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Specialized Knowledge of licensors in Mathematics in the context of formative practices for inclusion

Conhecimento especializado de licenciandos em Matemática no contexto de práticas formativas para inclusão

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Abstract

This article reports partial results of a qualitative research master's research, which investigates, in two subjects dedicated to the practice as a curricular component, focusing on teaching Mathematics from an inclusive perspective, the Specialized Knowledge of the Mathematics Teacher mobilized by undergraduate students in these practices. Data collection uses audio recordings and materials produced by the licensors. The analysis, based on the framework of the Specialized Knowledge of the Mathematics Teacher, shows the importance of training practices in the inclusive perspective so that the licensee articulates mathematical knowledge and pedagogical content knowledge with knowledge about Special Educational Needs, for inclusive teaching of mathematics. However, the possible gaps related to the Knowledge of the Mathematical Structure in these training practices indicate that the specific Mathematics subjects should approach the practice as a curricular component.

Keywords: Initial training; Inclusion; Specialized Knowledge of the Mathematics Teacher.

Resumo

Este artigo relata resultados parciais de uma pesquisa de mestrado de abordagem qualitativa, que investiga, em duas disciplinas dedicadas à prática como componente curricular, com enfoque no ensino da Matemática na perspectiva inclusiva, o Conhecimento Especializado do Professor de Matemática mobilizado por licenciandos nessas práticas. A coleta de dados utilizou audiogravações e materiais produzidos pelos licenciandos. A análise, fundamentada no referencial do Conhecimento Especializado do Professor de Matemática, evidenciou a importância de práticas formativas na perspectiva inclusiva para que o licenciando articule conhecimentos matemáticos e conhecimentos pedagógicos de conteúdo com conhecimentos sobre Necessidades Educacionais Especiais, para um ensino inclusivo da matemática. Porém, possíveis lacunas relativas ao Conhecimento da Estrutura Matemática nessas práticas formativas indicam que as disciplinas específicas de Matemática devem abordar a prática como componente curricular.

Palavras-chave: formação inicial; inclusão; conhecimento especializado do professor de matemática;

Introduction

This study aims to present partial results from a master's developing research which

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objective is to investigate Specialized knowledge (Carrillo, Climent, Contreras & Muñoz-Catalán, 2013) mobilized by future mathematic teachers on the concept of formative practices on inclusive perspective. The research involved two subjects that were focused on practice as curricular content, where teaching mathematics using inclusive perspective is the focus.

The elementary students constantly show low average in external evaluation, especially when mathematics is the point. According to Pacheco e Andreis (2018, p. 106), difficulties shown by the students in mathematics can be related to many facts. This problematic involves students with special needs (SEN)[3]. – with high difficulties in learning or limitations, related to organic causes or to disfunction, limitation or deficiencies; difficulties in communication and warnings; high skills/super gifted (Brasil, 2001) -, but also involves other without those difficulties.

Considering this context, the teacher has the role of finding new ways to make learning easier, using activities to include all students in the process of regular teaching. These innovative and more inclusive pratiques are beneficial to all and cannot be only thought as classes for SEN students. However, the teacher needs to know students' limitation and differences, so that, it is possible to be more inclusive, providing cognitive development and learning to all the students. Clearly, it is needed that the teacher must be prepared, having a previous graduation and knowledge about inclusion laws, therefore providing practical inclusive experience to develop the students in a real context (Ribeiro, 2018).

The Law of education basis guidance sees to assure to SEN students "Postgraduate teachers in each level of attention, to specialized service, as teachers from regular education able to the provide integration of these students to regular classes". (Brasil, 1996, p.44). So that, Passos, Passos e Arruda (2013, p. 5) claim: "each graduation course must provide to its students the knowledge about the most variety educational needs that they might face in their future professional living". Nevertheless, according to Souza (2016), most mathematics school programs do not have Inclusive Education in its content.

Therefore, Mathematics graduation courses should include, in its program, subjects that bring this thematic into the practical experience of teaching, as a curricular content. By the way, here we analysed results of a proposed subject which focus on Inclusive Education, thought and developed in 2019 by the researcher, first author of this article, in partnership to the subject' teacher, this research advisor, second author of this article. In the next section, we will bring the referential knowledge basis used, and the research methodological path. At the end, we are going to discuss about analysed data.

[3] There are more current legislation on inclusion (Brazil, 2011), however the term SEN, used here, encompasses all students who for some reason do not follow the class in which they are inserted, such as, for example, those with Disability Deficit Disorder. Attention to Hyperactivity (ADHD), attended by graduates in the scope of training practices.

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Knowledge about teachers' preparation/graduation

The precursor of Pedagogic content knowledge (PCK) idea was Shulman (1987), who assumed there is a new knowledge, produced in teacher's classroom, at the moment students are able to make a connection of theorical knowledge to practical: the PCK, which distinguish teacher's knowledge from a specialist' knowledge, both the same subject. Considering Shulman's research and contribution to teaching focused on PCK, some investigators decided to create their own models of each area of knowledge, which included Mathematics.

Carrillo coordinates a group known as Seminario de Investigación em Didáctica de la Matemática (SIDM), made by investigators from many universities around the world, located in the Huelva University – Spain, where they study about Mathematic teachers' professional knowledge. Flores-Medrano et al. (2016, p. 208, our translation), members from the group, explain this result:

> In the research that we developed collectively in the SIDM group on the professional knowledge of the mathematics teacher, working with the existing models has allowed us to know their limitations and potential and, consequently, has led us to propose a model centered on what is specific to the mathematics teacher, leaving aside aspects of knowledge that are shared with teachers from other disciplines.

In order to what was faced in these studies, the group created Mathematics Teacher's Specialized Knowledge (MTSK) model. They understand teachers' knowledge is not only shown by how much knowledge in Mathematics teachers have, but also in the ability to understand the pedagogical meaning in the subject. However, before MTSK, a specific model, made to analyse Mathematics teachers' knowledge, already existed – the Mathematical Knowledge for Teaching - MKT (Ball, Thames & Phelps, 2008) – it was used as main reference to MTSK. Due to Flores-Medrano, Escudero-Ávila, Montes, Aguilar e Carrillo (2014, p. 71-72, our translation), the MTSK came "in response to the difficulties detected in the MKT and taking it as basis for the potentialities of this and other models, which characterize Mathematics teachers' knowledge".

According to Flores-Medrano et al. (2014, p. 71-72, our translation), o MTSK "considers the character specialized knowledge from the teacher completely in all its subdimensions, avoiding to refer to knowledge from external reference (knowledge from other professions)". Furthermore, Ribeiro e Amaral (2015, p. 4, our translation) state this idea: "Teachers' knowledge is perceived on its particularities related to teaching job specificities, considering that these roles are developed with the objective of allowing students to understand what they do, why they do it and what for". The creators of MTSK based on two big knowledge domains (Content Knowledge and Pedagogic Content Knowledge) purposed by Shulman, but focusing on Mathematics. Taking it into consideration, the MSTK is divided in two domains: Mathematics Knowledge (MK) and Pedagogical Content Knowledge (PCK) which divide themselves in three other subdomains each, as it is shown in the Picture 1:

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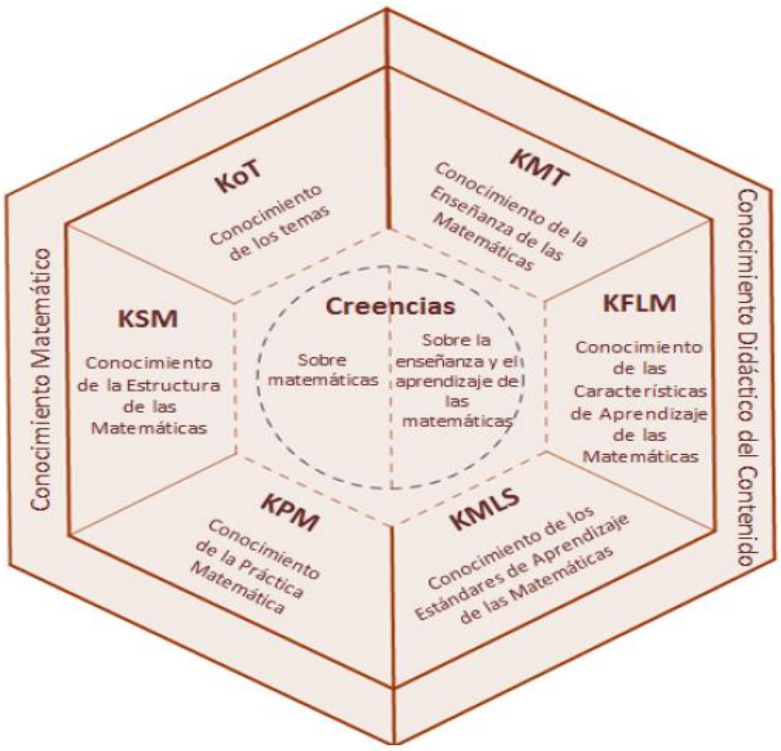

Figura 1 – Domains and subdomains of MTSK
Source: Flores-Medrano et al. (2016, p. 209).

The authors used acronyms in English, although the results were written in Spanish. The domains and subdomains from the MTSK model are interrelated, feeding itself mutually. In the middle, teachers' beliefs are highly considered. According to Bernardo et al. (2018, p. 116), "MK and PCK base teacher's practice and both influence teachers' believe". We will expose here two domains, its subdomains and its relation to its believes, according to the model. At the end, we will purpose a new subdomain which will make it possible to analyse specific knowledge to teach Mathematics in the inclusive perspective, in an articulative shape with subdomains existed in the model.

MK: Mathematical Knowledge

This domain has also three subdomains on it, composing and meaning the Mathematical Knowledge (MK) from the mathematical teacher: Knowledge of Topics (KoT), Knowledge of the Mathematical Structure (KSM) and Knowledge of Practices in Mathematics (KPM).

KoT: *Knowledge of Topics*

Refers to the knowledge of mathematics topics of the teacher, and according to Glores-Medrano et al. (2016, p. 210, our translation), "The knowledge of Topics does not refer only to the mathematical knowledge as subject, but also includes escolar mathematics". According to these researchers, the teachers know the mathematical content (concepts, procedures, facts, rules, theorems) and its meaning, but this knowledge includes the content the teacher desires the student learn more deeply.

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Flores-Medrano et al. (2014) states there are five characteristics:

- Phenomenology: Teacher's model knowledge related to a theme and its uses and applications.
- Property and fundamentals: Knowledge about properties and fundamentals relate to a specific procedure or topic.
- Representation records: Knowledge about the variety of ways to present any content (numbers, graphics, verbally, analytics) besides knowledge about appropriated vocabulary.
- Definition: Knowledge about specific objects' definition.
- Procedure: Knowledge about algorithm, conditions and fundaments to be used.

KSM: *Knowledge of the Mathematical Structure*

It is knowledge about relations that the teacher stablishes between different contents, according to Flores-Medrano et al. (2014, p. 77, our translation), "It can be from the course you are teaching, or it can be content from other courses or teaching levels. It is specifically about connections between mathematics topics". They purpose four characteristics of mathematics connections:

- Complexity Connections: It is the relation between taught content and future content.
- Simplification connections: It is the relation between taught content and what was taught even before. Example: Connection between the theme 'functions' and the theme 'sets'.
- Connection between transversal content: It is the relation between simple content and the most complex, besides the thoughts associated to them.
- Auxiliary connections: It is the relation to an auxiliary element in another theme, so that, interconceptual connection.

KPM: *Knowledge of Practices in Mathematics*

Due to Flores-Medrano et al. (2014), this domain includes the knowledge about how mathematics is described; the differences between a demonstration and a proof, evidence; the capacity to ratiocinate mathematically, to know about different thoughts and mathematical knowledge, and recognize which each context is appropriate to make connections, correspondence and equivalence. There are two characteristics for this subdomain:

- Practices related to mathematics in general: the knowledge about how independent from the previous content mathematics is, gives to teachers the structure of a logical thinking.
- Practices related to Mathematics' thematic: It is due to a particularity from the one before, when there is a specific theme.

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PCK: Pedagogical Content Knowledge

The PCK is the second domain and also sub share itself in three other subdomains: Knowledge of Mathematics Teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM) and Knowledge of Mathematics Learning Standards (KMLS).

KMT: *Knowledge of Mathematics Teaching*

It involves, according to Flores-Medrano et al. (2016), teacher's knowledge about sources, material and ways to present the content, besides their potential and the feeling to give the right examples to each subject. Flores-Medrano et al. (2014) state three, to better explain the ideas from the subdomain:

- Personal or institutionalized teaching theories: It is based on the knowledge about specific teaching theories for Mathematics.
- Material and virtual resources: It is the knowledge about the material an virtual resources – books, calculators, materials, games, software – as teaching element of mathematics content.
- Activities, homework, examples and help: Teacher's knowledge around the material or virtual object itself.

KFLM: *Knowledge of Features of Learning Mathematics*

Flores-Medrano et al. (2016) says that this subdomain and the last are basically what has been known as specialized knowledge from the mathematics teachers about the way students learn mathematical content. In KFLM are cited some students habits around certain contents, their difficulties, and their misunderstood or less attractive aspects, making a connection to learning theories. Flores-Medrano et al. (2014) suggest for characteristics to simplify the ideas from this subdomain:

- Learning ways: Teachers' knowledge about possible learning ways for the students, including cognitive development theories to them.
- Facilities and difficulties related to learning: The knowledge about mistakes, obstacles and difficulties related to mathematics in general.
- Student interaction to mathematical content: The knowledge about the process and students' strategies to communicate, through vocabulary to talk about a content.
- Students' conception about mathematics: The knowledge about their expectations and interests in relation to Mathematics.

KMLS: *Knowledge of Mathematics Learning Standards*

It is considered by Flores-Medrano et al. (2014, p. 85-86, our translation) the teacher's knowledge about what the student must or can achieve in a school year, or what he had achieved in a previous or later year - "it is what the teacher knows about conceptual, procedural capacities and mathematical reasoning that are promoted at certain educational moments". There are three characteristics:

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- Mathematical content needed to teach: knowledge about the contents taught in each grade / year, which is accessible in official documents (such as the National Common Curricular Base - BNCC).
- Knowledge of the expected level of conceptual and procedural development: It is to be able to evaluate the conceptual and procedural development expected for the student.
- Themes Sequence: It is basically the knowledge of a sequence of contents in a grade/ year, or in previous and subsequent years.

Beliefs in MTSK

To Flores-Medrano et al. (2014), the beliefs are related to the investigated teacher profile: his way of looking at mathematics, the process of teaching and learning constantly influences research. Therefore, beliefs are at the middle of the model rounded by dotted lines, it is because according to Flores-Medrano et al. (2014), this permeates the knowledge of each subdomain. However, they do not clarify the concept of belief adopted once they consider that beliefs cannot be measured. For this reason and because they are not the focus of the study, we will not take them into account in this research.

In order to relate knowledge to training practices focused on inclusion, we created a new subdomain - we named CEMI -, CEMI helps to identify a specific Knowledge for the Mathematical Teaching in the inclusive perspective. Here we describe it.

CEMI: *Conhecimento para o ensino de Matemática na perspectiva inclusiva (Knowledge for mathematical teaching in the inclusive perspective)*

In this new subdomain, we consider knowledge of specific resources and methods to teach effectively students with special needs (SEN), especially those that are different from the ones generally used for students without SEN, as well as organizing tasks and manage the class to ensure inclusion to all students, for example, a student with Autistic Spectrum Disorder (ASD), who cannot establish eye contact and needs a predetermined routine; or a student with Attention Deficit Hyperactivity Disorder (ADHD), who needs shorter and more sequential tasks to follow the classes. This knowledge is associated to other in the model and, when it is mobilized, it will be shown in an articulated way to others.

Based on this reference and on the adaptation of the characteristics presented by Ribeiro (2020), it was possible to compose a table to identify and characterize the Knowledge mobilized by the undergraduate students. In the next section, we will describe the methodological path of this research to present the analyses carried out so far.

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Methodological pathway

In this qualitative research[4], "the researcher warning is not about the numbers that will be shown along the researched group, but with the capacity to critically think about a social group, an organization, an institution, from a journey etc." (Goldenberg, 2004, p. 14). Thus, following Lüdke and André (2013), we made the study possible with descriptive data by the direct contact from the researcher to the object of study, prioritizing the process, not only the product.

Counting on disciplines designed to offer a variety of experiences with inclusive practices, students from a Mathematics course class, at a public university in Minas Gerais actively participated in the process: they defined which SEN they would like to investigate, the approaches and materials that could be used and negotiated among schools and teachers which content would be part of lessons plans, focusing on inclusion. Furthermore, we can affirm that the conception and development of the discipline, involving researcher, teacher-advisor and students influenced the education of all and that, certainly, this is a research-education, in which, according to Longarezi e Silva (2013, p 223),

> all the subjects involved actively participate in its process, investigating problem situations in the search to build answers and solutions for them; comprises academic research and pedagogical practice as a unit; it is developed by all its members through diverse discussions and interactions; part of the needs of the subjects involved, giving meaning to the process they are experiencing; it occurs in the school context; takes pedagogical practice as the content of the formative process; respects the different forms of knowledge that exist; and, fundamentally, it is a process of political formation.

At first, in the first subject, the undergraduate students had the experience of having two of their senses blocked, making them sensible about how it is to be deaf and blind students. After that, they developed small inclusive activities carried out with their own classmates. They were able to exchange experiences and clarify doubts and anxieties with teachers from the basic education who research and / or teach mathematics to SEN students, carrying out readings and theoretical discussions in parallel.

In the second subject, the main context of the analyses carried out here, the undergraduates were divided into small groups that, in negotiations to schools and teachers, should define a mathematical content to carry out theoretical studies, socializing their results to elaborate a lesson plan. This must be innovative for regular classes with at least one SEN student. Finally, graduates should give classes in public or private schools, within the scope of the internship or other projects and programs in which they were involved to implement these classes. This format for the discipline was designed to provide training practices in articulation to the basic education and focused on a present problem in a school routine - the inclusion of SEN students.

[4] The research project was submitted to the Research Ethics Committee (CEP), having been assessed and approved under the CAEE number: 18069719.9.0000.5094.

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Along this path, we proposed readings about inclusion, addressing the mathematics teaching to many SEN. These readings and studies based the students in the elaboration and implementation of the lesson plans, directing them to a more informed action, in relation to inclusion, in classes with SEN students. Lesson plans preparation and socialization took place in the discipline's classes, which made it possible to share ideas with colleagues, with the researcher, who fully follow the classes in the discipline, and also with the teacher-advisor. Thus, the lesson plans had valuable contribution from colleagues with a critic sight of a researcher and teacher-advisor. So that, we want to give opportunity to future teachers to understand all different needs of their students, SEN students or not; understanding the importance of respecting those needs; facing reflexions about manners to make a better mathematical learning to all.

There were 15 students that should organize themselves to elaborate and implement the lesson plan. We stipulated each group had at least one student in internship or in any teaching program. This student should negotiate with teachers from the school where they are teaching, the developed theme for the lesson plan, and the classroom should have at least one SEN student. If any formed group could not count on a member with these characteristics, they can look for any other possible partnership.

In table 1 are the group division with their number of participants, the content for the lesson plan, the grade/year involved, the kind of SEN student and the partnership context.

Table 1 - Groups

GROUP	MEMBERS	CONTENT	YEAR	SEN	ENVIRONMENT
ARE[5]	3	Area	6.º	3 ASD[6]	Internship
PRP	3	Point, straight and plane	6.º	3 ID[7] 1 ASD	Institutional Program of Scholarship to initiate (Pibid)
CAR	2	Cartesian plane	7.º	1 Deaf	Internship
PNO	3	Notable products	8.º	4 ADHD[8] 1 ASD	Partnership between researcher- teacher advisor
PRO	4	Probability	8.º	1 ASD	Scientific Initiation Program (PIC)

Source: authors' elaboration.

Each group made theoretical studies about the chosen topic, elaborating and implementing the lesson plan. The production from the student about theoretical studies, lesson plans and the class implementation process made the used data. The undergraduate students presented and audio recorded these moments, which complemented data collection.

[5] Each group chose an acronym, composed by the initials of the content, to facilitate analysis.

[6] Autistic Spectrum Disorder (ASD).

[7] Intellectual Disability (ID).

[8] Attention Deficit Hyperactivity Disorder (ADHD).

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In order to analyze the results, these productions were read and the audio recordings were transcribed, seeking to relate the discourse of the undergraduate students with characteristics which demonstrate the mobilization of each MTSK subdomain, as recorded in the theoretical reference on the MTSK, as well as the specific subdomain for Knowledge on Teaching Mathematics in the Inclusive perspective (CEMI), created by us to identify the links between this knowledge and the subdomains of the MTSK. For this, we prepared two tables with the excerpts, seeking to indicate the Expert Knowledge mobilized by the undergraduate students with highlights of the characteristics of each subdomain and of CEMI. Moreover, to identify more than one subdomain in excerpts, we highlighted in different colors each one. After that, we grouped those clues in a new table with small panoramas from mobilized knowledge. Numbering these excerpts in chronological order for done activities and keeping here this numbering.

The research allowed us to identify privileged knowledge in the process, besides pointing possible gaps in the formed practices analyzed: which knowledge must be prioritized in the initial graduation, focusing on the inclusive perspective in mathematics teaching, considering specific mathematics subjects or pedagogical nature. We decided to present here, in the next section, only data collected by ARE group.

Specialized knowledge mobilized by undergraduate students

When exposing the analysis of the specialized knowledge mobilized by the undergraduate students of the ARE Group in this research, we try to highlight its evidence in bold, in the excerpts transcribed here. ARE group is composed by Kim, Lulu and Woody, addressed in their lesson plan the content of areas, for a class of 23 students - three with Autism Spectrum Disorder (ASD) - from the 6th year of Elementary School II , in a public school. The group was organized as proposed activities, and the materials sought by them and the audio recordings transcriptions were reached in moments of preparation and socialization of the results. We have separated into two subtopics the analysis of students' activities: (1) moment of study and preparation; (2) moment of implementation. Each subtopic contains data that were collected from different data collection instruments.

Studying and preparation moment

This first moment involved theoretical studies: a comparative study between the curricular proposals from Minas Gerais and São Paulo; action of reading 12 books, which were 10 didactic books (6th grade), 1 paradidactic and 1 teaching material for the teacher (7th grade / 8th grade); 1 article study (Rodrigues e & Bellemain, 2016), about teaching flat figures areas in different environments, using paper and pencil, touching material and also the software Apprenti Géomètre 2. The group chose this article once they were looking for different strategies and methodologies proposal, indicated by several researches as facilitators for the ASD students learning process. This phase was finished with the lesson plan elaboration, subsidized by the studies that were done. Right below, we bring excerpts that indicate the mobilized knowledge by the graduates in these studies, for each subdomain.

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KoT – The Knowledge of Topics were mobilized for the three students in 12 excerpts. Woody shows knowledge about topics (KoT) when mentions in excerpts

> *that table that has the square meter, hectometer, dekameter, decimeter, meter, millimeter, centimeter and that table I sometimes see the teachers complaining to be useless [...] nobody uses it [...] it is that really dull content and I think it goes against the proposals once it is a sixth year content, I think that the main focus the proposals bring is to understand these contents, to understand what is a square meter, that it is a little square of a meter, but the square meter can be a triangle as long as the square meter fits within a triangle so it is not a matter of knowing how to transfer from square meter to square decimeter [emphasis added].*

When commenting the topics covered, he demonstrates knowledge about the theme "area" and its subthemes, in addition to the Knowledge found in manuals. It also presents phenomenological characteristics, about the applications in mathematics, exemplifying them. And it knows multiple equivalent definitions for the same concept, and uses this example, characteristic of KoT.

KSM – The mathematical knowledge was mobilized only for the student Woody in four excerts, we highlighted the 35:

> *I think it's really cool to use Tangram, then we'll make Tangram with paper. [...] At the same time that we are doing the Tangram to work in the area we are also using the Tangram to recap some concepts of Geometry.*

The knowledge about mathematical structure was mobilized, once the student assumes that, in cooperation to Tangram, it is possible to make a connection between what is being taught – area – to Geometry previous concepts, establishing simple connections.

KPM – Knowledge about the mathematical practice was present in five excerpts, mobilized by the students Woody and Lulu, as an example it can be observed on excerpt 12, expressed by Lulu: *"the books give us a support, a support, not only showing the formulas, but also showing where they came from, what is the importance of that [however] the books already give the formulas, which would be bad, thinking on the students side [emphasis added]".*

Therefore, the students state the books show where the formulas came from, without letting the students investigate and deduce, once it already shows the formulas ready to be applied. There are KPM clues that demonstrate to understand the importance to discuss how mathematical knowledge is built.

KMT – The mathematical teaching knowledge was the most mobilized for this moment: it appeared 22 times. In the lesson's plan the students show clues of KMT, in the excerpt 37, "with Geoplan's help, students separated in pairs or trios, must create different rectangles with different areas, the idea is that by creating several ones it is going to be better to observe the pattern [emphasis added]".

The undergraduate students mobilized knowledge about strategies and techniques because it suggests the students work in pairs or in trios, making possible to include ASD

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students in the activity. They also show resources, didactic materials and the manner of presenting the content and their potential, characteristics of KMT.

KFLM – Knowledge about mathematical learning characteristics was mobilized 17 times by all undergraduates. Kim, along excerpt 25, at the moment to talk about the chosen article results, states that "*they have different materials, for example, what they have they could touch made possible for them to cut, copy, decompose and overlap figures,* ***but for them****, the figure snip* ***looked like different in area*** *[emphasis indeed]".*

The undergraduate student presents KFLM characteristics: She demonstrated to know about the possible ways to learn from the students and refers to their possible difficulties; they believed the area changed just by cutting the figure.

KMLS – The knowledge of the mathematical learning parameters was mobilized 14 times, only by the undergraduates Woody and Lulu. Lulu presents, in excerpt 15, evidence of this knowledge: *"The curricular bases gave an orientation and the books complemented [added emphasis]".*

It is clear that the undergraduate student demonstrates knowledge of the curricular bases, by stating that they are present in textbooks and books complement it.

CEMI - It was possible to identify that teaching Mathematics knowledge in an inclusive perspective was mobilized twice, only by the undergraduate Woody. In excerpt 27 there is evidence of this knowledge.

> *The manipulative materials would help a lot the ASD student as well as other students to carry out the activities, making activities in pairs also brings the interaction to other students, which is very important for the student with ASD, because they have difficulty, two of our ASD students have much difficulty [emphasis added].*

Knowledge about teaching mathematics to ASD students is mobilized when the undergraduate warns themself about the need of manipulative materials to meet their needs. It reveals to articulate the ways of presenting the content and the strategies techniques, characteristic of KMT, to realize that the advantages of this material are not limited to the student with ASD, which can denote a broader perspective of inclusion.

Synthesis of specialized knowledge mobilized during the first moment

The table shows the excerpts distribution according to mobilized subdomains, highlighting to CEMI, it articulates with other subdomains.

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Table 21 – Excerpts distribution at the moment of study/preparation

Domains	Subdomains	Excerpts
MK	KoT	3, 5, 7, 16, 17, 20, 22, 24, 28, 29, 34 e 36
	KSM	6, 33, 35 e 37
	KPM	3, 12, **13**, 28 e 38
PCK	KMT	1, 4, 5, 6, 7, 8, **13**, 14, 15, 17, 19, 21, 23, 25, 26, **27**, 30, 32, 34, 35, 36 e 39
	KFLM	7, 8, 10, 11, 12, **13**, 14, 16, 17, 18, 21, 22, 24, 25, 26, 32 e 39
	KMLS	1, 3, 4, 5, 6, 7, 8, 15, 29, 30, 31, 32, 33 e 38

Source: authors elaboration

Data analysed show specialized knowledge mobilized by the group ARE manifested more in PCK domain. Besides that, KMT was the one the most appeared, the group made a study about materials and also prepared classes using resources and material. Whereas, KSM knowledge was less mobilized, what can indicate the fact they prepared the activities for an isolated group, in a classroom they had a brief contact, it made harder to stablish relation to other contents.

The KFLM, related to the students' learning modes, was highly mobilized, in view of the requirement to think about activities that would meet the needs of ASD students. This is an indication that the study of teaching proposals, focusing on the needs of students with ASD, mobilizes this type of knowledge. The KMLS represents the Knowledge about the curricular documents, which is also quite mobilized, since the undergraduates studied curricular proposals. The same for CEMI, although it has been little mobilized (only in two excerpts), it is noted that it appears in three subdomains, which relate the fact of placing this student as the center of the process (KPM), observing the necessary resources (KMT) so that they can learn with meaning (KFLM).

Implementation moment

This moment is part of the process of implementing the classes, whose analysis was based on the oral report and the narrative written by the undergraduate students. Using the same organization as the previous item, we will bring here some excerpts mobilized by the undergraduate student, according to subdomains.

KoT – Knowledge of Topics was mobilized in five excerpts in the context. The excerpt 50 mentions some of its characteristics:

> *From there we were able* ***to extract, with more intensity, the concept of standard unit of measurement, telling students that the measurement of the area changes in value depending on the unit of measurement we use and thus it was possible to explain about the square meter, its multiples and submultiples, and why, on different surfaces, we use different units of measurement*** *[emphasis indeed*].

There is evidence of properties' knowledge and fundamentals attributed to a topic, the subjects taught and its subthemes: the undergraduate students explain the concept of standard unit of measure, demonstrate knowledge of multiple equivalent definitions for the same concept and, when explaining why do so, they present characteristics of the results.

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KPM – The mathematical practice knowledge was mobilized only in the excerpts 52 and 54, extracted from the context. As they are related to each other, we will present both.

> *This was the activity that most seemed to work and, in fact, contributed a lot to the classroom, since it was an investigative and pattern-observing activity, where students should treat the information from a table created by themselves [emphasis added].*
>
> *The creation of the table, as shown in Figure 6, provided students were able to see the pattern for filling it out. The students did the activity in a group and it was possible to see some groups saying "oh, just do it twice", indicating that it was enough to multiply base and height to find the value of the area [emphasis added].*

In these two excerpts, the undergraduate students showed evidence of KPM, as they allowed students, by building a table, to observe patterns and conclude by themselves that the area is calculated by multiplying the base by the height. They revealed to know how mathematical knowledge is built. We chose to classify this knowledge as KPM, however, by adapting the mathematical practice for teaching knowledge, the undergraduate students also mobilize pedagogical content knowledge.

KMT – Knowledge about teaching mathematics was seen in 11 excerpts. In excerpts 53, we analyze some characteristics:

> *The principle of the activity was as it follows: in the **Geoplano** (Geoplan, translated literally), students built rectangles of their choice dimensions, then, in a pre-assembled table, in the first column the length of the base, in the second column the length of the height and in the third the area of the rectangle. It is important to realize that **this material makes it much easier to identify these data**, since to measure the lengths it was enough to look at the pins of the material and to measure the area it was enough to see how many squares existed within the created rectangle [emphasis added].*

In this excerpt, the graduates reveal that they know the resources and teaching materials, such as the Geoplano. And, when using this resource within an activity contextualized with the content being taught and affirming that the material facilitates the identification of the data, they demonstrate knowledge of its potential. The techniques presented by the licensors are also configured as KMT characteristics.

KFLM – The knowledge of mathematical learning characteristics was presented in ten excerpts along the narratives, including 46:

> *At this moment we noticed that **this student, and some others, were trying to use the idea of decomposition,** even though this method had not been explained, which made us very happy to see that **they themselves were building knowledge** and that, therefore, **it would have more meaning. the moment we were going to correct and formalize the concept** [emphasis added].*

The undergraduate students presented clues to know possible ways of learning from the students, relating their thinking, characteristics and strategies from them. They reveal knowledge about ways to interact to the content, simply becoming happy by using, by the students, from a formally non-presented strategy. Maybe what could have helped is that the

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students could get to know these strategies in previous grades – for example, to deal to basic equations.

KMLS – Indications of mathematical learning parameters knowledge emerge in two excerpts, among them, the 56, referring to Woody's speech in the presentation of the narrative:

> *In the normal test they had the table to complete it if it was multiplied or divided by 10, to use it, because we didn't think it was fair for students to decorate kilometer, hectometer, decimeter, because this sequence they will never decorate [emphasis added] [...].*

What the undergraduate student says about group's demonstration demonstrates knowledge about what a student should learn, the level of depth and the way the student is expected to learn.

CEMI – Knowledge for Mathematics teaching in an inclusive perspective was mobilized eight times, as it is in excerpts 56:

> *In the normal test they had the table to complete it if it was multiplied or divided by 10, to use it, because we didn't think it was fair for the students to decorate kilometers, hectometers, decimeters, because this sequence they will never decorate. [...] Then we arrived in the area activity, the first one had the standard and then they had to compare the area of the figure used, there is the area unit conversion table, two exercises just to convert and one exercise that used an agrarian measure. [...] Then, for students with ASD [...] we asked them to measure the table by their hands and measure their hand and then see how much the table measured in length. [...] we took the blocks for them to measure the perimeter of the figures and in the area of areas they used the Geoplano to build the rectangle and calculate the area of the rectangle using the material [emphasis added]*

The undergraduates were concerned about taking an adapted test for ASD students, as they have a limitation, when compared to other students. It is clear that they mobilized Knowledge about Teaching Mathematics to students with SEN, in conjunction with knowledge about ways to present content and its potential, with knowledge about resources and teaching materials, strategies and techniques used, KMT characteristics also.

Synthesis of Specialized Knowledge mobilized at the moment of implementation

The table 3 contains the excerpts distribution from that moment of implementation of the ARE group, according to the subdomains mobilized. Bold types refer to the CEMI subdomain, related to the others.

Table 3 2 – Excerpts Distribution at the moment of implementation.

Domain	Subdomains	Excerpts
MK	KoT	**41**, **43**, 44, 47, 48 e 50
	KSM	-
	KPM	52, 54
PCK	KMT	**41**, **42**, 44, 45, 47, 48, **49**, 50, 52, 53 e **56**
	KFLM	**40**, **43**, 45, 46, 47, **49**, 50, 51, 54 e **55**
	KMLS	**56** e **57**

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Source: authors elaboration.

The data produced at this time showed again that the Specialized Knowledge, mobilized by the ARE group, was mostly in the domain of the PCK. And KMT was the most evident, since the undergraduate students used resources, materials and strategies for the classes' implementation. The KFLM has also been mobilized a significant number of times, as it relates to the students 'modes of learning, which were the focus of the classes' narrative, which analysed the process of their development of activities. It is noted that at this moment of implementation, CEMI is more mobilized due to direct contact with students with SEN, appearing in four of the five subdomains mobilized.

The KMLS and KPM were not as evident as in the previous moment, because, in the implementation process, the concern is more directed to the students' learning, and not more to the curricular proposals and to the mathematical practice. At this moment of implementation, KSM is not mobilized, due to the fact that relations with other content are not considered.

Conclusion

The small clipping presented evidences it was the subject's proposal aroused the interest and concern in the undergraduate students with the need to prepare themselves adequately to teach mathematics to students with SEN, as future teachers, they mobilized a lot of knowledge, especially pedagogical content knowledge, in a way related to inclusion (CEMI). They were willing to look for methods and materials that facilitate the learning of these students, tried to place them at the center of the learning process and were able to reflect on the limitations of these materials, since not all mathematical content can be taught with the existing resources, which requires us to continue creating. At the moment of implementation, CEMI becomes even more evident, showing the importance of discussing SEN, promoting the articulation between university and school, in the mathematics degree courses, so that the future teacher relates mathematical knowledge and pedagogical content knowledge. Having knowledge about SEN will make teaching meaningful for these students and for the others.

The analysis revealed the most mobilized knowledge were related to PCK, what shows a concern about the subject developing Pedagogical knowledge of content, without disconnecting mathematical knowledge and specific knowledge to teach mathematics in an inclusion perspective. The experience in the elementary school, in a context of developed lesson plans' application, but in an articulated way with all planning necessity from the teacher and also taking in consideration all students' needs, especially the ones with SEN, it made possible an experience also of articulation between the content knowledge and pedagogical knowledge of this content, basically resumed by the what to present the subject to students.

Although we recognize the limitations in the initial teaching, formative pratiques such as this one can turn the teacher more prepared to teach in this diversity context the school

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presents. Thus, it is shown the necessity to have subjects which teach both domains (MK and PCK), in a connective way in order to the teacher acting competently.

To sum up, it is important to point that KSM had been few times mentioned, besides being relate to the fact of the undergraduate prepare an only content for a small time with the group, can indicates that a subject which adopts formative practices focusing on inclusion, or even other pedagogical subjects that keep focused on practice as curricular component that not always will mobilize all other subdomains in MTSK. It is important to say that undergraduate students take specific Mathematics subjects which the structure of mathematics is the main focus, in these disciplines this knowledge can be more mobilized. This allows us to question: what possibilities will these future teachers have to articulate the knowledge between the contents of academic mathematics with school mathematics (Moreira and David, 2013), if these disciplines do not also problematize practice as a curricular component?

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