISTEG).
- Ghasarian, C. (2008). *De La etnografia a La antropologia reflexiva: nuevos campos, nuevas prácticas, nuevas apuestas*. Buenos Aires: Del Sol.
- Hartwig, S. C., Pereira, E. C., Machado, C. C., & Miranda, S. A. (2016). Um olhar sobre as práticas pedagógicas na construção de conhecimentos geométricos. *REVEMAT*, 11(2), 243-258.
- Iezzi, G., Machado, A., & Dolce, O. (2010). *Geometria Plana: conceitos básicos*. São Paulo: Atual.
- Knijnik, G. (2004). Itinerários da etnomatemática: questões e desafios sobre o cultural, o social e o político na educação matemática. In G. Knijnik, F. Wanderer & C. J. Oliveira (Orgs.), *Etnomatemática, currículo e formação de professores* (pp. 19-38.). Santa Cruz do Sul: EDUNISC.
- Lorenzato, S. (1995). Por que não ensinar geometria? *A Educação matemática em revista – SBEM*, (4), 3-13.
- Martins, E. A. (2016). Retratos da cidade. E. A. Martins & R. G. Pinto (Orgs.), *Retratos da centenária Resende Costa* (pp. 45-70). Resende Costa: Amirco (coleção Lageana).
- Menegucci, F., Martins, E. & Menezes M. S. (2016). Design de superfície têxtil: um estudo sobre o conhecimento geométrico presente nos Freedom Quilts. *ModaPalavra*, 9(18), 71-95.
- Ministério da Educação (MEC). (2017). Secretaria da Educação Básica. *Base Nacional Comum Curricular (BNCC) do Ensino Fundamental*. Brasília.
- Montenegro, G. A. (1991). *Geometria descritiva*. São Paulo: Blucher.
- Pavanello, R. M. (1993). O abandono do ensino da geometria no Brasil: causas e consequências. *Zetetiké*, 1(1), 7-17.
- Resende, E. M. (2016). Retratos das origens. E. A. Martins & R. G. Pinto (Orgs.), *Retratos da centenária Resende Costa* (pp. 15-44). Resende Costa: Amirco (coleção Lageana).
- Rosa, M., & Orey, D. C. (2014). Interloquções Polissêmicas entre a Etnomatemática e os Distintos Campos de Conhecimento Etno-x. *Educação em Revista*, 30(03), 63-97.
- Silva, F. (2016). Retratos da arte, artesanato e ofícios. E. A. Martins & R. G. Pinto (Orgs.), *Retratos da centenária Resende Costa* (pp. 179-188). Resende Costa: Amirco (coleção Lageana).
- Silva, C. S. (2020). *Estudo da Matemática Presente na Tecelagem Artesanal de Resende Costa, MG*. Dissertação de Mestrado em Educação. São João del-Rei: Universidade Federal de São João del-Rei. Retirado em 26 de agosto, 2022, de [https://ufsj.edu.br/portal2-repositorio/File/mestradoeducacao/Versao%20final%20da%20dissertacao%20de%20Mestrado%20-%20Cleisiane%20de%20Sousa%20Silva\(1\).pdf](https://ufsj.edu.br/portal2-repositorio/File/mestradoeducacao/Versao%20final%20da%20dissertacao%20de%20Mestrado%20-%20Cleisiane%20de%20Sousa%20Silva(1).pdf).
- Vergani, T. (2007). *Educação Etnomatemática: o que é?*. Natal: Flecha do tempo.