



## Formative contributions of the History of Mathematics course developed from the perspective of the logical-historical movement

### Contribuições formativas da disciplina de História da Matemática desenvolvida na perspectiva do movimento lógico-histórico

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#### Abstract

The article discusses part of the data of a research carried out with mathematics teachers in initial training. The objective is to analyze and discuss the formative contributions of a History of Mathematics course, organized and developed from the perspective of the logical-historical movement of the constitution of mathematical concepts. This discipline is part of the 1st period of the degree in Mathematics at the State University of Goiás - Campus Sudoeste - Sede Quirinópolis. The methodological path of the research was a formative teaching experiment that had duration on one academic semester with the collaboration of 32 participants. The structure of the analysis is composed of unit, episodes and flashes. The results give evidence of the understanding that the proposed organization allowed teachers in training the assumption of the existence of the internal logic of mathematical concepts, being in close connection with the production and human-historical development of the logical-historical unit.

**Keywords:** History of mathematics; Mathematics teacher training; Logical-historical movement of concepts.

#### Resumo

O artigo discute parte dos dados de uma pesquisa realizada com professores de Matemática em formação inicial. O objetivo do mesmo é analisar e discutir as contribuições formativas de uma disciplina de História da Matemática organizada e desenvolvida na perspectiva do movimento lógico-histórico da constituição dos conceitos matemáticos. A referida disciplina faz parte do 1º período da licenciatura em Matemática da Universidade Estadual de Goiás – Campus Sudoeste – Sede Quirinópolis. O caminho metodológico da pesquisa foi um experimento didático formativo que perdurou um semestre letivo, contando com a colaboração de 32 participantes. A estrutura de análise é composta de unidade, episódios e flashes. Os resultados dão indícios da compreensão que a organização proposta permitiu aos professores em formação a assunção da existência da lógica interna dos conceitos matemáticos, estando a mesma em íntima conexão com a produção e desenvolvimento humano-histórico a partir da unidade lógico-histórica.

**Palavras-chave:** História da Matemática; Formação de Professores de Matemática; Movimento Lógico-Histórico dos Conceitos.

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## Introduction

The research that gave rise to this article was developed in the context of the initial training of mathematics teachers, more specifically about the organization of the subject "History of Mathematics", which was based on the logical-historical movement of concepts. Based on these conjectures, a proposal for the organization of mathematical concepts was developed within the framework of this subject, which was offered in the first period of the Bachelor's Degree in Mathematics at the Universidade Estadual de Goiás - Campus Sudoeste - Sede Quirinópolis, in 2018. The development of this subject became the empirical context of the didactic experiment that lasted one semester (64h/2006). However, for the construction of this work, the research was cut in order to highlight the understanding of the importance of the organization of this subject through the logical-historical movement of concepts. Emphasis was also given to its contributions to the understanding of the internal connections of the mathematical concepts studied. The specific purpose of this article was to investigate the contributions of the organization of the subject "History of Mathematics", based on the logical-historical movement of concepts, to the learning process of teaching mathematics in a Higher Education Institution in Goiás.

In this context, the actions developed according to the theoretical basis that supports the logical-historical movement of concepts, proposed here, is one of the possible ways to understand and overcome the current organization of this subject and the learning of teaching in mathematics as a whole. According to researchers such as Cedro (2008), Moretti (2014), Sousa, Panossian and Cedro (2014), Sousa (2004, 2009, 2018), Moraes (2008), Moura (2010) and Silvestre and Silva (2019) and others advise the adoption of the logical-historical construction of the concept in a formative process of mathematics teachers, for being the same able to meet the restlessness of the undergraduates and, subsequently, of their students about the "why learn certain content", allowing them to realize that mathematics, like the other sciences, was not "simply invented", but emerged from the need to solve a certain situation, as well as with advances and setbacks, not being the same finished, being subject to changes.

In order to understand the actions that preceded the elaboration of this article, we first understood the emergence and development of the subject "History of Mathematics" in undergraduate programs; then we discussed the theoretical conceptions that support the perspective of the logical-historical movement of concepts and its relationship with the training of mathematics teachers. Then, the didactic experiment developed, i.e. the methodology chosen, is addressed; then, the development of the analysis structure composed of unit, episode and flashes is highlighted and, finally, some reflections on the research carried out.

## History of Mathematics: emergence and development of this discipline

Moved by the need to unveil the processes and movements that led to the emergence of the subject History of Mathematics in the initial training of teachers of this area in Miguel

and Brito (1996), Nobre (2012), Oliveira and Fragoso (2011), and Pereira and Guedes (2016), there are researches that support such understanding.

The first formal evidence of this discipline is shown by Miguel and Brito (1996) in the incorporation of the same in American universities between the years 1920 and 1958, in initial training courses for teachers who worked in secondary schools, pointing out that in this same period, in universities that trained mathematics teachers in the Soviet Union, the subject was already required.

In the Brazilian scenario, Oliveira and Fragoso (2011) show that the first indications of incorporation of the subject in higher education courses were motivated from the end of the 1920s, by the historical notes in the book by Euclides Roxo, but it only appears as a concern to be effective in the presentation of scientific work in the I Mathematics Education Meeting of São Paulo - I EPEM, held in the city of Campinas in 1989. There was, then, the verification that very few teacher education courses had the subject in their curricular structures, arguing the defense of such and its relevance to the education of the Mathematics teacher. In the following years, in events dealing with the subject of mathematics education, new arguments emerged that this subject should be part of the curriculum of teacher education courses. Such justifications helped to later incorporate the subject in the curricular matrix of most higher education institutions.

[...] Ordinance No. 57, published in the Official Gazette of the Union on February 6, 1998 for the National Examination of the Mathematics Course, the profile of graduates, in which it is emphasized that these professionals should have a historical and critical view of mathematics in its current state as in the various stages of its evolution (Oliveira & Fragoso, 2011, p. 629).

Other documents also signal the incorporation of the subject History of Mathematics in the higher education courses, such as the Law of Directives and Bases of Education, the National Curricular Parameters, the National Common Curricular Base and the Curricular Directives for Mathematics Courses, more specifically in the *Parecer* CNE/CES 1.302/2001, in the undergraduate and bachelor courses (Oliveira & Fragoso, 2011). With the increasingly frequent defense of the insertion of the subject in teacher education courses in Brazil, driven by discussions in major scientific events in the country and also supported by the normative and regulatory councils, currently most of the higher education courses have already incorporated the subject to the curriculum.

Nobre (2012) noticed the structural development of the subject in undergraduate courses, problematizing its relevance in teacher education, especially in view of the understanding of the historical, social and cultural elements of mathematical concepts. Nobre (2012) emphasizes the non-mandatory nature of the subject in the above-mentioned courses, emphasizing that the subject happens in the institutions according to their perspectives based on the pedagogical political projects. As the author highlights some objectives should be taken into account when developing it:

[...] allow the knowledge of the history of mathematical concepts; provide the understanding that mathematical knowledge is the result of the work of several

generations of thinkers; establish relationships between the origin of a mathematical concept and the sociocultural context where this took place (Nobre, 2012, p. 511).

These objectives are in line with the theoretical defenses of Moura (2011) regarding the students' expectations in understanding the historical, social and cultural origin of mathematical concepts, by understanding them as part of the movement of individuals who, throughout the production of mathematical knowledge, verified their interests and needs so that a certain mathematical concept was produced, meeting the social and cultural demands of a people in a given historical period.

In this perspective, it is necessary to worry about the format of the organization of the contents approached by the teacher educators, so that there are conditions for the students to include the historical, social and economic aspects as essential elements in the construction of mathematical knowledge, understanding the historical objective conditions of the creation and contextualization of mathematical concepts, serving as a way for the teachers in training to understand the origins and historical processes of mathematics, and also to serve as a tool to organize the teaching and learning of school mathematics.

Understanding that the subject of History of Mathematics is part of a mathematics teacher's training process, the next topic will address the logical-historical movement of concepts and how such perspective can be implemented in the training of these teachers.

### **Logical-historical movement of concepts and mathematics teacher education**

The training of mathematics teachers is a theme whose research possibilities have not yet been exhausted. Researchers such as Bicudo (1999), D'Ambrósio (2007), Fiorentini and Lorenzato (2009), Libâneo (2004), Moura et al. (2010), among others, have made public the studies about how the education of mathematics teachers is taking place in Brazilian universities. The publications resulting from these studies have sparked discussions regarding the training of these teachers. These researchers point out that part of the problems concerning mathematics teaching is related to the initial teacher education process. They start from the premise that the training of mathematics teachers based on a reproduction of content that does not transform the objective reality that we have, nor contributes to the emergence of a new reality, only multiplies quantitatively qualitative changes already produced previously:

[...] for years, students enter and leave mathematics classes with the feeling that, mathematical concepts are fragmented, they have no history. When many of these students become mathematics teachers, they claim, with some reason that, theory and practice are not related" (Sousa, 2018, p. 41).

In the opposite direction and seeking to understand the training of mathematics teachers, it is observed solid foundations in the theoretical and methodological assumptions of the logical-historical movement of mathematical concepts in authors such as: Cedro (2008); Moretti (2014); Sousa, Panossian and Cedro (2014); Sousa (2004, 2009, 2018); Moraes (2008); Moura (2010); Silva (2018); Silvestre and Silva (2019), among others. These

conceive the training of mathematics teachers as "necessary mediation in the process of constitution of the subjects involved, and not only as an end in itself" (Moura 2010, p. 89). That is, anchored in the understanding of the logical-historical movement of concepts they see the possibility that, from the understanding of the problems that surround the training of mathematics teachers:

[...] training proposals should be developed that consider both the conceptual learning of the mathematical object in a logical-historical movement, as well as the learning of elements of the organization of teaching, based on the subjects' needs [...], where the teaching learning is linked to the appropriations resulting from the activity of the subject in training" (Moretti, 2014, p. 38).

Anchored in this understanding is the training of mathematics teachers as a process central to the formation of the teacher as a man in his historical specificity. Thus, the adoption of the logical-historical construction of the concept in a mathematics teacher education process may be able to answer the students' and, later, their students' questions about "why learn a certain content", allowing them to realize that mathematics, like other sciences, was not "simply invented" by someone, but emerged from the need to solve a certain situation and had its advances and setbacks and, furthermore, it is not finished, it is subject to change and reworking.

In this vein, it is noteworthy that mathematical concepts took a long time to be created and have undergone several changes over time. Both the historically established theoretical thoughts and the internal nexuses of such concepts were created by various civilizations. Some concepts had a greater level of depth with certain peoples and, therefore, were conferred to them, which does not mean that other civilizations did not hold or seek to systematize them. Therefore, by not considering the logical-historical aspect of mathematical concepts during the learning process of teaching future mathematics teachers, we run the risk of denying these subjects the possibility of:

[...] by means of a proposal for the organization of the teaching of mathematical concepts capable of triggering, in thought, this dialectic between the historical and the logical" consenting "that the formation of ideas compose the logic of the movement of thought" (Silvestre & Silva, 2019, p. 6).

Given this, Radford's (2011, p. 44) statement is pertinent when he says that the history of mathematics can give us a new perspective on the teaching and learning of mathematics; "Obviously, we are not saying that our students have to follow the same path as those of the ancient mathematicians. Instead, it is a matter of better understanding the nature of mathematical knowledge and finding, within its historical framework, new possibilities for teaching."

However, when analyzing the training processes of mathematics teachers and mathematics classes in schools, in Brazil, from the theoretical perspective defended here, it can be stated that, despite the suggested and implemented changes, in the last decades, the organization of most of these two processes has prioritized the training of the subjects: student and teacher. Both in educational institutions and in classrooms, the teaching and

learning movement is basically summed up in, first, showing the concept to the students; then, proving how it works; later, by means of task lists, the students are trained to 'learn' this concept and, finally, the individual is 'evaluated' to prove learning. This way of organizing teaching learning and, consequently, the teaching of mathematical concepts in schools coerces most of the subjects involved.

Thus, by exploring the logical-historical movement of the concept as a proposal for the organization of subjects that are part of the initial training of mathematics teachers, we seek to contribute to the learning of the mode of knowledge production. This enables the subject to understand the non-existence of absolute truths, frequent conceptions of students in relation to mathematical concepts, generated either by the teaching method based only on formal logic or a historiographical conception of positivist slant. Next, we will discuss the methodological organization required for the subject History of Mathematics to be organized and developed with the research subjects.

### **Formative Didactic Experiment**

Understanding the teaching of mathematical concepts during the learning process of mathematics teaching from the theoretical assumptions of the logical-historical movement implies understanding such concepts as appropriation resulting from a process of internalization in activity (Leontiev, 1983).

In this particular case of teacher learning in the context of initial training, more specifically during the development of the subject "History of Mathematics" for the 32 students enrolled in the 1st period of the Mathematics undergraduate course at Universidade Estadual de Goiás, Campus Sudoeste, Sede Quirinópolis, the theoretical contributions aimed to cover teacher training in Mathematics from the understanding that learning implies appropriation. In this scenario - among many others - of teacher learning, in which the learning of contents and the organization of its teaching should be valued, it is evident that human need is the engine of man's generic activity and, thus, educational processes aimed at learning to teach mathematics can and should be intentionally organized in order to break with the memorization and fragmentation of concepts. This is because this type of organization does not value the conceptual nexuses (internal<sup>3</sup> and external) that are present in mathematical concepts, but only highlights empirical thinking, that is, the study of the perceivable data of these concepts. "Here, the external nexuses of mathematical concepts are related to formal language because they are clean, stripped of contradictions, of cultural and social practices present in the history of the concepts" (Sousa, 2018, p. 41).

Anchored in these assumptions, a formative didactic experiment theoretically supported in the cultural-historical perspective according to Freitas and Libâneo (2022) was

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<sup>3</sup> In traditional didactics it is not considered that the mathematical concepts studied in the classroom are generated, for the most part, in human praxis. We forget that human activity is contextualized in a particular historical, cultural and institutional context. Thus, the contents contain internal and external links.



developed in order to follow the movement of teacher learning of mathematics teachers in training during a formative proposal developed in the subject of History of Mathematics.

The work with the future teachers was based on the proposition that, instead of receiving ready-made texts with historical information about the concepts of number and numbering system, fraction, polygon, angle and function, they would build historical syntheses of these concepts and, during the investigation, they would list the main internal connections of each concept to be presented to the class. In this way, they had the opportunity to approach two dimensions of learning to teach mathematics: learning the concept to be taught and learning how to organize their teaching.

On the way to understand the truth about the investigated phenomenon, a didactic experiment was planned and developed during one semester (64 hours per semester and 4 weekly classes). According to Davydov and Markova (1987, p. 326) the formative experiment "is a structure of investigation of the development of the human psyche, which has its bases in the work of Vygotsky". Vygotsky considered that only through experimental analysis it was possible to reveal the whole essence of the genetic process of development of the human psyche and this type of investigation would be the "key to understanding the process by which concept formation develops in real human life" (Vygotsky, 2003, p. 86). Thus, the essence of the formative experiment is expressed "[...] in the study of the processes and new forms of the psyche, in the study of the conditions of the emergence of the conditions necessary for them to arise" (Davydov & Markova, 1987, p. 326).

Confluent to this perspective of formative experiment, in this research, we use the term formative didactic experiment for understanding the reflection of the "conception of developmental teaching and, consequently, its logic of organization and structuring of the students' study activity" (Freitas & Libâneo, 2022, p. 7). For it is considered the organization of the teaching of history of mathematics in the mental actions of students when developing their study activity for learning teaching.

Thus, the entire development of the didactic experiment was recorded in audiovisual form, and the recordings were transcribed in full in order to become the universe of research data that were later analyzed to achieve the objective of the research as a whole and also of this article.

In order to facilitate the understanding of the structure and development of the didactic experiment, the didactic structure of the experiment was elaborated in its moments in Chart 1. It is didactic because it did not happen in these tight and delineated moments, being at all "times sustained by the premise of movement, where everything is determined by contradictory elements coexisting in a structured totality" (Silva & Cedro, 2019, p. 476).

Chart 1 - Moments of the formative didactic experiment

<b>1st Moment Theoretical appropriation principle</b>	<b>2nd Moment Development and presentation of the historical overviews</b>	<b>3rd Moment The movement of analysis and synthesis</b>
<p>Aimed at understanding what would be the theoretical basis that would subsidize the teaching activities developed during the course of the subject History of Mathematics. The intention was to offer them the means to understand the transformation of the objective reality to which they belonged. Therefore, we concentrated in the first classes and, later on, during the course of the experiment, on the theoretical studies that supported our theoretical and methodological choices.</p>	<p>A - The classes of the subject were in two days of the week, in the first day we always did theoretical studies about the logical-historical movement, in special to its application to the teaching and learning of mathematical concepts.</p> <p>B - The focus of this second moment that happened on the second day of the week was when the groups (there were 5 groups altogether: one responsible for numbers and numbering systems, another for fractions, another for polygons, another for angles and lastly the one involved with the concept of function) elaborated and brought the historiographies of these concepts. In the classroom and under the care of the researchers they would build the historical synthesis of each concept and, along the way, discover their main conceptual connections.</p> <p>C - At the end of the first bimester we had the presentation to the whole class of the groups of Numbers and numbering systems, fractions and polygons.</p> <p>D - At the end of the II bimester it was the turn of the groups responsible for the concepts of angles and functions. In the presentations the groups highlighted the main internal connections of each concept, highlighting the connection of this emergence to man's own history.</p>	<p>This was the occasion for the mathematics teachers in training to socialize the situations experienced in the didactic experiment. We wanted, in this way, to apprehend the path taken by them. At that moment, we wanted to apprehend the possibility of appropriating the mathematical concepts of numbers, numbering systems, fractions, angles, polygons, and functions, based on the assumptions of the logical-historical movement, so that they could have theoretical and objective conditions to understand the confluence between this movement, or even between the classes of mathematical knowledge that constitute the logic of the history of training, of the formal language of these concepts, of the movement of mathematical thought included in its historical development.</p>

Source: authors' production

Chart 1 exposes the organizational structure of the didactic experiment, a scope of how it happened, an attempt to present the reader with a form where he or she could apprehend it in its entirety. To this end, successive and increasingly comprehensive approaches are indispensable, as this makes the appropriation of the phenomenon accessible. "In this way, it is possible to detect the exteriorities of things, including the aspects of *Omni lateralitas*, a phenomenon in which man does not develop innate human potentialities, but creates them, as a product and producer of the various social determinations" (Silva & Cedro, 2019, p. 477). Such peculiarities will be highlighted in the analysis composed of unit, episodes and flashes in order to better understand the elected proposal.

## Data Analysis

In line with the theoretical discussions posed here, it was defined that the theoretical structure will be composed of units, episodes and flashes. For Aquino (2013, p. 247), the use of units "is based on Vygotsky's idea that the division in units, and not in elements, allows better knowledge of the concrete laws of the whole, because only the units contain the essential aspects of the whole. This method combines the advantages of analysis and



synthesis, and allows for the adequate study of complex whole" (Vygotsky, 1993, p. 4). In this path and appropriated from the concept of units proposed by Vygotsky we have the idea of episodes. According to Moura (2004, p. 267), these would be the moments that "may reveal interdependence among the elements of a formative action". From the episodes, the flashes were highlighted, i.e., the parts that would configure "the indications of the conscious and internalized reflection of reality, that is, the flashes in the sense elaborated here, implies much more than contextualizing the subjects in the collective discussions (Silva, 2018, p. 151), since by performing a spiral movement "we will dialogue with the theoretical basis, seek the implicit, the motives and needs, the meaning and the sense that are expressed in language, but are not reduced to it" (Silva, 2018, p. 151). However, the analysis structured in units, episodes and flashes do not, according to Silva (2018), have the obligation to show reality exactly as it is, but how it was perceived and felt by the researcher. From this procedural movement of exposing the development and understanding of the phenomenon the following structure of analysis is born:

Table 2 - Unit of analysis

<b>The organization of the subject 'History of Mathematics' from the logical-historical movement of concepts</b>	
<b>First Episode</b>	<b>Second Episode</b>
The understanding of the importance of the organization discipline 'History of Mathematics' from the logical-historical movement of concepts	The contributions of the organization of the subject 'History of Mathematics' to the understanding of the internal connections of the mathematical concepts studied

Source: authors' production

In this unit of analysis, we sought to understand the process of contribution of the subject "History of Mathematics" to the education of mathematics teachers based on an organization that emphasizes the logical-historical movement of concepts. The selected episodes seek to understand how these contributions were made so that the research subjects could understand mathematical concepts according to the assumptions of the logical-historical movement and, in this way, they give signs of appropriation of the internal nexuses of the concepts addressed in the development of the subject, which is the context of this research. Therefore, the two episodes have as a common feature the fact that they represent collective actions that demonstrate the path taken by the subjects. In the sequence, the indications of the transformations of the subjects in question were unveiled in the analysis of each episode and its component flashes.

*First episode - the understanding of the importance of the subject History of Mathematics: an organization grounded in the assumptions of the logical-historical movement.*

According to Sousa (2018) the history of mathematics has been used by many teachers as a source of motivation for teaching mathematical concepts, as well as a tool for demystifying this process. It has also been used as a tool to promote critical mathematical thinking and unification between fields of mathematics, such as pure mathematics and mathematics education. Even being in line with these issues, the reorganization of the subject

"History of Mathematics" proposed in this didactic training experiment was viewed with caution, because there is not a single history of mathematics of which the trainee mathematics teacher should be aware: **I thought we would discover in this subject that each content had been discovered by a person, in a single place, with a history** (Aline<sup>4</sup>, Flash1, Episode1).

It cannot be denied that this subject is important in the formative process of mathematics teachers. The analysis has revealed that the organization proposed here, based on the assumptions of the logical-historical movement, has enabled the construction of a differentiated view of mathematics, which has come to be perceived as a science capable of enabling the process of humanization of man, rather than a body of knowledge given to him a priori, or simply a set of problem solving techniques:

**[...] I actually thought that in the math course I was only going to learn how to do math. I didn't think we were going to learn where things in mathematics come from, much less why they are like that.... It all has to do with the history of man himself** (Luis, Flash 2, Episode1)

**I spent my whole life in school thinking: I don't know why they invented fractions; they only serve to complicate our lives. I never thought that, in fact, this concept was created to solve problems in our lives; that it is part of the process of man's life, of his very history** (Barbara, Flash 3, Episode 1)

As the didactic experiment unfolded, the trainee teachers showed signs of understanding that throughout their lives as students in basic education they had been deprived of a mathematics education that connected the historiographies of the contents and their teaching, and that this could have enabled them to have a different interpretative line, allowing them to approach the same mathematical object from another perspective and, thus, contributing to its better understanding.

**When I am a math teacher I will teach differently than I learned, because I was never told where the contents that we were taught came from and much less why they were created and what they were and are for** (João, Flash4, Episode1)

**In fact, there was never a math class with the history of the math content, the most there was a textbook that told a little bit about the life of an important mathematician.** (Marcia, Flash5, Episode1)

**But I don't remember any of my teachers using the history of mathematics to help us see a content, to help us learn, the history came as something separate and not very important, like this: whoever wants to read it** (Laura, Flash6, Episode1)

Thus, the trainee teachers perceived the emergence of mathematical concepts from the appropriation movement of the human cultural production itself, as well as they noticed the historical man as an active subject who organized his actions and selected instruments that allowed him to objectify the reasons for his activities so that they could provide answers to the most pressing needs and, little by little, in the course of his history, mathematical

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<sup>4</sup> The names used are fictitious to preserve the identity of the research subjects.

concepts originated.

**It makes perfect sense, so as man needed them he would create mathematical concepts and would develop his culture, in the historical period that he was, according to the needs he had he would create them** (Eduardo, Flash7, Episode1)

**And that had everything to do with the activity they had, that's why numbers, fractions, angles and others were not created by the same people all at the same time, it had to do with what these people did and what they needed to create** (Aline, Flash8, Episode1)

In this movement, the research subjects' appropriation of the understanding of the importance of the subject 'History of Mathematics' for their teacher education, from an organization based on the assumptions of the logical-historical movement of concepts, occurs through "a process of internalization of social meanings in activity and with attribution of personal meaning, which characterizes the dialectical unity between meaning and significance in the constitution of the psychism" (Moretti, 2014, p. 33).

The understanding of this process impacted the formative process of these mathematics teachers, since learning causes the appropriation of concepts and attribution of personal meaning (Vygotski, 2003). It is in this sense that the development of the didactic experiment was established. If, in turn, this relationship is established in the individual's activity and the activity only materializes from a need, it derives that this needs to be evaluated in the process of teacher training of mathematics teachers. This is because future teachers should not be deprived of realizing that human problems are part of the path to the emergence and development of mathematical concepts - which, in the course of pedagogical activity, will be transformed into school contents - and that these are closely linked to the need to create these concepts.

Therefore, organizing the subject "History of Mathematics" based on the conceptions of the historical-logical movement, within a Mathematics undergraduate course, had, among other contributions, the possibility of creating learning conditions for the subjects by proposing teaching situations that put them in front of the knowledge of the human need that generated a given mathematical concept. This structure proposed for this subject is not understood as an exercise in the application of mathematical concepts previously presented by the teacher based on historical facts.

The discipline's organization presupposed a first approach of the trainee teachers to the theoretical basis that would subsidize the teaching activities, so that they could appropriate these mathematical concepts as a product of the historical construction of human culture. They were intentionally guided to develop the actions proposed in the course in order to manifest the essence of the concepts that had been previously chosen, that is, they would have to be able to evidence their main internal connections and, thus, be impregnated with the need that led humanity to their construction. This movement of understanding the internal nexuses will be evidenced in the following episode.

Second episode - the contributions of the organization of the subject History of Mathematics for the understanding of the internal connections of mathematical concepts

The proposed organization of the subject "History of Mathematics" suggested in the research that gave rise to this article advocates a structure that is able to allow the subjects involved in the teaching and learning processes of mathematical concepts to appropriate the essence of mathematical concepts. Kopnin (1978, p. 161) defines this as "[...] clues and relations that go beyond the sensorially perceptible by means of authentic abstraction that generalizes not only the form but also the content of the object. Easy task? No. After all, establishing the principle of organizing mathematics teacher education processes that consider the essence of the mathematical concept may be more difficult than one might imagine. However, it is not impossible to situate in this process the possibility that subjects realize the existing relationship between the human production of mathematical concepts and cultural needs such as that of controlling variations of quantities, discrete or continuous quantities, interdependence between quantities, movement of variable quantities and their regularities, among others (Sousa, 2009, 2018).

The following flashes give us signs of understandings of such propositions by teachers in training:

**[...] when Barbara said that she spent a long time in school wondering why they created fractions, I thought I was just like her about the numbers with commas, the non-integers, because I didn't know about the things that exist inside the concepts, we only see them from the outside (Leandro, Flash1, Episode2);**

**[...] I also always wondered why so many numerical sets, as if the set of naturals was enough; the problem is that teachers, when they teach at school, don't understand what is inside the concepts, as Leandro said (Laura, Flash2, Episode2);**

**[...] and function then, I never understood this concept properly, but I've seen that it is not only me that doesn't know properly, because all these years and we didn't see that one of its inner secrets is the idea of movement, function only exists because it has movement, it was only created because this idea appeared (Paula, Flash3, Episode2).**

Thus, during the development of the subject, mathematical concepts were understood as living productions in direct relationship with the human needs and historical times that produced them. Therefore, appropriating a concept in this theoretical perspective suggests understanding it as a historical and cultural production, implying not only appropriating its logical-formal structure, but also the mechanisms of its historical constitution, the essence of human needs that moved the human species in the trajectory of the social-historical construction of mathematical concepts, that is, it implies seeing and apprehending it in the logical-historical movement. "The unity between the logical and the historical of the concept for the understanding of this concept is necessary, since the logical reflects not only the history of the object itself but also the history of its knowledge" (Kopnin, 1978, p. 186). Thus, understanding the process of production of the concept is an element of the movement of appropriation of the concept itself:

**[...] it makes perfect sense to have so much difficulty learning the math content. The way they are taught without connecting them to their history, how they were created, makes it difficult to understand why they are the way they are in math**

**books. Knowing how they are organized from the inside and how this interferes with the way they are today is very important for us who want to be mathematics teachers** (Pedro, Flash4, Episode2).

In these flashes the subjects denote realizing that in the case of mathematics teacher training the historical aspect of the concept has the possibility of revealing itself in the essence of the need for its human production. Moretti (2014, p. 38) points out that when "articulated with the logical aspect in the process of analysis and synthesis that aims at the solution of this need by the subject, one learns by internalizing the intersubjective movement of collective production of a solution to the need posed".

The knowledge of this essence of the concept - at school foreshadowed in contents - is only possible in the dialectical unity between the historical and logical aspects of the object of knowledge. Still according to Kopnin (1978) this is only realized with the necessary articulation between the logical and historical aspects of the object of knowledge that allows a movement of conceptual appropriation that is established in the unity between the essence and the theory of the object. Thus, the "study of the history of the development of the object creates, in turn, the indispensable premises for the deeper understanding of its essence" which is why "enriched by the history of the object, we must once again take up the definition of its essence, correct, complete and develop the concepts that express it"; after all, only in this way "the theory of the object provides the key to the study of its history, while the study of history enriches the theory, correcting, completing and developing it (Kopnin, 1978, p. 186). However, it does not mean that one should "teach mathematics by history, nor repeat the historical course in the formation of a mathematical concept, but seek in the historical process the movement of thought in the context of the formation of this concept" (Silvestre & Silva, 2019, p. 4). The authors warn, anchored in Sousa (2004), that it is not a matter of reinforcing the prevalence of the logical over the historical and vice versa, nor the foundations of mathematics over mathematics itself and its applications, but to privilege the elements that propitiate the construction of the mathematical concept to be taught by seeking a path that helps to outline a path that leads to the appropriation of the concept by the subject.

Let's take a look at the flashes:

**[...] I think that the way this discipline is being taught, that actually we thought it was going to be really boring, that we were going to have to memorize a lot of names and dates... actually, we are the ones who built it; everything with the help of the teachers, of course, but it was us who created everything. And for that we had to discover a lot of things that we didn't know** (Lúcia, Flash5, Episode2 );

**[...] well, actually, Lúcia, we didn't know almost anything about what we discovered to build the historical syntheses. But that was good because we had the opportunity to learn the history of the contents, but not the ready-made history. We were reconstructing and discovering that nothing was invented all at once. It happened little by little, and this is important to know and understand how it is today. Because I would never have known that angles have to do with the movement of planets. For me, they had invented it just to build houses** (Willian, Flash6, Episode 2).

The historical aspect evidenced in the above flashes manifests elements essential to the knowledge of this mathematical object. These elements, when appropriated by human thought, institutes the logical aspect of the concept. Thus, the logical is "the reproduction of the essence of the object and the history of its development in the system of abstractions" (Kopnin, 1978, p. 183), being the true appropriation of the historical aspect by human thought. Therefore, Kopnin argues that "the logical is the historical freed from the casualties that disturb it" (1978, p. 184), being in the dialectical unicity between them that the object builds the judgment of the correlation between thoughts: particular and general. Thus, when this discipline is organized in these molds it seeks to capture the real historical movement of emergence and development of concepts, "selecting and organizing them, in thought, in the way it conceives the historical, which will constitute the logic of this movement" (Silvestre & Silva, 2019, p. 4).

### **Some Conclusions and Final Considerations**

The analysis of the unit, episodes and flashes grounded on the exposed theoretical basis allows us to infer that the subject 'History of Mathematics' when intentionally organized from the logical-historical approach of the concept with the purpose of establishing the unity between the logical and historical aspects of mathematical concepts, here understood as objects of human knowledge, was epistemologically aligned with the understanding of the human psychic development instituted in the dialectical unity between the individual and the collective, in the midst of the intrapsychic and the interpsychic. Thus, it is assumed the defense that teachers in training, by assuming the logic of the concept in intimate connection with the production and human-historical development of this knowledge, the logical-historical unity allowed them, while they were part of the didactic formative experiment, to understand the process of emergence and development of concepts in relation to their internal structure, their conceptual nexuses. This defense is made in this article based on Kopnin (1978, p. 186), who highlights the "unity between the logical and the historical as an indispensable methodological premise in the solution of the problems of interrelation of knowledge and object structure and knowledge of history and its development".

Believing in the possibility that such an organization can contribute to the process of learning to teach mathematics by the teachers participating in the didactic educational experiment, that is, that the unity between the logical and the historical has important implications for the educational-training processes allowed us to glimpse the signs of appropriation by the subjects of the culturally produced objects of knowledge, the mathematical concepts. Therefore, "the logical-historical movement of the concept, dialectically considered in the process of knowledge of a given object, can guide the organization of the teaching work" (Moretti, 2014, p. 37), or even, as intended here, its learning.



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