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Final Years Mathematics Teachers' Probabilistic Literacy: Reflections of Continuing Education

Letramento probabilístico de professores de matemática do Ensino Fundamental: reflexões da formação continuada

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Abstract

The teaching of probability is seldom addressed in initial teacher education courses for teachers who teach mathematics, which means that continuing education becomes relevant. This article presents aspects of a completed doctorate study on probabilistic literacy in six continuing education meetings, which were filmed and had the participation of five teachers of the final years of elementary school. It brings the analysis of data collected from individual semi-structured interviews and from the first meeting. The results suggest that teachers had an elementary understanding of the concept of probability and its teaching, following a perspective focused on the application of the Laplacian form of probability, without developing in their practice a critical discussion about the concepts, nor fostering the idea of probabilistic literacy.

Keywords: Statistics education; Probability literacy; Statistics teacher education; Mathematics education.

Resumo

O ensino de probabilidade é pouco abordado em cursos de formação inicial de professores que ensinam matemática. Assim, a formação continuada adquire relevância. Este artigo apresenta aspectos de um estudo de doutorado finalizado, sobre o letramento probabilístico em seis encontros de formação continuada os quais foram filmados e contaram com a participação de cinco professores dos anos finais do ensino fundamental. Analisa-se os dados de entrevistas semiestruturadas individuais e do primeiro encontro. Os resultados sugerem que os professores tinham uma compreensão elementar sobre o conceito de probabilidade e seu ensino, seguindo uma perspectiva voltada para a aplicação da forma laplaciana de probabilidade, sem desenvolver em sua prática uma discussão crítica sobre os conceitos, nem que fomentasse a ideia de letramento probabilístico.

Palavras-chave: Educação estatística; Letramento probabilístico; Formação de professores que ensinam estatística; Educação matemática.

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Introduction

Probability refers to different topics, such as: weather forecasting, gambling, sports, health, and engineering. As this theme is inserted in the most diverse social media, it needs a deeper understanding. Probabilistic literacy is inserted within a larger field of research called statistical education, which relates to the teaching and learning of statistics, probability, and combinatorics.

Gal (2005) and Batanero (2005) point out that Probability has a primary role in understanding non-deterministic social phenomena, which are guided by decisions that must take probabilistic reasoning into account. For example, in deciding about a risky enterprise, an analysis of the possibilities and implications of taking one decision over another is required.

Probability teaching is related to demands arising from the need to understand the factors discussed in the education of mathematics teachers that teach statistics and probability. One of the challenges pointed out by studies is that little attention is given to the work with statistics and probability in initial teacher education courses (Kataoka et al., 2008). This may cause the verbal difficulties elementary school students find to describe probabilistic situations (Santos & Grando, 2011).

Campos and Petropaolo (2013) identified that most of the teachers that participated in their study stated that probability should only be studied in high school. This result emphasizes that it is necessary to foster mathematics teachers' understanding of the importance of probability for mathematics education, and studies investigating the teaching of probability within the final years of elementary school.

The National Common Curricular Base (MEC, 2018) is an official Brazilian normative document that provides for the teaching of probability and statistics from the initial years of elementary school to high school. The document points out that the work in a longitudinal way will provide students with the opportunity to understand the nature of probability and its different nuances involving random phenomena.

Considering the results of the research and the curricular recommendations, it is possible to identify the importance of investigations on how probability has been worked in elementary school, focusing on the perspective of the mathematics teacher.

Lopes (2008) conducted a study on statistics and probability teaching, articulated with teacher education in Brazilian basic education. The researcher indicates that there is a paradigm called linearity, which she defines as being similar to a form of culture in Brazil of teaching content focused on the following mathematical topics: Numbers and Operations, Quantities and Measures, Algebra, and Geometry. The author says that research by Shaughnessy (1992), Mendoza and Swift (1981), Machado (1997), and D'Ambrosio (1998) affirm that work with the statistics and probability teaching may assist in the rupture of a linear teaching practice that privileges the concepts cited and may broaden the understanding of students in different areas of mathematics and statistics, thus aiming at the development of

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citizenship and criticality. Thus, according to the research, we perceive that there is a need for studies that discuss the teaching of probability, a theme that has seemingly been neglected in the initial and continuing education of teachers who teach mathematics. We must reflect to understand the process in which the theme is found and what challenges for the expected advancement are.

International studies also point to the importance of initial and continuing mathematics teacher education. Contreras et al. (2010) conducted a study with 166 mathematics preservice teachers in Portugal, Mexico, and Spain. The research suggested that the teachers surveyed demonstrated erroneous intuitions on probability and failed to do well in situations involving problem solving in probability. The authors emphasize that both initial and continuing education must critically and reflexively address the relevant themes on the teaching of probability. The authors consider this to be a non-negotiable condition for the development of teachers so that they are able to carry out teaching processes, capable of developing their students' critical sense. From the discussions raised by those authors, we emphasise the importance of continuing education of the mathematics teacher.

To better understand how the process of probabilistic literacy takes place in Brazilian basic education, a doctoral research was developed aiming to analyse competencies for the probability teaching from a probabilistic literacy perspective within a group of mathematics teachers of the final years of elementary school.

This article analyses the teachers' previous knowledge of probability and its teaching in the final years manifested in the semi-structured interviews and at the first teacher Education Meeting.

In the next section, we present what we understand about probabilistic literacy and the view of researchers who have been developing research in the area of statistics education. We also delimit some demands and challenges in the Latin American context to achieve a statistics education focused on the critical performance of mathematics teachers.

Theoretical approach to probabilistic literacy

The primary function of statistics education, in which the discussion on the teaching and learning of statistics, combinatorics, and probability is inserted, is to develop people's critical sense. Lopes (2008) says that basic education students must experience the teaching of probability integrated with statistics. In that regard, statistical, combinatorial, and probabilistic elements that involve counting, data collection, and analysis, measurement of uncertainty, work with nondeterministic elements, are characteristics of the area in question. For example, the financial market that deals with the rise of the dollar, whose appreciation will influence the price of fuel, cooking gas, French bread that is consumed by most Brazilians; meteorology, violence, births and deaths, traffic accidents, voting polls in elections at the most different levels, among others, are just some of the contexts where we can observe the importance of statistics, combinatorics, and probability.

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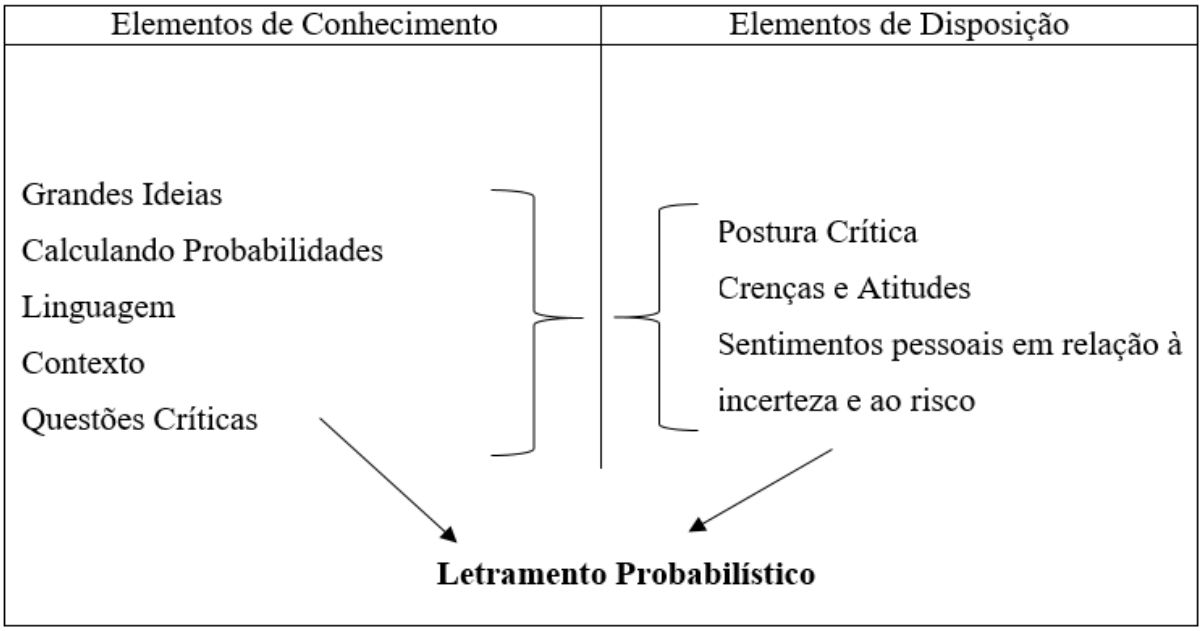
In Brazil, the official normative National Common Curricular Base (MEC, 2018) disconnected combinatorics from the thematic unit of probability and statistics, directing it to numbers. Although it is not the focus of this article to analyse this disconnection, we consider it a setback in relation to teaching and research in combinatorics, as this field of study is not limited to counting techniques, but contributes to the development of hypothetical deductive reasoning and combinatorial thinking that help understand probabilistic and statistical situations.

Gal (2000) states that there is a discussion about the term literacy, which is understood as the mastery of basic or minimal reading or writing skills that are articulated with the common life of individuals in society. Gal (2002) further develops his discussion on literacy, pointing out that there is statistical literacy, which is the ability to interpret, critically assess, express opinions on statistical information, and communicate through them.

In this same perspective, (Gal, 2005) develops the theoretical discussion about probabilistic literacy, which is inserted in two lines of discussion that he calls knowledge elements and dispositional elements, as we can see in Figure 01 below:

Figure 01: Gal's probabilistic literacy (2005).



Source: Adapted from Gal (2005).

As we can see in figure 01 above, Gal (2005) understands probabilistic literacy from two perspectives, the *cognitive* and the *dispositional* elements. Cognitive elements, as the word itself denotes, are related to probability calculations, the language used to communicate probabilistically, different contexts of use of probability, and critical questions about the concept. The elements he calls dispositional, on the other hand, would be linked to personal beliefs, attitudes, habits, and personal feelings in relation to uncertainty and risk in the context of probability.

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About the knowledge elements, Gal (2005) points out that the *"big ideas"* are: randomness, independence, variation, and other elements such as the ability of students to understand the derivation, representation, interpretation, and implication of probabilistic statements.

The second cognitive element that Gal (2005) points out is *calculating probabilities*, in which students would be familiar with the different approaches to probability, such as the classical, frequentist, geometric, subjective, and axiomatic approach.

Another cognitive element is *language*, which consists of the terms used to communicate about chance, randomness, etc. For example, words and expressions such as possible, probable, unlikely, right event, impossible event, chance, probability, random, may happen, may not happen, experiment, event, sample space, luck, bad luck, at random etc., are used to communicate what we want to discover or analyze about probability.

The fourth cognitive element, according to Gal (2005), is the *context*. For the author, being literate probabilistically means that the subject can develop elements of the big ideas, calculate probabilities, probabilistic language from a knowledge of the world (Gal, 2002). This knowledge is understood as contextual knowledge, where people have specific expectations about the role and impact on the ideas of chance and randomness in different processes and situations in which they are required.

The fifth and final cognitive element, according to Gal (2005), is the critical questions about probability. It is no use only working with the big ideas, calculating probabilities, language, and context if there is no work of reflection on what one wants to inquire probabilistically.

In Gal's literacy model (2005), there are also the dispositional elements: *critical stance, beliefs and attitudes, personal feelings in relation to uncertainty and risk*. In his article (Gal, 2005), he does not develop a broader understanding of these elements but makes some notes in relation to each element. For example, the author states that each individual had their experiences throughout their life, for example, living in different social sectors such as family, church, school, neighborhood, city, state, and country. In this sense, each individual will have an understanding of the world from the set of experiences they had access to, thus causing a plurality of interpretations, beliefs, and stances towards probabilistic data.

The multiplicity of experiences in various social experiences contribute to people having contact with different situations involving probabilistic contexts and that influence their decisions. So the critical stance fits each social context experienced.

Gal (2005) discusses the teaching of probability from some questions such as: "What are the main characteristics of probability? What do adults need to know about probability to deal with the most different daily situations?" These questions reflect what the author considers important in probability in the context of the real world, knowing that societies are

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requiring adults capable of resolving conflicts, innovating, and managing problem solving in a creative and sustainable way.

Gal (2005) argues that in the context of probabilistic literacy, the learning of probability would help students prepare for life in society. Since the understanding that there are several events that are deterministic in nature, such as the existence of non-deterministic events and that express themselves randomly, such as the chance that something happens, like raining tomorrow, a player hitting all dozens and winning the lottery, a patient being operated and surviving. These are elements that are studied and measured from uncertainty, but that we can measure from the perspective of probability. It seems that probability is a very important concept, in the sense of serving as a basis for other learning in the fields of statistics and mathematics.

Monteiro (2016) points out that the term "*letramento*" corresponds to *literacy* in English, which has the idea of reading articulated to the establishment of meaning, i.e., the subject reads and can extract the understanding of what was read. And this understanding that was established in reading can and should be linked to the social practices of any subject living in society.

Shamos (1995) uses the expression "scientific literacy." Other authors, such as Watson (1997), Wallman (1993), Watson and Callingham (2003), and Gal (2002) work with the perspective of statistical literacy. Niss and Jablonka (2014) present the term *mathematical literacy*, which can be translated into Portuguese as "*letramento matemático*". All these terms and expressions denote the way each author understands an education that acts from the perspective of literacy, but that specifically deal with elements of criticality, argumentation, scientific speculation, and that people should rely on mathematical, statistical, and scientific elements to make correct decisions regarding the phenomenon that is put to them for analysis.

Thus, we realize that different authors point to the development of a formation that takes into account statistical or probabilistic literacy, but that works with the conceptual elements and that articulates with the critical elements of the concepts addressed.

Education of teachers that teach statistics and probability

The initial education of teachers who teach mathematics in the final years of elementary school and high school in Brazil takes place from the undergraduate teaching degree courses, which last four to five years, and are structured depending on the political pedagogical project of each course. This formation takes place in different types of institutions, whether public or private. It is worth noting that within the mathematics teaching degree courses there are curricular components that address statistics and probability specifically.

Silva (2014) analyzed 78 mathematics degree courses from 48 higher education institutions in the five regions of Brazil based on the mapping of the number of curricular components addressing statistics and probability present in political pedagogical projects,

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syllabus, and course plans of the institutions. The study revealed that the approach to statistics and probability is infallibly conceptual and that, in most courses, only 2 or 3 curricular components offered those contents. This investigation shows us that there are a very small number of curricular components directed to reflection on the teaching and learning of statistics and probability in the initial education of teachers who will teach mathematics. Therefore, we conclude that the initial education has experienced difficulties regarding the workload allocated to teaching statistics and probability. As a consequence, teachers who undergo initial education in those institutions may present difficulties in addressing those concepts in their pedagogical practices. Costa and Pamplona (2011) point out that a higher workload must be allocated to statistics and probability curricular components in mathematics initial education courses. This proposition is necessary so that there is time available for the discussions that need to be held in this field of study. In this way, continuing teacher education can become a second opportunity for teachers to have access to content that could not mature at the initial moment of their qualification.

Almeida (2000) and Fiorentini and Nacarato (2005) indicate that continuing teacher education is necessary for a society in constant evolution and that education professionals need to be constantly studying to deal with the most diverse problems that appear in the educational sphere. Teaching must be accompanied by research, in constant evolution, given that, in this way, it can go through a path of discoveries, contributing to the growth of the research area. In this sense, continuing teacher education is a new horizon for education professionals, being necessary for their progress in the field, since most teachers qualified in mathematics degrees will work in basic education.

Method

In this research, we use a methodology from the qualitative approach, as it is characterized by concern in the meanings constructed by the subjects participating in the research. We also understand that qualitative research seeks to understand the construction process developed by the subjects in a research, and not just a look at the final results of the study (Trivinos, 1987).

Thus, the study was conducted from qualitative research, seeking to understand how the interrelations in continuing teacher education occurred. Next, we present in more detail our choices to carry out the field study, which took place with a private school.

The school for research environment was chosen from two perspectives. The first was because one of the researchers in this study had already taught in the educational institution and knew the school board and teachers. The second was because there was a group of teachers that already met periodically for continuing education moments in school. Those moments are divided between the institutional formation offered by the school and the discussion of mathematical concepts aimed at improving mathematics teaching at the school. The research to which this article is linked (Eugênio, 2019), was a completed doctoral study carried out in a centenary school of the private network of *Agreste de Pernambuco*, in the

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municipality of Garanhuns, which offers classes from kindergarten to high school. Data collection was carried out from 2018 to 2019, when the school had six mathematics teachers working in the final years of elementary school. They officially signed a consent form to participate in the research, in which the procedures provided for in the research method and the possible implications of participation were explained, thus carrying out an important step in the ethical dimension of the study.

Five mathematics teachers who worked in the final years of elementary school (6th to 9th grade) participated in the research. The students that volunteered to participate in the research were between 11 and 14 years of age. The teachers were given the acronyms: PF1, PF3, PF4, PF5 and PF6; PF2 was present in the initial interview but did not participate in the continuing education. The researcher was given the acronym PE. Table 01 presents some data that characterise the participants.

The participants were formally contacted through a letter of consent that was delivered to the school management, explaining the content of the research and the procedures that would be carried out in the research.

Table 01 - Academic profile of the research participants

Teacher	Gender	Age	Year graduation was concluded	Postgraduation	Attended statistics and probability course during under graduation
PF1	M	48	1994	Masters in Ed. Mathematics	Yes
PF3	F	50	1993	Mathematics Teaching Specialist	Yes
PF4	F	52	1997	Mathematics Teaching Specialist	Yes
PF5	F	30	2011	Mathematics Teaching Specialist	Yes
PF6	F	27	2016	Mathematics Teaching Specialist	Yes

Source: Authors.

As we can see in Table 01, most teachers are women with specialization in mathematics teaching, attended in their undergraduate courses a school subject that involved probability and statistics.

Interviews. Semi-structured interviews were conducted with teachers individually, according to each participant's availability, 15 days before the first formation meeting. The questionnaire, which lasted about six minutes, aimed to outline the teachers' profiles and acknowledge important elements of their formation and their trajectory as basic education teachers, besides gathering relevant information about probability teaching. Chart 01 below shows the roadmap of the interview.

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Chart 01 - Roadmap of the semi-structured interview

Block 1: Personal and education information:
1. How old are you? 2. Which college degree did you finish? What is your background? Postgraduate specialisation Master's degree? PhD? 3. Which year did you finish college? 4. What is the name of the higher education institution you attended? 5. In the higher education course you completed did you study any school subject on statistics? What about probability? 6. Has the topic been addressed in any other curriculum component?
Block 2: Information on probability teaching
In your opinion how would you define probability or what do you understand by probability? Could you give an example? Do you know another definition of probability besides the one you mentioned earlier? What concepts do you think are important for elementary school students to learn probability? Could you give an example? How do you evaluate that your student has learned probability? How do you start and conduct your probability class? In the didactic material used by the school is there a topic that specifically discusses probability from the 6th to the 9th grade? What are the places and situations where we can perceive applications of probability in our lives?

Source: Authors.

Soon after the interviews with the teachers, the researcher organized a schedule for the continuing education meetings. Six meetings were held to identify and analyze what teachers knew about the teaching of probability from a perspective of probabilistic literacy according to Gal's model (2005). The aim was to identify the teachers' posture and study what they were already doing in the final years of elementary school from the 6th to the 9th grade of the field school.

We emphasize that the teachers analyzed their own answers, which the researcher showed in a slide presentation in the first formative meeting, omitting the respondents' names, i.e., so they did not know who had given which answer. However, they knew that the answers from all colleagues involved in the research were represented there. As mentioned before, the teachers were interviewed 15 days before the first formation meeting.

In this article, we will discuss the most important elements analyzed in the first meeting with the five participating mathematics teachers. Due to the limited number of characters for this article, we will develop the other meetings in other publications. The questions raised in the meetings and the probabilistic concepts worked were elaborated from reading Gal's text (2005) and the literature review carried out. Chart 02 shows how the first formative meeting was organized and what the objectives proposed were.

Chart 02: Organization of meeting 1

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Meeting I	
Actions developed	*Objectives*
- Present of the formative proposal; - Discuss the definition of probability; - Resume the semi-structured interview.	- Discuss the formative systematics with teachers; - Discuss with teachers what they understood of probability at the time of the interview and contrast with the other colleagues' answers.

Source: Authors.

The meetings were filmed, and protocols were generated from the full transcription of the speeches.

Data analysis. The interviews with the teachers were transcribed in full and gave us an idea of the participants' profile and what they understood about probability. Our main objective was to be able to relate the answers given by teachers in the interview process with their respective analyses carried out in the first formation meeting.

We carried out a process of categorization and the analysis of the teachers' answers to the interview held in the first formation meeting. Analyzing Gal's probabilistic literacy perspective (2005). PF2 participated in the semi-structured interview but did not participate in the formative meetings. Nevertheless, his answers were included in our analysis.

Results and discussion

From the six questions of the second block of questions of the interview (Chart 01), we chose three questions that had also been addressed in the first meeting of the continuous education. In this article, we will not present all the questions asked in meeting one due to lack of space for discussion. However, they will be discussed in future articles. Chart 03 below brings the questions asked and the answers given by the six teachers. Next, we will analyze the answers according to the critical considerations of the teachers involved in meeting one, referring to the elements of knowledge and the elements of disposition.

Chart 03. Questions asked at the first meeting of continuing education

Questions asked in meeting 1	
Question 1	In your opinion, how would you define probability or what do you understand by probability? Could you give an example?
Question 2	What concepts do you think are important for elementary school students to learn about probability? Could you give an example?
Question 3	How do you asses your students to know whether they have learned probability?

Source: Authors.

The teachers' answers to the questions presented in Chart 03 will be analysed individually, as we can see in the dialogues presented below. We will also focus on the PE's and teachers' questions (PF1, PF3, etc.) in the discussions below, taken from the first formation meeting, about the answers given in chart 03.

PE: Do you agree with those definitions? Do you want to discuss in pairs, or would you modify anything you said? About what probability is...

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PF5: I found PF1's answer more complete; It has to do with chance, isn't it?...
PF4: I think each teacher... answered a little bit and we could sumarise here. I think everyone said something, right?... That you could put it together... and make one!
PE: Which element would you highlight in the answer given by PF1? What is the key element about what probability is?
PF4: The random, I don't know, the random...
PF5: Exactly! I would give only one answer. The randomness.
PF6: Study the possible events... Whether it happens or not.
PF4: As teacher PF4 said so much... that the probability of something happening... is a measure of chance. RANDOMS... PF5: Yes.
PE: Which are random events, since PF4 is saying it? What is something that happens randomly?
PF5: Something that is not defined... It is going to happen, but...That you cannot determine...That you cannot determine for sure.. .that it will happen, right?!! It's a possible...Possible event…
PF1: And that you only know the result after performing the experiment.
PE: In other words, do I need to carry out the experiment so that I know if that will happen or not?
PF1: This complemented…

As we can see in the dialogue above, the entire discussion between researcher and teachers was based on the concept of probability. At first, teachers were guided by the answer given by PF1, for which the key concept of probability would be randomness. But when the PE asked whether randomness was a key concept, at first they said yes, later they said no. This denotes a lack of consistency in relation to understanding randomness. The discussion ended with the idea that probability was the possibility of something happening, but at random, and that the person cannot perceive it before performing an experiment.

We can see that the conceptual discussion about probability is inserted in what Gal (2005) calls great ideas, referring to the discussion about "variation, randomness, independence, predictability, and uncertainty." It seems that teachers recognize that there is randomness within the concept of probability, but place it as a secondary element, initially prioritising the possibility of an event happening or not. They also pointed out that one cannot predict an event, but the experiment is necessary for probability to exist.

In the context of probability teaching, the experiment is not needed for probability to exist (Batanero, 2001). For example, if we want to know the probability of flipping a coin to land on heads, we do not need to carry out an experiment to know that the result is ½, or 50%. This is because we know that the sample space is 2, heads or tails, and the event we want is 1, represented by the result heads. Thus, we noticed that teachers had probabilistic notions, but that they needed a more in-depth discussion to be able to understand what the different probabilistic concepts represent.

Another interpretation of the teachers observed in relation to the first question was what Gal (2005) calls beliefs and attitudes, because the teachers made it clear that they understand that probability was the possibility of something happening or not. And that this event has a random characteristic, that according to them, the experiment is needed to prove the existence of probability. From this perspective, we realized that the teachers' beliefs led them to affirm that probability was largely restricted to empiricism, i.e., experience. In assuming this position, they did not consider the mathematization of the concepts and the possible axioms that sustain it as a mathematical object.

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Beliefs are very important elements to understand a multifaceted mathematical concept such as probability, but that sometimes can generate some misconceptions. Perhaps the teachers did not yet have, at this point in the research, theoretical elements to understand that probability is independent of experience. It can materialize through experiments and practical situations, but it has its mathematical support independent of empiricism.

Thus, the teachers' answers reflect an intuitive understanding of the concept of probability that needs to be improved so that there is a reflective teaching for students of the final years of elementary school.

The second question: "What concepts do you think are important for elementary school students to learn about probability? Could you give an example?" generated the following dialogue:

PE: We seee that PF1 is always focused on the concept of randomness.
PF3: PF4, PF3, PF5: Yes.
PE: Let's reflect on the PF1's answer!
PF1: I think it will depend on a situation...
PE: Accident and randomnes are the same thing?
PF5: Of the context. PF3: Yes.
PF1: I think there are things that I could classify as an accident, and there are things that depending...on my context, on my set, let's say, it's randomness, right!!! Right!... I think the accident is more unpredictable, so to speak...
PE: I see what you mean. Who could set an example for us to understand it better?
PF1: Accident?!!! Well, I get to the classroom, eee... suddenly, the ceiling falls down! It's an accident! From my point of view, it was an accident. Randomness would be... I have a somewhat predefined set, that I want to see an event that's going to happen in that set. And for me not to interfere with its results, I use, ahahah... I use random thinking. RIGHT?!! I want to choose a student here who's wearing a watch, in the classroom...So I have a set of students and I'm going to choose him/her randomly... I'll choose yours, yours, and yours... Then, accident would be different... There is a debate...
PF3: PF4 and PF5: I agree.
PF4: Accident is as if you weren't waiting! It is unexpected... No, predict, no!!!yeah, yeah...
PF6: You can't define the situation...At least choose the situation, right!!!! In an accident, you can't choose the situation...
PF5: Right. It's totally unexpected, right!!!!!!!!!!
PF1: It's where the textbook often confuses us a little...Because it says: Rolling a pair of dice accidentally! Or rolling a pair of dice at random?
PF6: Because if you rolled a pair of dice, it's no longer accidental, right!!!
PE: Is the rolling accidental? Since you know what might happen?
PF4: So it's not accidental!!! I already know what I'm going to do, which is roll the dice!
PE: Do you know how many possibilities are there?
PF6: The situation you are going to do is already predefined, isn't it?...You just don't know the final answer.
PE: So, is the way the book conducts [the matter] wrong?
PF1: I think IT IS! Because then you will create in the student's head two unclear situations! The difference between accident and randomness.
PE: OK!!!

We can see in the dialogue above that PF1 influenced colleagues to reflect on randomness as a basic element to understand the concept of probability. Question 2 focused on which the important concepts for students to learn in the final years of elementary school were. The discussion was based on randomness, which makes us reflect on the importance of

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a continuing education process, where teachers have a voice and can discuss mathematical concepts that are often silenced by the lack of dialogue between peers in the school context.

PF1 argued that something that happens accidentally is something without a plausible explanation, i.e., it simply happens. She gave the example of the ceiling falling down in a school. And counter-argued that the textbook brought that rolling the dice was an accident. From PF1's perspective, it would be to roll the dice at random. And the other teachers agree and point out that something that happens accidentally is something without a logical explanation, something unpredictable. On the other hand, something of a random is as if the subject clearly had the total number of possibilities (sampling space), but did not know specifically what would appear in that context, but could calculate its probability.

In the dialogue about the second question, we realized that the teachers' answers are contained in the big ideas pointed out by Gal (2005), which are knowledge elements and also express dispositional elements (beliefs and attitudes), as well as the critical stance, also defended by Gal (2005). Those elements are part of the construction of probabilistic literacy that was being developed from the dialogue of the participants, plus the questions and propositions made by the researcher.

Through the debate about question 2, teachers could advance their understanding of what Gal (2005) points out as big ideas, in which the mathematical concept itself is taken into account. But we can highlight the appearance of the dispositional element: critical stance. Through the critical analysis of a teacher (PF1), teachers could criticise the way the textbook brings the words *acaso* (accident) and *aleatoriedade* (randomness), raising a deeper discussion of the theme addressed.

According to Bennett (1998), randomness is the property of a result that you do not know a priori what will happen, i.e., it represents the break of pattern within the sample space at stake. By contrast, *acaso* (accident/accidental) comes from the Latin *a casu* and means without cause. Thus, we perceive that the teachers' understanding is correct and reflects an evolution in relation to the answers given to question 1. This implies that the dialogue process provided teachers with reflection and they changed their initial impressions of a mathematical concept.

Regarding the third question observed in table 4: How do you assess what your student has learned about probability?

PE:	Would you add or modify....???? Or didn't I say that??? First, assessing is a difficult thing, right?
PF1:	Difficult, and there is no consensus, right... There is no consensus. But, I think PF6, he brings something interesting! Why? What is the tendency of our teaching in general? It is example and reproduction, i.e., reproducing examples!
PF6:	If he can learn the school subject as a whole and create contexts....and the resolution itself, create a situation, where there is a creation of his, do you understand? A concept already created by him! (slide).
PF1:	What is the space where the child will create it?
PF4:	True!!!Other teachers: True!

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PF1: Because from the moment he/she starts to create it, there will be doubts about where the teacher will be able to mediate the thing. Isn't it? So, when they are in this process and create resolution contexts, they create a situation where there is a creation of his/her, then there was a creation of his/her... Because then it would open the child's mind so that he/she could think...
PF6: Developing his/her reasoning and not just reproducing it.
PF4: That it's different from just playing, playing content and doing this and doing that...At this moment the teacher encourages the student to create...and from this creation, stimulate him/her to develop the concept... what did you understand about that? What do you think it is? What happened? It is not easy...BUUUUT, it's interesting. It is very interesting for the students to create, but for the student to create, they must see examples too... they must have practical examples where the teacher must guide... it is not a question of copying from the teacher, it is guidance... they must understand...
PF5: They must understand the dynamics...
PF4: You must understand the dynamics, where am I going with this? Do I want them to read something? So the teacher could challenge them...
PF1: What I believe is that teacher PF6 thought of a problematizing situation in the classroom...

Teachers raise the discussion about assessment and point out that evaluation is difficult and there is no consensus on this topic. But PF1, PF4 and PF6 point out that if the students can create contexts to apply probability, that will learn this content. This will make the students protagonists in their knowledge rather than reproducers. They raise the discussion that in school there is little room for creation and consequently for critical thinking, since students are used to reproducing teachers directions. In this third question, we noticed that teachers met what Gal (2005) classified as the knowledge element called context. The teachers' answers were classified within what Gal (2005) points out as context, given that they mention the word context and assert that students could only develop the learning of the concept if they experienced the content in different situations, from their individual or collective creation.

Regarding the dispositional elements, teachers once again developed their critical stance, which is one of the assumptions of probabilistic literacy defended by Gal (2005). Their ability to correlate the assessment in probability, with the students being able to create contexts where they will use probability critically, denotes an initial understanding of probabilistic literacy.

Final considerations

In the light of Ga's (2005) theoretical contribution, we noticed that the answers given during the semi-structured interview reflected a superficiality in the conceptual understanding of probability. Thus, the teachers started the first teacher education meeting with a very intuitive understanding of probability and its teaching. Apparently, the teachers had a very simplistic view and focused on the classic definition, i.e., the number of favourable cases over the number of possible cases.

In the first meeting, we noticed that the participating teachers were clearly following a school model, i.e., when the mathematics teacher defines a concept mathematically but does not offer conceptual reflections or possibilities for students to discuss and question. The

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participants said that they were obeying the interests of the school, the family, and the textbook. Apparently, the teacher ends up being voiceless, in the middle of a process of multiple understandings of what education and the teaching of probability should be. However, during this first meeting, with the researcher's mediation, the teachers experienced the reflective practice of questioning what was being put to them and evaluating whether it should happen that way or not. This approach is in line with Gal's (2005) probabilistic literacy perspective, which emphasises that teaching and learning processes take place through dialogue and debate that articulate aspects of teachers' and students' knowledge and dispositions.

During the first teacher education meeting, with the researcher asking the teachers about their respective answers, there was a process of reflection on what the initial and solid concepts of probability would be and how to approach them critically in relation to them. This process took place without the researcher giving ready answers, but always raising questions for teachers' reflection.

Working with teachers from the perspective of probabilistic literacy can promote educational processes in which both teachers and students are encouraged to exercise a critical and reflective citizenship, contributing to the analysis of social problems through assessments of probabilistic situations. This type of work, from the perspective of probabilistic literacy, can mobilise the subjects involved to make decisions based on the understanding of the big ideas about probability defined by Gal (2005) and, thus, solve problems more effectively in different social contexts.

This study was a contribution to a better understanding of the processes related to the teaching of probability, in an area that still has few studies and discussions in Brazil. In this sense, the uniqueness of the study gives relevance to the research but also gives rise to new aspects to be investigated. We emphasise that we do not intend to exhaust the theme of probabilistic literacy. Rather, we want to encourage reflections on the formation of the mathematics teacher of the final years of elementary school. In future articles, we will complement the results and reflections about mathematics teachers' probabilistic literacy. Such complementation will provide a reflection on pedagogical and didactic knowledge about teaching and probability and the importance of continuing teacher education.

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