ZETETIKÉ

DOI: 10.20396/zet.v29i00.8661731

Knowledge mobilized in a collaborative context about algebraic generalization in the early years

Conhecimentos mobilizados em contexto colaborativo acerca da generalização algébrica nos anos iniciais

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Abstract

In this article we discuss the professional teaching knowledge of the teachers who Teaches Mathematics for the development of algebraic thinking in the early years, especially regarding the generalization of patterns in recursive sequences, which were mobilized by professors-researchers in a study group recognized by its members as collaborative. This article is part of a qualitative longitudinal research developed at the doctoral level with a research-formation characteristic. Discussions and analyzes were based on narrative research and indicate that this education space provided a dynamic that favored interaction and mathematical communication between Future Teachers, University Teachers and Teachers who Teach Mathematics in different segments of Basic Education; the resignification of the idea of generalization of patterns in recursive sequences articulated to the questions of the teaching profession and the reflection on the participants' own practice.

Keywords: Teacher Professional Development; Mathematical Knowledge for Teaching (MTK); Development of algebraic thinking.

Resumo

Neste artigo discutimos os conhecimentos profissionais docentes do Professor que Ensina Matemática para o desenvolvimento do pensamento algébrico nos anos iniciais, sobretudo relativos à generalização de padrões em sequências recursivas, que foram mobilizados por professoras-pesquisadoras em um grupo de estudos reconhecido pelos integrantes como colaborativo. Este artigo é parte de uma pesquisa qualitativa longitudinal desenvolvida em nível de doutorado com característica de pesquisa-formação. As discussões e análises foram pautadas na Pesquisa narrativa e sinalizam que este espaço de formação oportunizou uma dinâmica que favoreceu a interação e a comunicação matemática entre Futuros Professores, Professores da Universidade e Professores que Ensinam Matemática em diferentes segmentos da Educação Básica; a ressignificação da ideia de generalização de padrões em sequências recursivas articulada às questões da profissão docente e à reflexão sobre a própria prática dos participantes.

Palavras-chave: Desenvolvimento Profissional Docente; Conhecimento Matemático para o Ensino (MTK); Desenvolvimento do pensamento algébrico.

Submetido em: 29/10/2020 – **Aceito em**: 30/11/2021 – **Publicado em**: 31/12/2021

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DOI: 10.20396/zet.v29i00.8661731

Introduction

To meet the demands that emerge today, it is necessary to have a conception of teacher education based on critical reflection in the face of different knowledge that are in continuous evolution. And the recognition that, in addition to the knowledge for subject's teaching (Shulman, 1986), there is a need for teachers to delve deeper into questions about their profession. This perspective of teacher education, called professional teacher development, is understood as a continuous learning process that starts with questions about the pedagogical practice, which contributes to a change in the way of thinking and acting of teachers, because in addition to individual reflection, teacher reflection is also conceived as a social practice (Garcia, 1999; Imbernón, 2010; Nóvoa, 2008; Zeichner, 2010; Rodrigues, Cyrino & Oliveira, 2018). In contrast to the formalist conceptions of teacher education, based on technical rationality[3], the perspective of professional teacher development recognizes the teacher as the protagonist of his own education process, thus, he himself seeks the knowledge that makes sense and brings meaning to the teaching practice and professional learning (Fiorentini & Crecci, 2013).

In the field of teacher education of Mathematics Teachers, there is a trend of investigations that consider aspects related to the professional development of these teachers, such as their professional practices, teaching knowledge, professional identity, trajectory, beliefs and conceptions. Likewise, studies about hybrid formative contexts, which provide opportunities for the exchange of knowledge between Future Teachers, Professors and in-service Teachers, emerge. They also provide the approximation between academic references and practices materialized in schools and contribute to the constitution of knowledge and the resignification of beliefs and conceptions of teachers who teach mathematics (Fiorentini, Passos & Lima, 2016). Collaborative study groups can be characterized as hybrid and alternative education contexts for the search for solutions or the understanding of situations that emerge in the dichotomies between initial education and continuing education, theory and practice, academic and school knowledge, among others. These aspects were signalized in studies by Fiorentini (2019) on collaborative work between beginning and experienced teachers. For this author, when beginning and experienced teachers collaboratively participate in common intellectual work, they raise problems, identify discrepancies between theories and practices, challenge routines, mutually support each other in the development of knowledge that make visible what is considered implicit in the teaching-learning process in mathematics classes (Fiorentini, 2019).

Studies show that teacher's knowledge is relevant for students' learning and that their initial and continuous education is significant for the improvement of their learning (Shulman, 1986; Fiorentini, 2003). In the field of Mathematics Education, studies on

[3] It is a conception of teacher education that does not consider teaching knowledge focused on the professional experience of teachers, therefore they must assume a passive posture in relation to the related choices about their learning.

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professional teacher knowledge argue that the knowledge mobilized by the Mathematics Teachers in their actions characterizes the specificity of this professional. And that this specialized knowledge can be explained in the way the Teacher who Teaches Mathematics prepares, develops and analyzes the tasks he proposes to students (Ball *et al.*, 2008; Carrillo *et al.*, 2013; Ribeiro, 2017). Therefore, it is necessary that the education spaces of Teachers who Teach Mathematics provide discussions that allow the articulation between the theoretical, epistemological and pedagogical references of the mathematical contents worked in Basic Education, and provide opportunities for social interactions, mathematical communication and exchanges of experiences among the participants, so that the specialized knowledge of these teachers is mobilized and developed (Trivilin & Ribeiro, 2015).

We argue that the Teacher who Teaches Mathematics needs to experience, throughout their education process, experiences that favor the discussion of the diverse knowledge related to the teaching of mathematical content, critical reflection on issues involving their teaching profession and their own pedagogical practice. However, in this article we focus on knowledge for teaching mathematical content, in particular, on the development of the idea of pattern generalization in recursive sequences. All information brought in the discussions comes from a qualitative and longitudinal study, at the doctoral level, developed in a study and research group recognized by its members as collaborative, called ICEM - Creative Insubordinations in Mathematics Education, linked to the Federal University of Santa Catarina.

Associated with the understanding and connection of different mathematical ideas, the algebraic thinking, when properly explored in the actions of generalizing, abstracting and formalizing, has a transversal impact on the learning of mathematics and other areas of knowledge. For this, it is necessary that the education spaces consider it as a different way of seeing and acting mathematically that supports the understanding of other concepts and favors the deepening of children's cognitive and linguistic abilities (Boavida *et al*., 2008; Canavarro, 2009).

We start from the following investigative question: How do the actions developed by teachers, during the study moments in a collaborative group, mobilize knowledge for the development of algebraic thinking in the early years? Considering the space limitation, we chose for this article to bring the discussion of the teachers participating in the ICEM, in the study of one of the texts chosen by the group to support the discussions about the generalization of patterns in recursive sequences with the use of exploratory tasks. The information provided was guided by the contributions of Narrative Research, according to Clandinin and Connelly (2011) as we understand that its assumptions are aligned with the qualitative approach, longitudinal studies and research on the practice itself.

Discussions about the knowledge mobilized for the teaching of mathematical content were based on the model proposed by Ball *et al*. (2008) of Mathematical Knowledge for Teaching, reorganized from the knowledge that constitutes the basis for teaching (Shulman, 1986). The choice for this reference is linked to these authors' research about the knowledge

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mobilized by teachers *in* and *for* teaching. This concern with understanding the knowledge in pedagogical practice transcends the conception of the preponderance of academic knowledge in a given area for the teaching of this subject in Basic Education and is in line with the assumptions that underlie the actions of ICEM group members.

With this article, we intend to share our experience and contribute to fostering discussions about the development of algebraic thinking in early years students and Specialized Knowledge of Mathematical Content Ball *et al*. (2008), especially with regard to generalization in recursive sequences.

Knowledge to teach mathematics

According to Shulman (1986), teachers need to demonstrate in the teaching activity a set of understandings, skills, knowledge and motivations necessary for this profession. This knowledge necessary for teaching derives from a body of knowledge acquired during the academic, professional and personal trajectory, which he calls the knowledge base for teaching[4]. For the author, this knowledge provides the recognition of the specificity of teaching practice and has intrinsic aspects that constitute it through the interrelation of knowledge in the specific area and pedagogical knowledge to teach it. Such knowledge is possible to identify, although difficult to be explained and theorized by the teachers themselves. For Shulman (1986) this content knowledge from different sources can be gathered into three categories of knowledge, which he called Specific Content Knowledge, Pedagogical Content Knowledge and Curriculum Content Knowledge.

The model proposed by Shulman (1986) refers, in general, to the knowledge necessary to teach, but without focusing on a certain area. From the perspective of Mathematics Education, Ball *et al*. (2008) developed the Mathematical Knowledge for Teaching (MKT) model, which proposes a reorganization of the knowledge categorized by Shulman, from the perspective of Mathematical Knowledge for Teaching. Thus, based on Shulman's approach (1986), they divided the knowledge for teaching mathematics into two domains: the domain of Content Knowledge (CK) and the domain of Pedagogical Content Knowledge (PCK). In the Content Knowledge (CK) domain, there are three subdomains. The Horizon Content Knowledge (HCK) subdomain[5], which is not used directly at the level the teacher will teach, corresponds to a more advanced knowledge of the subject, but it supports the teacher in the mediations and counterpoints that are considered important for emphasize what is mathematically relevant and broaden connections about the taught content. The other two subdomains of the Content Knowledge (CK) can be more easily perceived in the teaching practice. The Common Content Knowledge (CCK)[6], which is related to "know-how", is also used in other contexts by different professionals, such as the mathematical knowledge which an engineer uses to calculate the load that a concrete beam will support.

[4] Knowledge Base for Teaching

[5] Horizon Content Knowledge.

[6] Common Content Knowledge.

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And the subdomain of Specialized Content Knowledge (SCK)[7], which according to the authors, is typical of teachers when exercising the action of teaching mathematical content and can be perceived in the teaching practice of these professionals when bringing different representations of a mathematical concept or by understanding the properties that support a given resolution procedure. This knowledge is associated with the knowledge that assigns meaning and seeks to interpret students' solving strategies, especially those discussed in their class or foreseen in their planning. Thus, Specialized Content Knowledge (SCK) is associated with understanding of the possible causes that lead students to incorrect resolution strategies, in other words, the interpretation of the nature of errors in students' responses. This knowledge is called Interpretive Knowledge (IK).

In the Pedagogical Content Knowledge (PCK), of the (MKT) model, three subdomains were also allocated. The subdomain of Knowledge of Content and Students (KCS)[8], associated with the identification of difficulties in learning the taught content, signaled in other classes that have the same level of education and can help the teacher to anticipate the aspects that need more attention and to make mediations that will mobilize the learning of this content. The subdomain of Knowledge of Content and Teaching (KCT)[9] refers to the knowledge that the teacher mobilizes to plan his pedagogical approach, to make choices of materials that will support the explanations and the tasks that will be proposed to the students. And Knowledge of Content and Curriculum (KCC)[10], is related to the teacher's recognition of the distribution of mathematical objects and their connections in the mathematics curriculum according to the official documents and the learning objectives of these contents for each year throughout of Basic Education. This knowledge allows the teacher to anticipate or relate the taught subject to ideas, properties, or concepts covered in previous or later years by students in their class.

According to what Ball *et al.* (2008) proposes, we understand that, in relation to the contents of algebraic thinking in the early years, the Mathematical Common Content Knowledge (CCK) is related to the understanding of concepts and to the recognition of the processes that develop the idea of generalization in repetitive and recursive sequences and to the solving process of arithmetic expressions. To Specialized Content Knowledge (SCK) is concerned with understanding why a certain property is used or not, and how to explain this understanding to students, so that they understand the concepts and make connections with other ideas already consolidated. This understanding is important for: the teacher to make choices about the anticipations and concepts that are considered necessary for the recognition of the motive and the identification of terms that are distant from repetitive sequences; to recognize the part that varies or is invariant of close terms; to identify the formation law of recursive sequences; and choose the approach and resources that will be used in order to

[7]Specialized Content Knowledge.

[8]Knowledge Content and students.

[9]Knowledge Content and teaching.

[10]Knowledge Content and curriculum.

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promote the learning of these mathematical objects by students, which are associated with Knowledge of the Content and Teaching (KCT). Knowledge of Content and Students (KCS) includes knowledge of probable solving strategies that can be used by students in the class, as well as possible obstacles that may eventually emerge for the understanding of these objects with the use of exploratory tasks with students who are in classes that are close or in the same grade. And the Knowledge of Content and Curriculum (KCC) is associated with the teacher's recognition of the BNCC's (Brasil, 2017) conception, for the work of the Algebra axis contents in the early years, with the guidelines to develop the idea of generalization in this segment and the distribution of the contents of the Algebra Axis throughout Basic Education.

Among the approaches associated with Specialized Content Knowledge, we point out the perspective of Carrillo *et al*. (2013), which considers as specialized all the Knowledge of the Teacher who teaches Mathematics. The conceptualization of MTSK[11] proposed by Carrillo *et al*. (2013), considers that the three subdomains of the Content Knowledge (CK) domain are constituents of Interpretive Knowledge (IK). We comprehend that this understanding expands the comprehension proposed by Ball *et al*., (2008), that Interpretive Knowledge (IK) is associated only with the subdomain of Specialized Content Knowledge (SCK). Another aspect signaled in the (MKTS) model is that (Ball *et al*., 2008) do not consider the beliefs of Future Teachers and Teachers who Teach Mathematics. In the MTKS model, they are considered, because in the perspective of Carrillo *et al*. (2013), the beliefs and knowledge of these professionals affect the learning of the subjects, even if they are not related to mathematical questions, such as class management (Carrillo *et al.*, 2013). In relation to this proposition, we understand that the studies of Ball *et al*. (2008) consider in their research the mobilization of Mathematical Knowledge *for* and *in* teaching. Therefore, they are concerned with the understanding of mathematical contents and the discussion of resolution strategies in mathematics classes aiming to interpret the development of students' reasoning, the expansion of ideas about the taught object, the beliefs and conceptions about mathematics and the ability of these authors to learn mathematics, and similarly, in the contexts along the lines of ICEM, in which knowledge is understood as a collective production that is gradually constituted by the exchange of ideas, discussions and reflections between people and the cultural artifacts produced by them in a given time and space.

Development of Algebraic Thinking in the Early Years

Blanton and Kaput (2005), pioneer researchers of the *Early Algebra*[12] recognize that the development of algebraic thinking is associated with the process by which students generalize mathematical ideas from particular situations. For these authors, these generalizations can be demonstrated initially by oral language through the argumentation of their own ideas and gradually by more formal expressions during the educational process.

[11]Mathematic Teachers Specialized Knowledge.

[12]Term used by some authors of Mathematics Education to designate the field that studies the contents of Algebra in the early years.

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This understanding is corroborated by researchers in this field (Radford, 2006; Canavarro, 2009), who understand that in this approach, generalization is considered the core of the development of algebraic thinking, because as students, from particular situations, recognize the existence of relations between structures, terms and procedures to identify and explain what is common, they expand their reasoning and communication skills.

The BNCC (Brasil, 2017) indicates some dimensions of work with Algebra to be explored and developed in the Early Years of Elementary School. These are: the study of regularities; the generalization of patterns; and the properties of equality. And argues that the contents in the early years of this unit should be worked from the perspective of presenting generalization ideas using varied ways of expressing these regularities, without resorting to the use of letters (Brasil, 2017).

According to Vale *et al*. (2011, p. 9) definition, “we use the term pattern to refer to a placement or arrangement of numbers, shapes, colors or sounds where regularities are detected”. Recursive sequences have a recursive relationship, which makes it possible to identify variant and invariant aspects of a term to the next term and, therefore, calculate close terms within a sequence (Fiorentini, 2003). According to Van de Walle (2009, p. 300) the “description that tells how a pattern is modified from one step to the next is known as a recursive relation”. In Stacey's (1989) perspective, generalizations can be classified as close and distant. The "close generalization" occurs when the student chooses as a strategy to identify the terms requested in a certain sequence that involves patterns, drawing, writing or counting the terms "one by one" or "step by step", or using a table to record what varies from one term to the next, that is, looking at the recursive relation that is happening along the sequence. Meanwhile, the “distant generalization” occurs when the student elaborates a rule or law of formation, which makes it possible to identify the characterization of any term in the sequence. Vale *et al*. (2011) also conceive that generalizations can be understood as close or distant. However, Rardford (2006) uses the term arithmetic generalization for the strategy of counting or drawing “step by step”, and the expression “algebraic generalization” to refer to the distant generalization.

Research context and trajectory

ICEM is a community recognized by its members for having a collaborative dimension, based on the references of creative insubordination, which is concerned with learning mathematics in Basic Education, especially in the early years. ICEM aims both to investigate of school pedagogical practices and to train its participants. The theoretical and methodological assumptions of the studies developed in the ICEM group are aligned with the perspective of teacher professional development and with the research references that consider the teacher's reflection *for* and *in* their own practice. In the context of Mathematics Education, creative insubordination is associated with the creative practices or acts of teachers in education that seek to alleviate the oppression of the profession and the improvement of student learning, even if these actions are contrary to the culture and pre-

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established norms of the school context (D'Ambrosio & Lopes, 2015).

Throughout 2019, teachers in education at ICEM, decided to elaborate, develop and discuss class narratives with audio and video excerpts from the stage of development of exploratory tasks according to the approach defended by Canavarro (2009) discussed by the group. These discussions, agreements and negotiations at ICEM gradually consolidated the collaborative dimension of this context and brought evidence that our investigation was being characterized as education research in a context between equals (Nóvoa, 2008; Imbernón, 2009), because for all of us, teachers in education, participating in ICEM was an opportunity to learn and share knowledge.

We participated in the group meetings, held in the second semester of 2018, in the year 2019, and in the first semester of 2020, which characterizes this as a longitudinal study, with the objective of recording through a logbook, videos and audios, the discussions about the listed mathematical contents, the materialized pedagogical practices and the resignifications that emerged in this context. Below, we present a synthesis of the activities that were developed in the group in 2019. We emphasize that in all activities there were movements of study, discussion and reflection of theoretical references, however we also emphasize our analyzes at the meeting held on 04/24/2019, especially in the study stage of the references in Chart 1 below.

Chart 1 - Meetings and activities developed by teachers in the collaborative study group in 2019.

Dates of meetings	Activities carried out in the group
2/13 3/7	Presentation; definition of objectives, selection of references for the study of Algebraic Thinking and exploratory tasks.
4/10 4/24 5/22 5/08	Resumption of the references of Creative Insubordination and the research of one's own practice for new members; Study of algebraic thinking references and exploratory teaching.
5/29 6/5	Elaboration of exploratory tasks for generalization in repetitive and recursive sequences, and on the different uses of the equal sign; Preparation of papers and presentation at events in the area.
6/19 6/26 7/3 7/17	Discussion on the use of class narratives and audio and video excerpts from the development stage (Canavarro, 2009) of the exploratory tasks developed at ICEM to support the group discussions.
8/7	Welcoming new members and resuming/reviewing the references studied in the 1st semester/2019
8/21	Discussion and finalization of task sequences to be developed in the early years' classes of teachers in the ICEM group.

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9/11 9/18	Organization of a scientific event at the university; Systematization of points to guide the selective attention[13] of the group's teachers in the discussion of class narratives.
11/13 11/20 11/27 12/4 12/11 12/18	Development of exploratory tasks with students, elaboration and discussion about class narratives in the group.

Source: The authors 2020.

In relation to the professors who collaborated in this research and who were members of ICEM, we recorded the presence of 23 members since the group was formed. However, some came to the meetings occasionally, as participation in this space was voluntary. Among the members who regularly attended the meetings, two of them are undergraduate students in Mathematics; one is an academic in Pedagogy; six are graduate students in the Mathematics Education’s area; four are professors who teach Mathematics; and seven are teachers who teach Mathematics in Basic Education, three of them in the early years. In this text, the names used to represent the participants of the group do not correspond to their real names and the investigation was approved by the Research Ethics Committee14 of the Federal University of Santa Catarina.

To support the systematization and analysis of the information in this text, we used narrative research from the perspective of Clandinin and Connelly (2011), the same approach used to analyze the constructs throughout the longitudinal study. Narrative Research conceives that education is intrinsically related to experience and life. According to the authors, to think narratively about the phenomenon and compose research texts, researchers need to be in the field of investigation, because from this perspective, research is a way of understanding and questioning experience through collaboration between researchers and participants over time, and in social interactions (Clandinin & Connelly, 2011).

The texts selected by the group's teachers to discuss algebraic thinking were shared *online* among the members and in addition brought theoretical references and learning experiences of Future Teachers, Professors and Teachers who Teach Mathematics from other communities or study groups. To make the readings and discussions more dynamic, the teachers first performed an individual reading and signaled what would be discussed collectively. To record the dialogic interactions of the group, which subsidized the production of data for this text, we recorded the aforementioned meeting and transcribed the audio and video of the discussions. For the analysis of the information, we made several readings and the collation of speeches with the teachers' reflections and their interpretation in the

[13] The expression “selective attention” was used at ICEM with the same understanding that Future Teachers used to discuss the knowledge mobilized by the teacher and students in the multimedia case presented in Rodrigues, Cyrino and Oliveira (2018).

14 CAAE: 13727819.5.0000.0121; Opinion Number: 3,397,17.

DOI: 10.20396/zet.v29i00.8661731

perspective of the theoretical references of the knowledge base for teaching (Shulman, 1986); of Mathematical Knowledge for Teaching (Ball *et al*., 2008), in particular the aspects that refer to Interpretive Knowledge (IK) associated with Specialized Content Knowledge (SCK) (Carrillo, 2013; Ribeiro, 2017).

Knowledge mobilized for the development of the pattern generalization idea in recursive sequences

In the ICEM group, in the study of references about the development of algebraic thinking, we found the following three interconnected study themes: 1) the use of the equal sign with the idea of equivalence; 2) the recognition of patterns and motifs in repetitive sequences; and 3) the identification of terms, close and distant generalization in recursive sequences. However, the focus of this article consists in analyzing how Mathematical Knowledge for Teaching was mobilized in the study of patterns in recursive sequences, we briefly describe below some aspects about the study of generalization in recursive sequences, because we understand that they contribute to introduce the generalization of patterns in recursive sequences.

In the continuity, the group studied the contents related to the exploration of patterns and generalization in repetitive and recursive sequences presented in the book of Van de Walle (2009). According to this author, activities, in figurative contexts, about repetitive patterns of growth and decrease, provide a diversity of situations where the teacher can make rich and varied explorations together with the students to characterize the motif or group of repetitive sequences, the recursive processes used to find the terms close to a sequence and the law of formation that allows identifying any distant term, even if it does not have the previous term as a parameter in a recursive sequence.

To study the concepts of pattern, motif and repetitive sequences, teachers used logic blocks. Each pair would start a sequence and the other members would identify the motif for the sequence created. In the course of the sequences constructed by the pairs, the number of elements of the repetition groups and the complexity of the arrangements of these groups increased. This contributed to the understanding that the complexity of patterns depends on the number of motif elements and the typology of repetitions present, as indicated in Van de Walle (2009). Next, among the texts studied by teachers in education at ICEM to support the questions that emerged about the identification of terms, close and distant generalization, in recursive sequences, we bring the article "Algebraic thinking and the discovery of patterns in teacher education” (Vale & Pimentel, 2013). This text presents a discussion about the importance of patterns for the development of algebraic thinking, through an experience of a third-year teacher in the development of a didactic sequence. However, the authors indicate that this activity can be developed with the necessary adaptations in other classes of the early years. The article begins with a theoretical foundation focused on teacher education, related to the teaching of algebra in the perspective of developing algebraic thinking from the exploration of patterns. Next, the authors present a didactic proposal in order to support the

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approach of tasks with patterns, aiming at the development of algebraic thinking. And they argue that if we teachers intend to develop the ability to think and solve students' problems, we need to provide them with tasks that do not limit them to the mere application of procedures, but that favor the establishment of connections and communication opportunities (Vale & Pimentel, 2013).

Throughout the text, the authors present a task, characterized by the authors of level two, developed with third grade in the early years, and had as an objective the determination of the number of stars of any term of a recursive sequence, as shown in Figure 1 next.

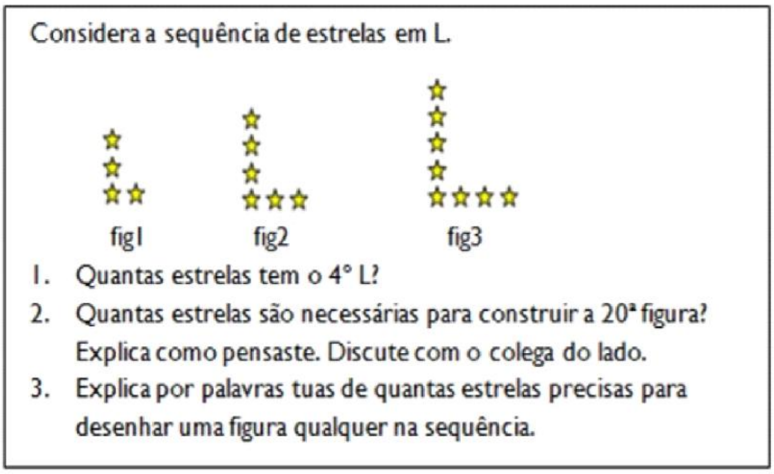

Figure 1 - Task on sequences that supported the group discussions.
Source: Vale e Pimentel (2013, p.111).

The respective task, according to the text, aimed at pattern recognition and generalization through rules formulated by the children themselves, using symbology or not. According to the authors, usually in this type of task, students perform a numerical conversion to identify some regularity or pattern between the figures. And, they point out that they can easily discover the rule from the recurrence, that is, each term is obtained by adding two units to the previous term. This would be a close generalization, or according to Radford (2006), arithmetic. The challenge lies in the identification of distant terms, from the elaboration of a rule without having to resort to the previous term, called by the authors as distant, or also as algebraic generalization (Radford, 2006). Therefore, the authors recommend that the teacher builds a table together with the students on the board (Figure 2), based on the observation of what is invariant in the structure of each figure. The construction of the table may help elementary students to understand what varies and what is invariant in the subsequent terms and in solving the proposed challenge.

From the perspective of the development of Knowledge for Mathematics Teaching, we understand that in addition to the Common Content Knowledge (CCK), the Teacher who teaches Mathematics needs to evidence in the teaching practices, for the teaching of mathematical content, the other subdomains of the Conten Knowledge (CK) and the MKT Pedagogical Content Knowledge (PCK) proposed by Ball *et al*. (2008), from the knowledge base for teaching (Shulman, 1986). And, in particular, what gives meaning to students'

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solving strategies, the recognized as Interpretive Knowledge (IK), associated with Specialized Content Knowledge (SCK).

When we read this part of the text, teacher Marcelo[15] suggested that we could try to identify other possibilities to solve the challenge proposed by the authors, that is, to determine any distant term, without having to resort to the previous terms. So, teacher Sophia, suggested that each pair could be constituted by teachers who worked in the same segment of Basic Education, or by undergraduates, so the possibilities brought would be more consistent with the segment in which these teachers would be teaching and could "think" in a different way, according to the knowledge that their students had already accessed. Teacher Vera added that this organization would also provide more opportunities for exchanges between the participants of each pair and then we could compare whether there were similarities or differences between the strategies presented by the different compositions (early years, final years, undergraduates and professors). The teachers present felt challenged and grouped themselves in pairs or trios, with the challenge of making a generalization and elaborating a law of formation different from that presented by the authors Vale e Pimentel (2013).

After a few minutes, a pair, made up of specialist teachers, who teach in the final years of Elementary School and High School, asked if they could share their strategy. The other pairs indicated yes, and then, teacher Marcelo began to expose the pair's strategy orally, but there was difficulty in understanding, so teacher Kátia, one of the early years' teachers, commented: *Orally, I am not able to understand your strategy. I cannot understand it this way.* So there was a consensus among the group that it was necessary to present on the board the strategies found. When teacher Marcelo went to the board, wrote and explained the way he and his colleague, teacher Rose, thought. In this situation, the participants experienced the students' difficulties in understanding such strategies, without the proper systematization of ideas and realizing different hypotheses for solving the same task. And we realized that this clash of professional realities made the mathematics teacher, Marcelo, who was presenting the first strategy, used to teaching Algebra in the final years of Elementary and High School, reflect about the importance of the register so that his ideas could be understood. This is related according to the model proposed by Ball *et al.*, (2008), to Knowledge of Content and Teaching (KCT) and Knowledge of Content and Students (KCS). And that the teacher of the Early Years, Vera, who although has knowledge about Algebra, was not used to teaching it, could deepen and expand the ideas associated with the Common Content Knowledge (CCK) which is related to "knowing how to do". Then, one by one of the strategies were demonstrated on the board by the pairs. This dynamic was important so that we could experience the students' difficulties when trying to understand the teachers' strategies or explanations, when they are not accompanied by the proper systematization of ideas on the board. Another aspect that was very evident in the comments made in the group when the pairs' hypotheses were shared, was the perception of the possibility of different solving

[15]The names used do not correspond to the real names of the teachers participating in ICEM.

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strategies for the proposed task, which at first seemed to have only one solution.

We systematize in Table 1, below, the different algebraic expressions discussed in the group from the socialization of the strategies developed by the pairs. And, following, we bring the details of each of these strategies validated by ICEM teachers. Each of the pairs as we will present, was made up of teachers with different backgrounds and performances.

Table 1 - Solving strategies developed by teachers.

Pair	Strategies represented by algebraic expression
1	2+2.n
2	2. (n + 1)
3	$(n+1)^2 - (n^2) + 1$
4	(n+1). (n +2) - [(n(n +1)]
5	4 + 2. (n - 1)

Source: The authors, 2020.

The **pair number 1**, composed of Elementary School II teachers, set the number 2 as a solving strategy, as an invariant part of all figures, being the two stars in the column, as illustrated in Figure 2, below.

Algebraic expression: 2 + 2.n

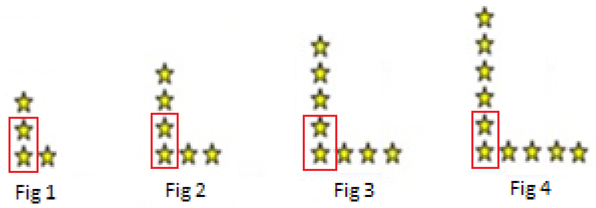

Figure 2 - Solving strategy of the pair 1.
Source: The authors 2020.

When considering the two fixed stars, the pair showed the group that there was a correspondence between the elements that varied, both in the row and in the column, and the position number of the figure. That is, the number of remaining stars, in the row and in the column, increased successively, characterizing a growth pattern, and was corresponding to the number of the figure in the sequence. Therefore, the part that varies is always twice the figure number. Which corresponds to the algebraic expression 2+2.n.

The pair also commented that they had chosen this strategy because they understood that such an expression would be more accessible to third-year children, since by selecting the two stars in the first column as the invariant part, the perception that the other stars remainders of the row and column correspond to the order of the term in the sequence becomes more evident. According to Rose, one of the teachers of the pair, "*the children would notice that in the first term, in addition to the two selected stars, there is one more star left in the column and in the row. While in the second figure there are two stars left in the column and another two in the row, and so on. Then, the children would easily perceive the correspondence between the figure number and the number of stars that should appear in the*

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row and column of that term." (Audio Transcript, 04/24/2019).

In teacher Rose's argument, the movement of "trying" to put herself in the place of students in the early years is evident in order to interpret possible reasons for choosing their solving strategies. This concern explained by the teacher, evidences the mobilization of Interpretive Knowledge (IK). This knowledge is associated with the teacher's action *for* and *in* teaching mathematics, and is considered one of the knowledge that makes up the Specialized Content Knowledge (SCK), as it is characteristic of the teacher who teaches mathematics (Ball *et al*, 2008).

In the excerpt with the explanation by teacher Rose, we can still see evidence of the mobilization of the three knowledge of the Pedagogical Content Knowledge (PCK) domain, because when trying to develop a strategy thinking about the third-year students, the teachers of this pair demonstrate to recognize how the school mathematics is organized in the early years' curriculum and this perception is associated with Knowledge of Content and Curriculum (KCC); when evaluating the gains of using the two fixed stars in the first column, the teachers show evidence of Knowledge of Content and Teaching (KCT), and, finally, evidence of the mobilization of Knowledge of Content and Student (KCS) when anticipating that the children could notice after fixing the two stars in the first column, that the other elements that vary both in the row and in the column increase successively according to the figure number.

It also allows us to infer that the exchanges with the teachers of the early years in this education space favored that such solving strategies were thought and considered by this pair, made up of teachers who teach in the final years and in High School of Basic Education.

However, when listening to teacher Rose's argument, the teachers who teach in the early years stated that probably, the children of the third, fourth and fifth years would not identify as the invariant part, only the first two stars of the column. In the teachers' understanding, the children would select all the stars of the first figure and then check what would be varying in the next ones. In this positioning of the early years' teachers, the appropriation and mobilization of Knowledge of Common Content (CCK) and of Knowledge of Content and Student (KCS) can be seen, as they can safely anticipate the possible strategy to be used by the children. In the conceptualization of Ball, *et al*. (2008), these perceptions are related to the knowledge of students' learning expectations, according to the year they are attending, the solving hypotheses, which probably emerge in that year's classes, and the possible obstacles that may eventually be constituted in the teaching process of a given content. And, they corroborate the evidences indicated in the studies developed by Ball, *et al*. (2008) on the exchange of experiences between mathematics teachers from different segments and modalities of Basic Education. The research of these authors points out that Specialized Content Knowledge (SCK), mobilized *for* and *in the* teaching of mathematics, can be developed and shared by teachers. And the research on hybrid formative contexts, for these education contexts that provide opportunities for the constitution of non-formal relations, favor the expansion of dialogic interactions between teachers, and consequently, the

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mobilization and development of knowledge for teaching through discussions about the teaching action. These perceptions have been considered in studies in the field of Mathematics Education in education spaces constituted by beginning and experienced teachers. And, they signaled that the exchanges between these teachers, whether at the beginning or at another stage of the teaching profession, contribute to the awareness that the teaching knowledge of the Teacher who Teaches Mathematics has an impact on the students' learning of mathematics (Fiorentini, 2019).

The pair, which we identified by the number 2, was composed of postgraduate students and teachers of Elementary School II. And it presented another reasoning, different from the one previously demonstrated, as we can see Figure 3, below.

Algebraic Expression: 2. (n + 1)

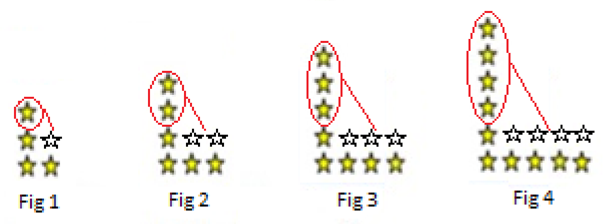

Figure 3 - Solving strategy of pair 2.
Source: The authors 2020.

This pair considered that the number of stars in the first row corresponds to the figure number plus one star, and that from the second figure it is possible to obtain a rectangular distribution by adding to the second row the number of stars corresponding to the figure number. Another question that the pair pointed out was that the number of stars to be added in the second row corresponds to the number of stars left in the first column. The algebraic expression 2. (n + 1) is equivalent to the expression 2 + 2.n found by the pair 1. Thus, due to the similarity with the expression of the previous pair, this strategy at first did not generate many discussions in the group. But in the course of socialization, the teachers understood that this perception brought by the pair, which shows that from the second figure on the configuration of the stars of the two rows has a rectangular representation, expanded the geometric perception and the recognition that the development of geometric thinking contributes to expand mathematical ideas, and that it is important that teachers who teach mathematics work on the development of this thinking together with the contents of algebraic and arithmetic thinking. This perception shows evidence of the mobilization of Horizon Content of Knowledge (HCK), (Ball *et al.*, 2008).

In turn, the strategy developed by the pair number 3, presented by the undergraduate students Pedro and Rodrigo, academics of the degree in mathematics, was considered the “most complex” by the teachers of the group, for having in its expression a remarkable product and this subject is not present in the curriculum of the early years. The undergraduates mentioned that they started from the geometric solution represented in Figure 4, below, to arrive at the algebraic expression $(n+1)^2 - (n^2) + 1$, according to Table 1.

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Algebraic Expression: $(n+1)^2 - (n^2) + 1$

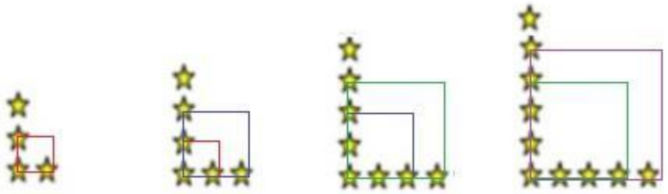

Figure 4 - Strategy resolution of pair 3.
Source: The authors 2020.

They considered two quadrangular distributions from the last two stars in the row of one of the figures in the sequence. The regularity perceived by the undergraduates was that the basis of the quadrangular distribution of the internal figure, from figure two, corresponds to the number of the figure in the sequence, and that subtracting the area of the internal quadrangular distribution from the external one, from any term in the sequence, with the exception of the first, there are always three stars left, two stars in the column and one star in the row of this figure. In this way, it would be possible to identify the number of stars of any term of the sequence without having to resort to the previous figure.

This strategy generated the need for further explanation. The undergraduates had to re-explain several times the way they thought to make their strategy accessible to the teachers in the group, and the more experienced teachers had to help, so that the other colleagues could understand the strategy of this pair. Some teachers suggested the *geometric demonstration* on the board to facilitate the understanding of this strategy by the group. This is in line with what was signaled in the studies carried out by Rodrigues, Cyrino and Oliveira (2018), that Future Teachers, compared to More Experienced Teachers, are starting the process of building knowledge to explain and predict how information will be assimilated by the students. This knowledge is related to Specialized Content Knowledge (SCK). The knowledge of this subdomain is made explicit in the teaching practices of the teacher who teaches mathematics and is increasingly constituted and developed throughout teaching. And it has repercussions on the actions focused on the choice of pedagogical approach and resources chosen by the teacher to promote the learning of the taught content, mobilizing the Knowledge of Content and Teaching (KCT), and, consequently the Knowledge of Content and Students (KCS), and the Knowledge of Content and Curriculum (KCC), (Ball *et al*, 2008). Therefore, in our understanding, we agree with the broader perspective indicated in the (MKST) and in other studies about teacher professional development in the context of Mathematics Education, which shows that all the Knowledge of the Teacher who teaches Mathematics is Specialized, and that the Knowledge Interpretive (IK) that characterizes the specificity of this professional is associated with the three subdomains of Content Knowledge (CK) of the MKTS (Carrillo *et al.*, 2013; Ribeiro, 2017).

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However, although academics and other members of the group recognized that the algebraic expression of this solving hypothesis would be complex for students in the early years, they considered its geometric representation accessible and important to promote discussion with children. The development of this task in the group allowed the undergraduates to experience the practice of teaching and making students understand what is taught, and to reflect on different ways to explain, argue and expose their strategies. The discussion about the recognition of the geometric distribution as a support for the arithmetic formulation and algebraic generalization of the pattern, shows the articulation of different fields of mathematics (arithmetic, algebra, geometry), expanding the understanding of mathematical objects and concepts by teachers in formation at ICEM, and indicates that the undergraduates, when elaborating this strategy, mobilized the Common Content Knowledge (CCK) associated with Algebra and the algebraic language, but not for teaching. This knowledge is not specific to the teacher, it can even be used by other professionals. And by making connections between the content of patterns in recursive sequences, with more elaborate concepts of Algebra itself covered in the final years of Elementary School and concepts of Geometry, the undergraduates showed evidence of the mobilization of Horizon of Content Knowledge (HCK), (Ball *et al.*, 2008). Regarding the difficulty indicated by the undergraduates in explaining their strategy in a way that other teachers could understand, and the contribution of more experienced teachers to the understanding of this strategy, we comprehend that there is evidence that confirm that Specialized Content Knowledge (SCK) is constituted and developed in the teaching action (Ball *et al.*, 2008).

The pair 4, composed of professors from the university, started from the geometric solution, shown in Figure 5, below.

Algebraic Expression: (n +1). (n +2) - [(n(n +1)]

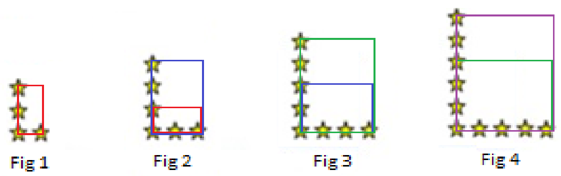

Figure 5 - Solving strategy of pair 4.
Source: The authors 2020.

This pair realized that by completing the missing stars in the figure, it would be possible to form a rectangular distribution of stars outside and another rectangular distribution inside. The subtraction between the amounts of stars in the two rectangular distributions would be equivalent to the number of stars in the figure. Therefore, it is necessary to consider that the number of stars in the row is always one more than the figure number (n + 1), while the number of stars in the column is always two more than the figure number (n + 2), finding out the algebraic expression (n +1). (n+2) – [(n(n+1)].

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To set up the algebraic expression, the pair emphasized that it is necessary to consider that the number of stars in the row is always one more than the number in the figure (n + 1), while the number of stars in the column is always two more than the figure number (n + 2), finding the algebraic expression (n +1). (n+2) – [(n(n+1)].

The pair itself, when presenting, explained that this strategy would not be a solution found by children in the early years, it would be more present within the scope of students in the final years. However, geometrically it would be interesting for children to perceive other possibilities of resolution. These considerations show evidences of the three knowledge mobilization of the domain of Specific Content Knowledge (SCK): the Common Knowledge of Content (CCK), which is associated with the use of algebraic language knowledge; the Horizon of Content Knowledge (HCK), because there is an appropriation of mathematical knowledge beyond the content discussed, considering that the pair made relations between this content and the development or expansion of algebraic and geometric thinking and with other topics in mathematics, and signaled knowing how to “situate this concept” at other times and make connections with other mathematics concepts that will be addressed in later years; and Specialized Content Knowledge (SCK) when they realized that, even though it is not a solution to be found by children, this strategy contributes to the identification and recognition of the recursive relation between the sequence and the expansion of the mathematical ideas involved. And also, that the possibility of expanding the discussion from the geometric representation of the recursive relation, may have repercussions on the understanding of other contents covered in the mathematics curriculum. This perception is associated with “Specialized” Content Knowledge, because this is not needed for purposes other than teaching. And its mobilization involves knowing mathematics in detail, understanding the “whys” of a procedure, the importance of different strategies and interpretations for solving a problem situation, and the possible gains for the understanding and connections of the concepts from them.

The considerations brought by this pair, led the group to discuss that perhaps seventh, eighth or ninth grade students would be able to reach algebraic generalization. This discussion shows evidence of the mobilization of Knowledge of Content and Curriculum (KCC). This knowledge is related to the recognition of the mathematical contents' distribution in school programs and allows the teachers to anticipate which contents of the course the students of their classes have had access or should have had access, and in which later years of Basic Education the content they will teach will be approached or used for new conceptual appropriations, according to official documents such as the BNCC (Brasil, 2017).

The pair number 5, composed of teachers from the early years, as we mentioned in the discussions of the first pair, considered that the children would fix the first figure and try to see what varied in the next terms. Thus, according to this understanding, they would realize that each term would increase in the row and column a number of stars corresponding to a number less than the figure number, as shown in Figure 6 below.

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Algebraic Expression: 4 + 2. (n - 1)

Figure 6 - Solving strategy of pair 5.
Source: The authors 2020.

The teachers of this pair showed in their arguments evidence of the mobilization of Common Content Knowledge (CCK) - which is associated with the knowledge of the algebraic language - and expanded the discussion in the collective bringing the knowledge of their experiences as teachers of this level of teaching. They argue that this would be the strategy that children would tend to use, as they would probably utilize the strategy of “stamping” the first figure on the others to identify what would vary or be invariant in the second figure, then in the third, in the fourth, and so on successively. However, the teachers of the present early years recognized that the algebraic expression 4 + 2. (n - 1) would not be the "most accessible" to children, as it is a more elaborate expression compared to the expressions presented by pairs 1 and 2, but even with the need to make use of addition and multiplication, and probably greater pedagogical intervention, this would be the most likely strategy that children would develop. This argument from the early years shows evidence of the mobilization of Common Content Knowledge (CCK), which is associated with the knowledge of the use of algebraic language and expanded discussion in the collective by sharing the knowledge of their experiences as teachers of this level of education.

In the movement to verify the most accessible strategy to students, the group realized that even the strategy that had the most complex algebraic expression, (n +1) . (n +2) – [(n(n +1)], had a geometric representation, which seemed to be possible to be built by children. In this way, the teachers understood that it was important to promote the collective socialization of the strategies created by the children, but also to provide opportunities for other strategies different from those they bring. Because the movement of trying to elaborate explanations for their understanding, contributes to the expansion of mathematical ideas. This reflection evidences the understanding by teachers in formation at ICEM, that the connection of mathematical content is importante to the expansion of students' ideas.

This movement of elaboration of strategies in pairs, systematization and validation of the hypotheses in the collective contributed for the ICEM teachers to experience the stages of the exploratory teaching approach proposed by Canavarro (2009). This experience provided an opportunity to reflect on the importance of communicating students' mathematical ideas in their classes. Another aspect that the group noticed was the need for the teacher to know in advance the students' strategies to organize the order of their socialization on the board, and

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thus, to anticipate mediations that allow the development of ideas that may emerge in the discussions, in order to enhance understanding of the content by the class. These actions help students understand that the "error" is part of the learning process, and is a constituent part of the solving process, in addition to promoting dialogic interactions and students' participation in these moments of socialization and discussion in math classes. These understandings show the mobilization of evidence of Interpretive Knowledge (IK), which is associated with Specialized Content Knowledge (SCK) in the conceptualization of Ball *et al*. (2008). And the recognition of the geometric distribution as a support for the arithmetic formulation and algebraic generalization of the pattern, evidences the group's concern with the articulation of different fields of mathematics (arithmetic, algebra, geometry), and this allowed the expansion and understanding of mathematical objects and concepts, as well as evidenced the mobilization of knowledge in the Content-Specific Domain (CK), (Ball *et al.*, 2008).

Final Considerations

In this study we sought to analyze how the knowledge for mathematics teaching was mobilized in moments of studies of a collaborative group, about the development of Algebraic Thinking in the early years. Excerpts, discussions and analyzes about the knowledge mobilized by the teachers in formation at the moments of studies, show evidence of knowledge in the domain of Content Knowledge (CK), and of the domain of Pedagogical Content Knowledge (PCK). For example, when they are concerned with explaining their strategies in a way that is accessible to children, they show evidence of the mobilization of Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), Knowledge of Content and Teaching (KCT), and Knowledge of Content Student (KCS). And, when teachers establish connections with other mathematical content and recognize in which later years certain expressions could emerge from the strategies presented by students, they signal indications of the mobilization of Horizon Content Knowledge (HCK) and Knowledge of Content and Curriculum (KCC) (Ball *et al.*, 2008).

We emphasize that the pairs of undergraduates, professors and teachers of Elementary School II showed more evidence of knowledge of the Domain Specific Knowledge (CK), while the pairs of teachers of the early years showed a greater concern in how to approach and contextualize these contents, evidencing in their reports how relevant they consider it to make the relation with the subdomains of the Pedagogical Content Knowledge (PCK), in particular, the Knowledge of Content and Student (KCS), since it is from this that they elaborate the tasks to be developed with the children. This concern of the early years' teachers, consequently, provoked the mobilization of the Specialized Knowledge of the Content (SCK). It is this knowledge that will give the opportunity for the teachers to make the best choices regarding the teaching approach, the type and level of tasks they will propose to the students, to reach the objectives proposed in their planning and to make possible the learning of the mathematical contents by the students. These perceptions and teaching actions of the Teacher who Teach Mathematics are a result of Interpretive Knowledge (IK), and it is this Knowledge that characterizes the specificity of actions *for* and *in* the teaching of

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mathematical content (Ball *et al.*, 2008). And, because it belongs to the Teacher who Teaches Mathematics, and constitutes the core of the Specialized Knowledge of the Teacher who Teaches Mathematics (Carrillo *et al.*, 2013; Ribeiro, 2017).

Another evident perception was the teachers' concern in using different representations of the same mathematical object to provide an opportunity to understand their ideas. The pairs sought to make the transition between the algebraic, geometric representation and natural language of their strategies. This movement evidences the mobilization of the three knowledges of the Content domain (CK), the knowledge (CCK, SCK and HCK), and the three knowledge of the Pedagogical Content domain (PCK), the knowledge (KCS, KCT and KCC) according to with Ball *et al.* (2008). The mobilization of this knowledge points to the importance of considering, in the formation of Teachers who Teach Mathematics, time and space to provide opportunities for mathematical communication, argumentation and discussion of strategies developed by students on the mathematical contents that, in this study, were limited to the development of pattern generalization in recursive sequences. This dynamic, based on collaborative work and collective discussion about the understanding and teaching of mathematical content, also contributes to the reflection on the practice itself and the problematization of the teaching profession. Thus, we understand that these moments of study in the ICEM group provided the mobilization of knowledge aimed at the professional development of Teachers who Teach Mathematics.

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