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Meanings produced by future mathematics teachers when studying different geometric models

Significados produzidos por futuros professores de Matemática ao estudar diferentes modelos geométricos

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Abstract

Recurring in academic scientific discussions and, consequently, in investigations in the area of Mathematics Education, the study of non-Euclidean geometries in the initial formation of the mathematics teacher can increase the undergraduate students' understanding of Euclidean geometric concepts. In the present article, the results of a Teaching Activity applied to future mathematics teachers presented, with the aim of analyzing the production of meanings of the participants in the study of non-Euclidean geometric concepts. Vygotsky's Historical-Cultural Theory, complemented by Leontiev's Activity Theory and Moura's Teaching Guidance Activity, are the theoretical contributions that supported the investigation and pedagogical actions in this study. By accepting the existence of other geometric models, the undergraduate students investigated changed the way they relate to the space in which they live and, thus, develop greater autonomy in the process of constructing the concepts by assigning different meanings to the studied geometric objects.

Keywords: Mathematical Education; Teaching Guidance Activity; Teacher training; Non-Euclidean Geometries.

Resumo

Recorrente nas discussões científicas acadêmicas e, consequentemente, nas investigações da área em Educação Matemática, o estudo de Geometrias não Euclidianas na formação inicial do professor de Matemática pode ampliar a compreensão dos licenciandos acerca de conceitos geométricos euclidianos. No presente artigo, apresentam-se os resultados de uma Atividade de Ensino aplicada a futuros professores de Matemática, com o objetivo de analisar a produção de significados dos participantes no estudo de conceitos geométricos não euclidianos. A Teoria Histórico-Cultural de Vygotsky, complementada pela Teoria da Atividade de Leontiev e pela Atividade Orientadora de Ensino de Moura são os aportes teóricos que subsidiaram a investigação e as ações pedagógicas neste estudo. Ao aceitarem a existência de outros modelos geométricos, os licenciandos investigados modificaram a maneira como se relacionam com o espaço em que vivem e, assim, desenvolvem maior autonomia no processo de construção dos conceitos atribuindo significados diferenciados para os objetos geométricos estudados.

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Palavras-chave: Educação Matemática; Atividade Orientadora de Ensino; Formação de professores; Geometrias não Euclidianas.

Introduction

The mathematics built by society has been used to explain, understand, and solve everyday problems of people in different cultures at different historical moments. Among the fields of mathematical research that offer a model for analyzing physical space, Euclidean Geometry has been used for centuries as the main area of human knowledge to denote space and its properties. However, is Euclidean Geometry the only geometric model capable of analyzing our physical Earth space?

From this questioning, the proposal of this study is reflected in the interest for the history of facts linked to the construction of Euclidean geometric knowledge and its limitations, the possibility of deepening the studies on the non

Euclidean metrics and, in particular, in the alignment about the study of these geometries in the initial training of mathematics teachers.

Thus, this paper seeks to reflect on the challenges related to the development of geometric knowledge from the analysis of meanings produced by future mathematics teachers about the study of non-Euclidean geometries. To this end, the theoretical and methodological assumptions of the Guiding Teaching Activity proposed by Moura (1996, 2001) were considered as a context for the negotiation of meanings among the research participants. The analysis of the production of meanings was based on the theoretical contributions of the Cultural-Historical Theory, whose exponent is Vygotsky (1987), and on the Activity Theory of Leontiev (1978).

Some theoretical assumptions

Cultural-Historical Perspective and Its Developments

The central idea of the Cultural Historical Theory (hereafter, THC) proposed by Vygotsky guides us that the development of higher psychic functions occurs in social interaction and through the use of signs. From this perspective, the core of cognitive development is related to the "internalization of cultural mediators, that is, the transformation of the cultural into psychological" (Abreu, 2000, p. 108).

For Castorina (1996), any transformations in the interpsychological plane provoke transformations in the intrapsychological plane, disregarding in this process the mere "transmission" from one plane to the other, but valuing, yes, a "transformation". Thus, the subject internalizes by means of social interactions that, upon noticing them, he appropriates them, recreates them, and incorporates them into his structures. The construction of meanings made by the subject transforms him and, consequently, transforms reality.

In Vygotsky's view, every intrapsychological function is a social-historical construction that manifests itself from an interpsychological function, that is, every passage

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from the external to the internal plane in the individual is mediated by a symbolic system that produces meanings for his experiences and serves as a stimulus to guide his way of feeling, thinking, and acting.

In the investigations on the development of the human psyche, concept formation is defended by Vygotsky and collaborators as an extension of the internalization process and, consequently, of the appropriation of knowledge. In other words, the construction of concepts is, par excellence, an activity mediated by signs, which occurs in the individual's cognitive field, with the purpose of directing and controlling his own actions, creating new forms and processes of representation of the objects of reality.

Another relevant aspect in THC concerns spontaneous concepts and scientific concepts: the former are those that the individual learns in his daily life, arising from experiences with certain objects, facts, phenomena, etc. Scientific concepts, on the other hand, are systematized and transmitted intentionally, based on a specific methodology. Therefore, we recognize the social role of the school to enable the formation of scientific concepts in individuals, so that they can be inserted in practices of production of goods and services necessary for the social, political, economic, and technological development of society.

According to Moysés (1997), the existence of a scientific concept is always linked to a hierarchical system of which it is part; consequently, the teacher is responsible for mediating the construction process of this type of concept by the student, in order to "establish an indirect link between the student and the object through abstractions about its properties and the understanding of the relations that it maintains with a broader knowledge" (p. 35). Thus, the formation of a scientific concept is only established through an intentional and systematized action, which enables the subject to have a conscious and consented relationship with the object of knowledge at stake.

In short, the process of relating the learners' experiences (spontaneous concepts) with the historically systematized knowledge referenced in human practices (scientific concepts) requires that the mediating agent (teacher) understands the different meanings of these concepts for the learners. It is through the adequate organization of teaching that the teacher can perceive in which contexts such concepts emerge and in which meanings they are being employed (Moysés, 1997).

For Vygotsky (1993), the essence of concept formation is constituted in the relations established between subjects and objects within a cultural-historical context that gives them meanings, that is, concepts materialize in a historical development process, by means of a shared action among subjects, acquiring, in a given context, a personal meaning for each individual.

As we can see, understanding the development of higher psychic functions is the object of study of THC, and, in order to carry out this study, Vygotsky tried to clarify how the production of meanings and senses by the subjects occurs, in order to deal with the

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relations between language and thought. In this way, the formation of concepts is intrinsically linked to the evolution of thought and language, because it is in the connection between them that one modifies and develops the other (Vygotsky, 1987).

Vygotsky (2003) also states that there is no thought without language: "the moment of greatest significance in the course of intellectual development, which gives rise to the purely human forms of practical and abstract intelligence, occurs when speech and practical activity, then two completely independent lines of development, converge" (p.33).

In Vygotsky's words, thought cannot be found directly in the word, but it takes place or ends in it. The relationship between thought and language is a two-way movement, in which one transforms and develops the other during the course of man's own historical development. Thus, we must understand that the materialization of thought through language requires an exhaustive search for capturing the system of relations that produced a meaning shared by different subjects, up to the investigation of how it was used, the context in which it arose, producing a meaning all its own and personal to each one (Vygotsky, 1993).

For Vygotsky (2010), meaning becomes a conscious human activity, which arises in the operationalization of signification, in a historical and cultural process in search of the production of concepts, since

> The meaning of a word is the sum of all the psychological facts that it awakens in our consciousness. Thus, meaning is always a dynamic, fluid, and complex formation that has several zones of varying stability. The meaning is only one of these zones of meaning that the word acquires in the context of some discourse and, moreover, a more stable, uniform and exact zone. (p. 465)

In this way, the guideline of Vygotsky's work guides us that conceptual thinking occurs by means of socially shared meanings and not only reproduced in the process of constructive transition that is structured from the dialogue between the social, interpersonal, and individual intrapersonal dimensions. By sharing meanings, the subject builds its own referentials, what we call "senses".

The role of the teacher as a mediator in the teaching and learning process requires that he or she seeks to know the scope of the meanings and senses attributed by the students to their speeches, since, "depending on the context, a word can mean more, or less, than it would if considered alone: more, because it acquires a new content; less, because the context limits and restricts its meaning" (Vygotsky, 1987, p. 125).

Thus, in the development of higher psychic functions, the meaning of each word is a generalization, a concept. It is up to the teacher to adequately organize the activities focused on teaching and to always be aware, during the process of sharing meanings, of the possibility of misunderstandings, distortions, and other problems that can be seen as didactic obstacles in the learning process.

Activity Theory: concepts and perspectives

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Leontiev, based on the assumptions of THC, was concerned with studying mental activity and its relation to the development of man's own personality (Moysés, 1997). The studies developed gave a deeper meaning to the term "activity", proposing the activity as a human process psychologically characterized with the purpose of satisfying a need, distinct from a simple action. The meaning assigned to this term inspired the genesis of the theory by Leontiev (1978), who named it Activity Theory.

He deepened the concept of activity and transposed the central vision of a mediated action to a process of collective development, in which the function articulated by individuals becomes a mode of organization of each subject, and the interpsychic action is transformed into intrapsychic action (Moysés, 1997). Therefore, the second generation of Activity Theory (hereafter, AT) emerged, with Leontiev as its main representative.

According to Braida (2012), Leontiev brought important contributions to the concept of activity and centered his focus on the complex interrelationship between the individual subject and his community. Thus, the model proposed by Vygotsky was not extended to a system of collective activity, but adapted, in such a way that the focus of a system of human activity does not focus only on the micro level, referring to the individual subject and the instruments he operates, but also makes it possible to perform analysis at the macro level, collective and linked to the community.

Thus, Leontiev (1978), in the context of AT, defines activity as follows:

> those processes which, by carrying out man's relations with the world, satisfy a special need corresponding to it. By activity, we designate the processes psychologically characterized by that to which the process as a whole is directed (its object), always coinciding with the object that stimulates the subject to perform this activity, that is, the motive. (p. 68)

Leontiev (1978) appropriates Vygotsky's studies and considers that the activity of man is a two-way process between the extremes, subject and object, where reciprocal movements are manifested, whether this is a material or symbolic object. In this way, the author points out the objective aspect of the activity, emphasizing the quality of always and necessarily being guided by an object.

For Leontiev (1978), "activity" designates the processes that, effecting the relations of man with the world, satisfy a special need of that world, that is, its motive; and other processes that do not meet this assumption are called "actions" and "operations" by him. In this regard, Kozulin (2004) explains that Leontiev suggests that the activity corresponds to a motive; the action, to a goal; and the operation depends on conditions.

The structuring elements of human activity are fundamental for us to understand the relations between meaning and sense as structural components of consciousness, which guides the activity at the same time that it is regulated and determined by it (Leontiev, 1978). The author focuses on two important concepts in his studies from Vygotsky's cultural-historical perspective: meaning and meaning; and provides a new look for such terms,

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exposes new subtleties in relation to them and calls them "social meaning" and "personal meaning".

The outline of this study is set against the backdrop of initial training of mathematics teachers and intends, based on the AT, to find theoretical and methodological bases to analyze the production of meanings about the study of non-Euclidean geometries by undergraduate mathematics students. And, to carry out this study, we used the terms "social meaning" and "personal meaning" as defined by Leontiev (1983). The author considers that the human being, from the activities he performs, is immersed in a world of meanings, which translate and present the norms, the knowledge to the culture of a particular society, giving them personal meaning.

Leontiev (1978) proposes a definition for the concept of social signification:

> Meaning is the generalization of reality that is crystallized and fixed in a sensible vector, ordinarily the word or the locution. It is the ideal, spiritual form of the crystallization of humanity's experience and social practices. Significance belongs, therefore, first of all to the world of objectively historical phenomena. (p.94)

In other words, social meanings should be understood as the generalization of reality through joint social practices, the ideal form of human existence, immersed in an objectively historical and cultural world.

Asbahr (2014) understands that the process of social signification is constituted as a phenomenon of individual consciousness, with the purpose of producing, within its subjectivity, the social content. The way the subject appropriates or not certain meanings depends exclusively on the personal meaning apprehended by him.

According to Leontiev (1978), the personal meaning is produced subjectively by the subject and results objectively from the link between the reason for performing the activity, that which moves the action, and its immediate objective, that which guides it to its end. Thus, according to Duarte (2004), human consciousness is structured and functions based on the relations between meaning and meaning, since human activity is realized through the psychically mediated relationship between its reason, the content of each action that composes it, that is, the meaning of the action, and its final objective.

In this perspective, Duarte (2004) points out that contemporary school education has as one of the great challenges to enable the learning of school contents to provide a certain meaning for students. Teaching as the main activity of the teacher should be organized, intentional and systematized, in order to trigger in the students, the need for appropriation of scientific knowledge, in a process that gives personal meaning to the activity of studying, for the student, and teaching, for the teacher.

Thus, the intention of this research is to analyze the production of meanings of future mathematics teachers about the study of non-Euclidean geometries in undergraduate courses, in order to formalize, through exploration and experience of Teaching Activity, scientific concepts that can expand the knowledge in the field of Geometry, to the point of establishing

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personal meaning for the insertion of these ways of understanding the different geometric models in mathematics classes.

In the next section, as an unfolding of THC and TA, we will discuss the theoretical and methodological contributions of the Guided Teaching Activity to the development of mathematical concepts and, consequently, to the initial education of mathematics teachers.

Guiding Teaching Activity: a context for negotiating meanings

The studies conducted so far on THC and AT have revealed the concerns of the creators and researchers of these theories with the formalization of concepts, especially those that are assigned to school, called scientific concepts. Thus, they propose as a social necessity to rethink education and, consequently, the process of teaching and learning mathematics.

In the educational field, the appropriation of scientific knowledge emerges as a challenge for the professionals involved in the schooling process. In this perspective, Vygotsky (1987) emphasizes the importance of mediation for the process of schooling of individuals, since "properly organized learning results in mental development and sets in motion various developmental processes that would otherwise be impossible to occur" (p. 101).

Therefore, we rely on the concept of activity proposed by Leontiev (1978, 1983) to better understand the importance of the teaching work in the organization of teaching and its effective contribution to the development of students' higher psychic functions, in the relationship mediated by cultural instruments between subjects and objects.

Moura et al. (2010) state that AT can ground the teacher's work in the organization of teaching, as it involves an action that should be intentionally focused on the appropriation of historically produced knowledge, thus achieving the social goals of the school curriculum. The organization of teaching is considered by the authors as an activity, because it implies that the teacher defines actions that consider the objective conditions of the school; chooses instruments, processes of mediation of the subjects with the objects; evaluates the teaching and learning process, objectives related to teaching and learning; and, finally, verifies the appropriation of historically accumulated knowledge by the students, relative to their need and motive.

Based on the assumptions of AT, Moura (2010) reveals that the teacher's teaching activity should produce and promote student activity. His involvement with his teaching activity can help him become aware of his own work and of his teaching object, and the product of knowledge construction will be transformed into an object of learning for students. Thus, in organizing teaching, the teacher also performs actions that promote the theoretical knowledge at stake, making it a necessary object for his learning activity, which simultaneously creates in the student the need to appropriate the concept in question.

For Lopes (2009), organizing teaching assumes an important role in the teaching activity, as far as learning is concerned, because both the teacher and the students will be mobilized to appropriate knowledge. Thus, the Guiding Teaching Activity (AOE) becomes

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the unity of formation between teacher and students, for the teacher, by organizing the teaching process, also qualifies his knowledge (Moura, 1996, 2001).

Based on the cultural-historical principle of the activity, Moura (2002) defines teaching as an activity that should involve the student in a reflective process, based on the experience of problem-situations that lead him to develop meanings proper to the concept at stake. For Moura (2002), an AOE can be defined as

> that which is structured in such a way as to allow subjects to interact, mediated by a content, negotiating meanings, with the objective of collectively solving a problem situation. It is a guiding activity because it defines essential elements of the educational action and respects the dynamics of interactions that do not always reach the results expected by the teacher. The teacher establishes the objectives, defines the actions, and chooses the teaching aids, but does not control the whole process, precisely because he accepts that the interacting subjects share meanings that change when faced with the object of knowledge under discussion. (p. 155)

Therefore, at the heart of AOE is the problem situation as a triggering aspect of the need that led man to the construction of a certain concept, promoting the sharing of meanings and experiences among students, in an environment that collectively seeks solutions to the problem situation and enables the exchange and production of knowledge among those involved in the dynamic process of teaching and learning. Thus, AOE is configured through the educator's intentionality, by articulating instruments and strategies that will allow the subjects to produce meanings with the object of knowledge (Moura, 1996).

In this perspective, Moura points out three indispensable aspects for the structure of AOE: the historical synthesis of the concept, which enables the teacher to appropriate the pedagogical aspect of the history of the concept (Moura, 1996, p. 20); the problem-situation, which enables the teacher to identify the pedagogical aspect of the history of the concept (Moura, 1996, p. 20). 20); the problem situation or the learning triggering situation, which should contemplate the genesis of the concept and can be materialized in different ways, with three methodological resources for such approach: the games, the situations that emerge from everyday life and the virtual history of the concept (Moura, 2010, p. 103); and the collective synthesis, which is the "mathematically correct" solution of the problem situation developed by the students collectively (Cardoso et al, 2012, p. 4).

Thus, this study focuses on the initial training of mathematics teachers, understanding the importance of the problematization of the pedagogical actions of the teacher, since it seeks, through situations of experience and exploration of teaching activities, to develop the psychic functions of students, future mathematics teachers, about the study of non-Euclidean geometries as a way to expand the knowledge in the geometric field and, consequently, to give new meaning to the teaching of geometry.

Description of the experiment conducted with the future mathematics teachers

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The work was developed with undergraduate students of Mathematics of the Universidade do Estado da Bahia - UNEB/Campus X, located in the city of Teixeira de Freitas, in Bahia. A priori, the proposal was of a theoretical-practical nature and had the purpose of building, for the study, documents that would allow the analysis of the production of meanings, by future Mathematics teachers, from the experience and exploration of Teaching Activities with different geometric models. To do so, the following data collection tools were used: a diagnostic questionnaire, Teaching Activities, audiovisual recordings and an observation script.

The Teaching Activities were based on the theoretical and methodological assumptions of the Guided Teaching Activity and used as a context to stimulate the negotiation of meanings of future mathematics teachers about the study of non-Euclidean geometry concepts. The geometric systems addressed in both the diagnostic questionnaire and the Teaching Activities were: Euclidean Geometry, Spherical Geometry, and Hyperbolic Geometry.

The participants were divided during the research into four groups with four members each and called themselves: Beta, Delta, Geodesic and The Four Postulates. To identify the subjects during the course of the research, we used the following coding:

- LB 1, LB 2, LB 3, LB 4 (Undergraduates of the Beta group);
- LD 1, LD 2, LD 3, LD 4 (Undergraduates of the Delta group);
- LG 1, LG 2, LG 3, LG 4 (Undergraduates of the Geodesics group);
- LO 1, LO 2, LO 3, LO 4 (Graduates of The Four Postulates group).

To identify, in the categorization process, the origin of the data from this research, we denoted the following coding: L (accompanied by the letter of the group and the number) - Graduating student and the group to which he belongs; AE (accompanied by the number) - Teaching Activity; PP - Teacher Researcher.

Teaching Activities

Based on the theoretical framework adopted in the research, six Teaching Activities (TE) were planned and developed: AE 1 - What does Euclid say? AE 2 - Questioning Euclid? AE 3 - Is the sum of the internal angles of any triangle always 180? AE 4 - Is the Earth flat?; AE 5 - Is the Earth flat?; AE 6 - A question of curvature? These AE were composed by means of adaptations of materials produced by other researchers.

The AE served as a thread to build a space for negotiation of meanings about different geometric models that were constituted from discussions, dialogues, and reflections produced by the students and the teacher-researcher. The process of constructing geometric knowledge was investigated by the need for its production, analyzing the limitations of the Euclidean geometric model so that new ways of perceiving and understanding the space we live in could emerge.

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In this paper we will discuss the undergraduates' production of meanings during the application of AE 2: Questioning Euclid?

ACTIVITY 2 - QUESTIONING EUCLID?

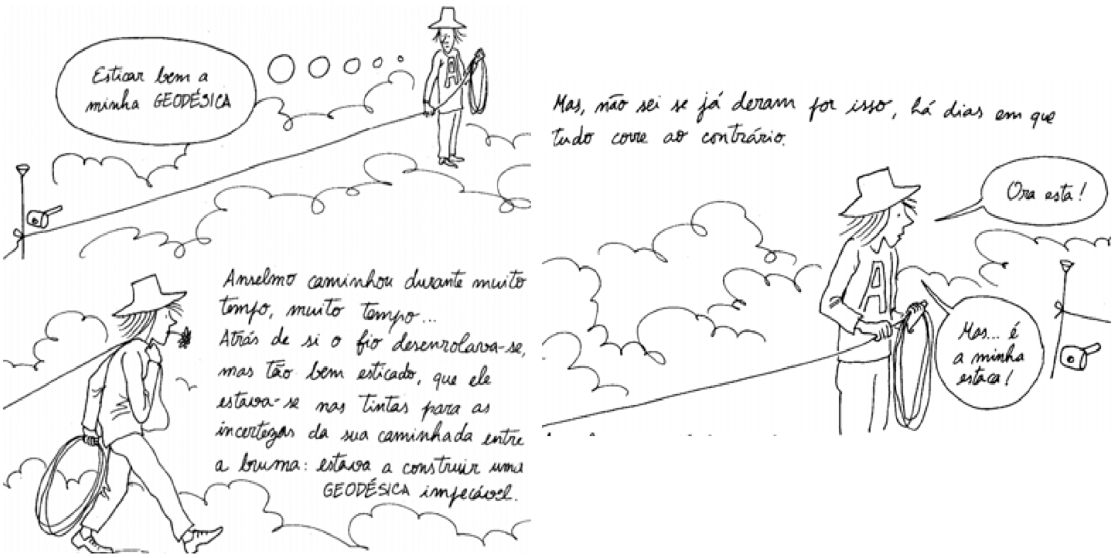

Figure: Material adapted from "The Adventures of Curious Anselmo: The Mysteries of Geometry
Source: http://www.mat.uc.pt/~alma/escolas/alice/OS_MISTERIOS_DA_GEOMETRIA.pdf

1. **If Anselmo departed from the stake and walked 10 km to the south. Then he turned west and walked another 10 km. Then turned again and walked 10 km to the north. Is it possible for Anselmo to find the starting point? Justify it with drawings and/or explanatory texts.**

2. **Imagine that Anselmo, decided to leave the stake and walk in a straight line infinitely.**

   a) **Draw the path Anselmo crossed on a sheet of paper.**
   b) **According to the path drawn on the sheet of paper, is it possible for Anselmo to return to the starting point?**
   c) **Using tape, represent the path Anselmo took on the Styrofoam sphere? What geometric figure did you find? What can you tell from this representation about the path Anselmo crossed?**
   d) **Compare the representations made on the sheet of paper with the representations made on the Styrofoam sphere and write down your reflections considering their relationships.**

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3. Based on the reflections proposed so far:

a) Mark a point on the sphere.
- How many "lines" can you draw passing through this point?
- On the Styrofoam ball, draw one of these lines.

b) Euclid's first postulate states that only one line can pass through two points.
- Does this postulate also apply on the spherical surface? Why or why not?
- On the spherical surface are the lines infinite and unlimited as on the plane surface?

c) Two lines are called competing lines when they are in the same surface and have a point in common.
- On the spherical surface are there competing lines?
- If there are, on the Styrofoam ball, represent two competing lines.

d) Two lines are parallel if they are in the same surface and have no point in common in Euclidean geometry.
- Are there parallel lines on the spherical surface?
- If there are, on the Styrofoam ball, represent this idea.

e) Let X and Y be two distinct points. Into how many parts do they divide:
(a) a Euclidean straight line?
b) a spherical line?

REFERENCES:

NASCIMENTO, Anna Karla Silva do. Geometrias não-euclidianas como anomalias: implicações para o ensino de geometria e medidas. Dissertação (Mestrado) – Universidade Federal do Rio Grande do Norte. Pós-Graduação em Ensino de Ciências Naturais e Matemática. Natal, 2013.

PETIT, Jean-Pierre. *As Aventuras de Anselmo Curioso – Os Mistérios da Geometria*, traduzido por Luís Pignatelli, Lisboa: Publicações Dom Quixote, 1982.

PRESTES, Irene da Conceição Rodrigues. Geometria esférica: uma conexão com a geografia.201 f. Dissertação (Mestrado em Educação) – Pontifícia Universidade Católica de São Paulo, São Paulo: 2006. Disponível em: https://tede2.pucsp.br/handle/handle/11101. Acessado em: 18/07/2018.

Figure 1 - AE 2: Questioning Euclid?
Source: the author himself (2018)

Constitutive aspects of the production of geometric meanings about the study of different geometries

Based on a careful and thorough reading of the material from the observations, the written record produced by the participants, and the transcripts of the events that occurred in the meetings, we grouped the information by similar, complementary or contradictory content, in order to apprehend the production of meanings of future mathematics teachers about fundamental concepts of Non-Euclidean Geometries.

The triangulation of the data led to the emergence of the following categories of analysis: Conflict - from logical validity to empirical validity and Rupture - from Euclidean space to other spaces.

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The category "Conflict - from logical validity to empirical validity" manifests itself firstly in the participant's prior knowledge, when he assumes Euclidean geometry as a logically consistent system, considering it as the only way to interpret and represent the real physical space. And it is completed when the participant understands that the empirical validity of Euclidean geometric knowledge depends on the context in which it is used.

The category "Rupture - from Euclidean space to other spaces" is revealed when there is a break from the paradigm of a single Geometry, that is, in the acceptance of non-Euclidean geometric models and in the movement of construction of new knowledge from this understanding.

Next we will present the process of analysis of the production of meanings of the research participants based on the categories proposed for this purpose.

AE 2 Meaning Production: questioning Euclid?

This Episode is the cut of the third meeting of the application of the intervention proposal that took place on May 04, 2019. The episode has its structure captured at the moment of the plenary session of AE 2, and we highlight the movement of the undergraduates to formalize the concept of parallel and coincident lines in both Euclidean Geometry and Spherical Geometry.

In the Episode, we highlight the students' movement during the discussion about the construction of the concept of parallelism between lines on a spherical surface using manipulative material. Thus, each group received a kit containing the following materials: Styrofoam ball, different colored adhesive tapes, scissors, colored pens, rubber money (rubber band), ruler, and pins.

Scene 1: Discussion about the parallelism between lines on the spherical surface

PP: Two lines are parallel if they are in the same plane and have no points in common. On the spherical surface are there parallel lines?
LB 3: So... Our group had a big fight (laughs). But we measured it right; the distance between the lines was 2.5 cm, but not with maximum circumferences.
LB 4: We can make several parallel lines, but with smaller circumferences.
LB 3: But they will have the same distance between them. So it exists!
LO 4: We also discussed a lot... Our initial question was about coinciding lines, if coinciding lines were parallel. Because in spherical geometry, the line, or geodesic, is only parallel if it passes through the greater circle. So, if you have coinciding lines, you have parallel lines. But we defined that coinciding lines are not parallel, for our discussion here there are no parallel lines in the spherical.
LB 3: I still don't understand. Why wouldn't the coinciding lines be parallel on the spherical?
LO 4: The coinciding lines are not parallel even in plane geometry.
LB 3: But we consider coincident lines parallel because they have the same distance. The distance is zero! If the distance is zero, they are parallel. Because... because it says that, given two points, the points must have the same distance.
(Participant dialogue, 2019)

In the discussion present in Scene 1, we observe that the LB group proposes the existence of parallelism between lines in Spherical Geometry, relating this concept with the idea of distance between lines, that is, there is parallelism between lines when the distance

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between the points of a line to another is the same measure successively, as LB 3 suggests: "So... Our group had a big fight (laughs). But we measured it right; the distance between the lines was 2.5 cm, but not with maximum circumferences. (LB 3, Scene 1, 2019).

In Figure 2, we present the construction, performed by the LB group, of parallel lines on the Styrofoam ball to justify the existence of this concept in Spherical Geometry.

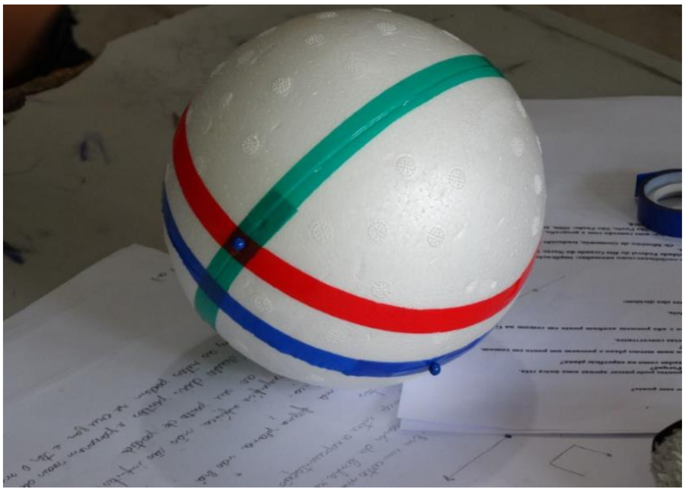

Figure 2 - Construction of parallel lines on Styrofoam ball

Source: Research data (05/04/2019)

The representation proposed by the LB group in Figure 2 suggests that the blue and red ribbons represent two geodesics that are parallel by keeping a distance of 2.5 cm between them in the course of their finitudes. We observe that the LB group tries to give coherence to the concept of parallelism on the spherical surface based on one of the consequences of Euclid's Postulate of Parallel Lines, which suggests the existence of a pair of equidistant lines. In this context, we analyze that there is a rupture, by the participants, in relation to the existence of a single geometric model, because they accepted the idea of parallelism on the spherical surface and attribute meanings to this new concept, considering only the equidistance between lines as a value judgment for the formalization of this concept.

The idea constructed by the LB group is refuted by participant LO 4, when she signals that "in Spherical Geometry, the straight line or geodesic, it is only if it passes through the larger circumference" (LO 4, Scene 1, 2019), which evidences a conceptual error committed by the LB group, when proposing that there is parallelism on the spherical surface, because they admit that the red ribbon denotes a geodesic in the representation exposed in Figure 2. In our analysis, participant LO 4, with the acceptance of the spherical model, not only breaks with the idea of a single geometric model but also transcends the concept of parallelism by internalizing the concept of a geodesic on a spherical surface. Producing new meanings, it makes them able to question the existence of parallelism on the spherical surface by saying:

> Because in spherical geometry, the straight line or geodesic, it is only if it passes through the larger circumference. So, if you have coinciding lines, you have parallel lines. But we defined that coinciding lines are not parallel, for our discussion here there are no parallel lines in the sphere. (LO 4, Scene 1, 2019)

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The movement made by the participants in Episode 3 was manifested in the first moment by the **rupture** the paradigm of the existence of a single geometric model, capable of representing physical reality in its entirety. Thus, the participants, by noticing some limitations in the Euclidean geometric model to model the various situations that emerge on a non-planar surface, began to assign new meanings to the concept of parallelism, which reveals that geometric knowledge is no longer presented as something ready and finished, but as something alive and passive to be built and/or reconstructed.

The categories of analysis of the participants' production of meanings emerge in a spiral process, through revolutions, in which the internalization of concepts can be captured through the **conflict** between the logical consistency of Euclidean Geometry and its empirical application, which may lead or not to the **rupture** the paradigm of a single geometric model. We also emphasize that the participants' production of meanings was captured and represented in our research through a spiral process, in which the order of the categories does not emerge in a fixed and predefined way, and the rupture may lead to the emergence of new conflicts and vice-versa.

That said, we understand that the participants, by breaking with the idea of a single geometry of reality throughout the interactive process presented in Scene 1, were refining their thought process and making decisions that led them to correctly state that in Spherical Geometry there is no parallelism between geodesics. This statement was proposed by the LO group, which conditioned the existence of parallelism between geodesics on the spherical surface, by proposing as true the premise that coinciding lines do not constitute parallel lines.

In the dialogue presented by the participants in Scene 1, we observe that the premise proposed by LO 4: "Coinciding straight lines are not parallel even in Plane Geometry" (LO 4, Scene 1, 2019) generated a kind of **conflict** for the participants who defended the existence of parallelism between geodesics on the spherical surface. Participant LB 3 confronts LO 4's statements by proposing, "But we consider coincident lines parallel because they have the same distance. The distance is zero! If the distance is zero, they are parallel. Because... because it says that given two points the points need to have the same distance" (LB 3, Scene 1, 2019).

In our analysis, the statements of participant LB 3 express that the participants of this group attribute meanings to the parallelism between lines in Euclidean Geometry, considering the equidistance between the points belonging to the lines as the only necessary aspect to determine the parallelism between lines. Thus, we understand that the members of the LB group did not internalize the concept of parallelism between lines from the understanding of the currently accepted Postulate of Parallelism for Euclidean geometry - **given a line and a point outside of it there is only one-line parallel to the given line** - but from one of the consequences imposed by the Postulate of Parallelism, which results in equidistance between lines.

In this context, we attribute the participants' difficulties in verifying the existence of parallelism in Spherical Geometry to the fact that they have not internalized the concept of

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parallelism in Euclidean Geometry, and the deficient formation of this knowledge contributed to the consolidation of a conceptual error, when they propose that coincident straight lines are parallel lines.

In Scene 2, we highlight the participants' attribution of meanings for coincident straight lines based on their acceptance that there is parallelism between these coincident lines. The idea of coinciding lines, linked to the concept of parallelism, is slowly deconstructed by the participants, at the moment they resort to the Postulate of Parallel Lines as proposed by Play fair and currently used - **given a line and a point outside it, one can draw through this point a single line parallel to the given line.**

Scene 2: Discussion about parallelism from the Parallel Postulate

> PP: So, let's go to Euclid's V postulate.
> LO 3: But there are two conditions for them to be parallel.
> LO 4: That caused a confusion in my head, because coinciding lines were parallel. But, by definition, parallel lines cannot have points in common, and if they are coincident, they have all points in common.
> PP: Come on, then! The 5th postulate that we work on today in basic education, which is a transformation of Euclid's postulate, one of the ideas is that, given a line and a point outside this line, you can draw a single line parallel to the given line. So you have to have, in a sense, a point outside of that line.
> LB 3: Ohhh! because I learned that way. In my whole course and even in the mini-courses I took, we learned that coincident lines are parallel lines.
> LB 2: I am with the thought of LO 4, the lines can't have points in common... If they have points in common, it already eliminates the chance of being parallel.
> (Participant dialogue, 2019)

In Scene 2, we observe the movement of participant LO 4 to deconstruct the idea that coinciding straight lines are parallel lines. The participant assumes the conflict as a way to question something that seemed true and defined to her, by saying, "That caused a confusion in my head because coinciding straight lines were parallel" (LO 4, Scene 2, 2019). In the sayings of participant LB 3, we find evidence that this conceptual error was propagated during college: "It is ... because I learned it that way. In my whole course and even in the mini-courses I took we learned that coinciding lines are parallel lines" (LB 3, Scene 2, 2019). In this context, we understand that the participants misappropriated the concept of parallelism between lines and that the studies carried out during the course of Bachelor of Mathematics were not enough to understand the conceptual dimension that involves the idea of parallelism.

The participants' movement towards the construction of the concept of parallelism on the spherical surface motivated the undergraduates to rethink the concept of parallelism in Euclidean Geometry, in which the existence of parallelism in Geometry is something accepted and consolidated. The acceptance of this concept by the participants led them to seek at least one particular case to validate the idea of parallelism, considering the existence of this concept regardless of the type of surface under study.

Final considerations

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We understand that the manifestations present in the episode indicate that the research subjects are unaware of geometric models other than Euclidean, recognizing Euclidean Geometry as the only way to describe and interpret the physical space in which they live. Moreover, we found that the undergraduates, by accepting the existence of other geometric models, modify the way they relate to the space in which they live and, thus, develop greater autonomy in the construction process of the geometric concepts under study.

When studying the concept of parallelism on the spherical surface, the students broke with the logical consistency of the Euclidean geometric model, when they verified that there is no parallelism between geodesics in Spherical Geometry. Thus, they began to assign new meanings to the concept of parallelism in Euclidean Geometry, when they understood that coinciding lines cannot be considered parallel lines.

Finally, we believe that among the contributions of this research is the reflection about the training of future mathematics teachers, regarding the appropriation of non-Euclidean geometric knowledge as a possibility to change the way teachers understand and carry out the practice of Geometry teaching.

References

Abreu, G. (2000). O papel mediador da cultura na aprendizagem da matemática: a perspectiva de Vygotsky. *Educação, Sociedade & Culturas*, (13), 105-117.

Asbahr, F. S. F. (2014). Sentido pessoal, significado social e atividade de estudo: uma revisão teórica. *Revista Quadrimestral da Associação Brasileira de Psicologia Escolar e Educacional, 18(2), 265-272*. DOI: https://doi.org/10.1590/2175-3539/2014/0182744

Braida, F. C. (2012). Interferência do enquadramento de trabalho nas representações sobre o ensino de leitura no contexto da formação inicial docente sob o entendimento da Teoria Holística da Atividade. Tese de Doutorado em Letras - Estudos Linguísticos. Santa Maria, RS: Universidade Federal de Santa Maria.

Cardoso, N. P., Hundertmarck, J., Perlin, P., & Lopes, A. R. L. V. (2012). Atividade Orientadora de Ensino: uma experiência utilizando trocas nos anos iniciais. In *Anais da 3ª Escola de Inverno de Educação Matemática (pp. 1-11). Santa Maria, RS: 1º Encontro Nacional PIBID-MATEMÁTICA*. Recuperado de https://www.ufsm.br/app/uploads/sites/534/2020/03/RE_Cardoso_Naise.pdf

Castorina, J. A. (1996). O debate Piaget-Vigotsky: a busca de um critério para uma avaliação. In J. A. Castorina, E. Ferreiro, D. Lerner & M. K. Oliveira. *Piaget – Vigotsky: novas contribuições para o debate (pp. 7-50)*. São Paulo: Ática.

Duarte, N. (2004). Formação do indivíduo, consciência e alienação: o ser humano na psicologia de A. N. Leontiev. *Caderno CEDES*, 24(62), 44-63. Available at: http://www.scielo.br/ pdf/ccedes/v24n62/20091.pdf

Leontiev, A. N. (1978). *O desenvolvimento do psiquismo* (2a ed.). São Paulo: Centauro.

Lopes, A. R. L. V. (2009). *Aprendizagem da docência em matemática: o Clube de Matemática como espaço de formação inicial de professores*. Passo Fundo: Ed. Universidade de Passo Fundo.

DOI: 10.20396/zet.v29i00.8661597

Moura, M. O., Araújo, E. S., Ribeiro, F. D., Panossian, M. L., & Moretti, V. D. (2010). A atividade orientadora de ensino como unidade entre o ensino e a aprendizagem. In M. O. Moura (Org.), *A atividade pedagógica na teoria histórico-cultural (pp. 81-109)*. Brasília, DF: Liber Livro.

Moura, M. O. (1996). *Controle da variação de quantidades: atividades de ensino*. São Paulo: FEUSP.

Moura, M. O. (2001). A atividade de ensino como ação formadora. In A. D. Castro & A. M. P. Carvalho (Orgs.), *Ensinar a ensinar: didática para a escola fundamental e média (143-162)*. São Paulo: Thomson Learning.

Moysés, L. (1997). *Aplicações de Vygotsky à educação matemática*. Campinas: Papirus.

Vygotsky, L. S. (1987). *Pensamento e linguagem*. São Paulo: Martins Fontes.

Vygotsky, L. S. (1993). *Pensamiento y palabra*. In L. S. Vygotsky, Obras escogidas (vol. 2) *(pp. 287-348)*. Madrid: Visor.

Vygotsky, L. S. (2003). *A formação social da mente: desenvolvimento da percepção e da atenção* (6a ed.). São Paulo: Martins Fontes.

Vygotsky, L. S., Luria, A. R., & Leontiev, A. N. (2010). *Linguagem, desenvolvimento e aprendizagem* (11a ed.). São Paulo: Ícone.